A Classically-Based Theory of Impossible Worlds*

Edward N. Zalta†
Center for the Study of Language and Information
Stanford University

The appeal to possible worlds in the semantics of modal logic and the philosophical defense of possible worlds as an essential element of ontology have led philosophers and logicians to introduce other kinds of 'worlds' in order to study various philosophical and logical phenomena. The literature contains discussions of 'non-normal worlds',[1] 'non-classical worlds',[2] 'non-standard worlds',[3] and 'impossible worlds'.[4] These atypical worlds have been used in the following ways: (1) to interpret unusual modal logics, (2) to distinguish logically equivalent propositions, (3) to solve the problems associated with propositional attitude contexts, intentional contexts, and counterfactuals with impossible antecedents, and (4) to interpret systems of relevant and paraconsistent logic.

However, those who have attempted to develop a genuine metaphysical theory of such atypical worlds tend to move too quickly from philosophical characterizations to formal semantics. For example, one of the best attempts to develop such a theory can be found in Priest [1992]. In

*This paper was published in the *Notre Dame Journal of Formal Logic*, **38**/4 (Fall 1997): 640–660 (Special Issue, Graham Priest, Guest Editor).

†This paper was presented at the meetings of the Australasian Association of Logic at Macquarie University in July 1998. I am indebted to those in attendance for their interesting comments. In particular, comments by Greg Restall, Graham Priest, Allen Hazen, Ed Mares, Daniel Nolan, and Cian Dorr were especially helpful.

[1]See Kripke [1965], Cresswell [1967], Rantala [1982], Priest [1992], and Priest and Sylvan [1992].

[2]See Cresswell [1972].

[3]See Rescher and Brandom [1980] and Paśniczek [1994].

[4]See Morgan [1973], Routley [1980] (p. 291), Yagisawa [1988], Mares [1997], and Restall [1997].

that work, we find such claims as: (1) that non-normal worlds are those where logical theorems, that is, semantically logical truths, may fail, (2) that worlds where the laws of logic are different are 'logically impossible worlds', and (3) that the worlds where statements of entailment may take on values other than their usual values are exactly the non-normal worlds (pp. 292-293). These claims are then 'cashed out' in terms of a formal semantics. However, cashing out philosophical claims in terms of a formal *model* is not the same as giving a genuine philosophical *theory* of non-normal or impossible worlds. In Priest's model, the atypical worlds are just assumed to exist and the behavior of the 'logical truths' at such worlds is just stipulated. Furthermore, Priest goes on to point out that the non-normal worlds employed in the semantics are worlds where the laws of logic differ, not worlds where the logically impossible happens (p. 296). So it seems that there is more work to be done if we are to have a genuine account of impossible worlds, i.e., worlds where the logically impossible happens.[5]

In this paper, I derive a metaphysical theory of impossible worlds from an axiomatic theory of abstract objects. The axiomatic theory is couched in a language with just a little more expressive power than a classical modal predicate calculus. The logic underlying the theory is classical. This system (language, logic, and proper theory) is reviewed in the first section of the paper. Impossible worlds are not taken to be primitive entities but rather characterized intrinsically using a definition that identifies them with, and reduces them to, abstract objects. The definition is given at the end of the second section. In the third section, the definition is shown to be a good one. We discuss consequences of the definition which take the form of proper theorems and which assert that impossible worlds, as defined, have the important characteristics that they are supposed to have. None of these consequences, however, imply that any contradiction is true (though contradictions can be 'true at' impossible worlds). This classically-based conception of impossible worlds provides a subject matter for paraconsistent logic and demonstrates that

[5]Priest takes his account of non-normal worlds to be compatible with any account of possible worlds (so we are free to assume our favorite theory of possible worlds). But it may be that a theory of impossible worlds, as opposed to a model, will establish a fundamental connection between possible worlds and impossible worlds which turns out to be incompatible with certain conceptions of possible worlds. That is, it may not be the case that a good theory of impossible worlds is compatible with every conception of possible worlds.

there need be no conflict between the laws of paraconsistent logic (when properly conceived) and the laws of classical logic, for they govern different kinds of worlds. In the fourth section of the paper, I explain why the resulting theory constitutes a theory of *genuine* impossible worlds, and *not* a theory of ersatz impossible worlds. The penultimate section of the paper examines the philosophical claims made on behalf of impossible worlds, to see just exactly where such worlds are required and prove to be useful. We discover that whereas impossible worlds are not needed to distinguish necessarily equivalent propositions or for the treatment of the propositional attitudes, they may prove useful in other ways. The final section of the paper contains some observations and reflections about the material in the sections that precede it.

1. Compressed Presentation of the Background Theory

Readers familiar with Zalta [1983], [1988a], or [1993] (among others) may skip this section, for it contains a compressed presentation of the logic of encoding and the metaphysical theory of abstract objects. This system was developed not only to derive precise theories of such abstract objects as Platonic forms, Leibnizian monads, Fregean senses, fictions, possible worlds, and mathematical objects, but also to produce an intensional logic suitable for representing and systematizing philosophically interesting truths and inferences of ordinary language. The system enhances the second-order modal predicate calculus simply by adding an extra atomic formula to the basis of the language. The formula 'xF' asserts 'x encodes (the one-place property) F', where this is to be understood as a mode of predication distinct from the classical mode of predication known as 'instantiation' or 'exemplification'. When x encodes F, the property F characterizes the object x in an important new sense. Statements of exemplification are represented in the usual way by formulas of the form 'Fx' and (in the general case) '$F^n x_1 \ldots x_n$'. The resulting calculus can be used to assert the existence of special abstract objects which encode properties as well as exemplify properties. From the fact that such an object x encodes property F, it does not follow that x exemplifies F, nor does it follow either that x encodes properties implied by F or that x fails to encode properties *excluded* by F. (Further motivation and explanation of the encoding mode of predication can be found in the previously cited works.)

Thus, the special abstract objects may encode incompatible properties; i.e., the language gives us a way to talk about objects which are *characterized* by incompatible properties. But the logic of encoding and theory of abstract objects preserves the classical axioms of propositional, predicate and modal logic.[6] It is a theorem of the system that $\neg(xF \;\&\; \neg xF)$ and that $xF \vee \neg xF$. Moreover, abstract objects (and all ordinary objects as well) are classically behaved with respect to the properties they exemplify: for every property F and any object x whatsoever (whether ordinary or abstract), $Fx \vee \bar{F}x$ (where $\bar{F}$ just abbreviates $[\lambda y\, \neg Fy]$). However, there are 'incomplete' abstract objects x and properties F such that $\neg xF \;\&\; \neg x\bar{F}$, as well as 'contradictory' abstract objects y such that $yF \;\&\; y\bar{F}$.

To be more precise, the theory of abstract objects is couched in a second-order modal predicate calculus without identity that has been modified only so as to admit encoding formulas. We assume the usual primitive logical notions standardly represented using $\neg$, $\rightarrow$, $\forall$, $\Box$, as well as the usual axioms and rules for fixed-domain, S5, quantified modal logic (with first- and second-order Barcan formulas). The language also includes two special kinds of terms: rigid definite descriptions of the form $\imath x\phi$ and n-place λ-expressions of the form $[\lambda y_1 \ldots y_n\, \phi]$ (in λ-expressions, ϕ may *not* contain encoding subformulas). These terms behave in the standard ways. Once the primitive predicate $E!$ (which denotes the property of having a spatiotemporal location) is added to this system, the ordinary objects ($O!x =_{df} [\lambda y\, \Diamond E!y]x$) are distinguished from the abstract objects ($A!x =_{df} [\lambda y\, \neg\Diamond E!y]x$). Identity for ordinary objects is defined:

$$x \!=_E\! y =_{df} O!x \;\&\; O!y \;\&\; \Box\forall F(Fx \equiv Fy)$$

and the ordinary objects are subject to the proper axiom:

$$O!x \rightarrow \Box\neg\exists F xF$$

[6] The system is typically formulated with primitive definite descriptions of the form $\imath x\phi$ and these may fail to denote. So the axioms of free logic are employed for any terms containing such descriptions. Moreover, in modal contexts, these definite descriptions rigidly denote the object satifysing the description at the actual world. To accomodate such terms, the laws of modal logic have to be adjusted in well-known ways (for example, the Rule of Necessitation may not be applied to any theorem that depends on the contingent logical axiom which governs the rigid descriptions). None of these adjustments constitutes a departure from 'classical' axioms and rules of propositional, predicate, and modal logic.

In other words, ordinary objects necessarily fail to encode properties.

The theory of abstract objects consists of a proper axiom schema and a definition:

$\exists x(A!x \;\&\; \forall F(xF \equiv \phi))$, where ϕ has no free xs

$x = y =_{df} x =_E y \lor (A!x \;\&\; A!y \;\&\; \Box\forall F(xF \equiv yF))$

The axiom schema has an infinite number of instances; each instance asserts the existence of an abstract object that encodes just the properties satisfying the supplied condition ϕ. The definition implies that when x and y are abstract objects, they are identical whenever they necessarily encode the same properties. Note that these two principles guarantee that definite descriptions of the form $\imath x(A!x \;\&\; \forall F(xF \equiv \phi))$ are always well-defined—there couldn't be two distinct abstract objects that encode exactly the properties satisfying ϕ, since distinct abstract objects must differ with respect to an encoded property.

This proper metaphysical theory of abstract objects is supplemented by a logic which includes a theory of properties, relations, and states of affairs (where properties are 1-place relations and states of affairs are 0-place relations). The fundamental axiom of this logic of relations is λ-conversion:

$[\lambda y_1 \ldots y_n\, \phi]x_1 \ldots x_n \equiv \phi^{x_1,\ldots,x_n}_{y_1,\ldots,y_n}$

From this axiom, it follows that:

$\exists F^n \Box\forall x_1 \ldots \forall x_n(Fx_1 \ldots x_n \equiv \phi)$, where ϕ has no free Fs and no encoding subformulas.

This is a comprehension principle for relations and it holds for all n, $n \geq 0$. A definition tells us that 1-place properties F and G are identical whenever they are necessarily encoded by the same objects:

$F = G =_{df} \Box\forall x(xF \equiv xG)$

This definition can be generalized for the case of n-place relations when $n > 1$ (the case of $n = 0$ is dicussed below).[7] Note that given this definition, we may consistently assert that both $\Box\forall x(Fx \equiv Gx)$ and $F \neq G$. We have, therefore, a more fine-grained, intensional conception of properties, though as we shall soon see, their identity conditions are in fact extensional.

[7] I omit the details here; they can be found in the previously cited works.

We conclude our very brief summary of the system by mentioning two final important principles. The first is the substitution of identicals. Since both '$x = y$' and '$F = G$' are defined, we adopt the unrestricted rule of substitution:

$\alpha = \beta \rightarrow [\phi(\alpha,\alpha) \equiv \phi(\alpha,\beta)]$, where α,β are both object variables or relation variables and $\phi(\alpha,\beta)$ is the result of substituting β for α in one or more occurrences of the latter in $\phi(\alpha,\alpha)$

Secondly, it is a logical axiom of the system that encoded properties are rigidly encoded:

$\Diamond xF \rightarrow \Box xF$

Thus, the properties that characterize an abstract object in the encoding sense do so in a way which does not vary with the contingent circumstances. Since encoded properties are rigidly encoded, the identity of abstract objects x and y follows immediately whenever $\forall F(xF \equiv yF)$ and the identity of properties F and G follows immediately whenever $\forall x(xF \equiv xG)$. This means that our properties and abstract objects have extensionally defined truth conditions.

The above system has been given a precise semantics and models of the theory show that the system as a whole is consistent.[8] Though the models of the theory are developed in ZF, the theory itself presupposes no set theory and offers an ontology free of primitive mathematical objects. Mathematical theories, however, can be interpreted in the language of the present theory so that the terms (constants and predicates) of the mathematical theories refer to abstract objects and abstract relations. A fuller discussion of this, however, would take us too far afield.[9] The foregoing brief and compressed presentation should suffice for the remainder of the article and I shall assume that the reader, if necessary, can examine those works in which the logic of encoding and the theory of abstract objects is explained, motivated, and developed in more detail. On occasion, however, I will explain other features of the logic and theory when they are essential for understanding what follows.

[8] See Zalta [1997] in addition to the works cited previously.
[9] See Linsky and Zalta [1995].

2. The Theory of Impossible Worlds

Before we present the definition of 'impossible world', it will serve well to review the definition of a 'possible world' and its supporting definitions. In previous work, the notion of a possible world was defined in terms of the notion of a situation:

$$Situation(x) =_{df} A!x \;\&\; \forall F(xF \rightarrow \exists p(F = [\lambda y\, p]))$$

In this definition, the variable 'p' ranges over states of affairs (i.e., 0-place relations). The definitions says that a situation is an abstract object that encodes only properties of the form *being such that p* ('$[\lambda y\, p]$'). In what follows, we use the variable 's' to range over situations.

We also introduced the notation '$s \models p$' to formalize a notion which can be expressed in any of the following ways: s makes p true, p is true in s, p obtains in s, and p holds in s. This notion can be defined as follows:

$$s \models p \;\; =_{df} \;\; s[\lambda y\, p]$$

This tells us that s makes p true (p is true in s, or p obtains in s) just in case s encodes being such that p.

These definitions may become more vivid if we briefly explain expressions of the form $[\lambda y\, \phi]$ and $[\lambda\, \phi]$. We use the latter (i.e., λ-expressions with *no* variables bound by the λ) to denote complex states of affairs. We may read the expression $[\lambda\, \phi]$ as 'that-ϕ'. These expressions denoting states of affairs are subject to (the 0-place instance of) λ-conversion:

$$[\lambda\, \phi] \equiv \phi$$

This degenerate instance of the λ-conversion principle should be read as follows: that-ϕ is true (or that-ϕ obtains) iff ϕ. From this principle, we may derive a comprehension principle for states of affairs (where p is not free in ϕ and ϕ has no encoding subformulas):

$$\exists p \Box (p \equiv \phi)$$

This comprehension principle for states of affairs ensures that every state of affairs has a negation, that every pair of states of affairs has a conjunction, that every state of affairs has a necessitation, etc.

Now the comprehension principle for properties guarantees that for every state of affairs p, there exists a property F which is necessarily such that something exemplifies F iff p:

$$\exists F \Box (Fx \equiv p)$$

Expressions such as '$[\lambda y\, p]$', which we used above, denote such properties. They are perfectly well-behaved, for by λ-conversion, necessarily, an object x exemplifies being such that p iff p:

$$[\lambda y\, p]x \equiv p$$

We use these properties to say when states of affairs are identical:

$$p = q \; =_{df} \; [\lambda y\, p] = [\lambda y\, q]$$

So states of affairs p and q are identical whenever the property of being such that p is identical to the property of being such that q (property identity, recall, has already been defined). Note that given this definition, we may consistently assert that necessarily equivalent states of affairs may be distinct. The claims that $\Box(p \equiv q)$ and $p \neq q$ are consistent with one another. Although the comprehension principle and the above identity conditions offer a simple theory of states of affairs, we shall, in what follows, supplement that theory with the minimal assumption that there exists a pair of states of affairs p and q such that both $p \neq q$ and $p \neq \neg q$.[10]

With this understanding of states of affairs, the above definition of 'situation' tells us that situations are objects that encode only properties 'constructed' out of states of affairs. Indeed, we may now extend our notion of encoding so that we may speak of a situation s as encoding a

[10] We have intentionally left our theory of relations and theory of states of affairs relatively weak so that we are free to answer the question, 'How fine-grained are relations and states of affairs?', in ways that match our intuitions. Our definition of identity for states of affairs tells us the extensional conditions that must obtain for p and q to be identical (they are, therefore, not creatures of darkness). But the theory doesn't decide the question of identity for arbitrarily chosen states of affairs; rather, it is meant to be consistent with our pretheoretic judgements on the matter. For example, although the following principle is derivable from the theory we have so far:

$$p \;\&\; \neg q \rightarrow p \neq q,$$

the theory doesn't rule on the following claims:

$$p \neq (p \;\&\; p)$$

$$(p \;\&\; p) \neq (p \;\&\; p \;\&\; p)$$

$$(p \;\&\; q) \neq (q \;\&\; p)$$

If there is data which suggests that, in these cases, the states of affairs flanking the identity sign are indeed distinct, then principles such as these could be consistently added to the theory. But, for our present purposes, we shall need to appeal only to the obviously true assumption just mentioned in the text.

state of affairs p whenever s encodes $[\lambda y\, p]$. So a situation encodes the states of affairs that obtain in that situation.

Given these remarks, the definition of a possible world offered in previous work can be understood:[11]

$$PossibleWorld(x) =_{df} Situation(x) \ \&\ \Diamond\forall p(x \models p \ \equiv\ p)$$

That is, a possible world is any situation s which might have been such that all and only the states true in s are states that obtain. (In what follows, we sometimes say that state of affairs p obtains *at* (or holds true *at*) w whenever $w \models p$.) In Zalta [1993], the reader may find evidence that this definition is a good one. From this definition and a few other standard definitions, we may derive the *a priori* truths of world theory, including such claims that every world is maximal, that every world is consistent, that there is a unique actual world, that a state of affairs necessarily obtains iff it obtains in all worlds, and that a state of affairs possibly obtains iff it obtains in some possible world.

Now the leading intuition that underlies our theory is that an impossible world is any maximal situation s such that it is not possible for all of the states of affairs that s makes true to be true. The notion of a 'maximal' situation is a straightforward one:

$$Maximal(s) =_{df} \forall p(s \models p \ \vee\ s \models \neg p)$$

It is important to emphasize here that while other philosophers sometimes talk of 'incomplete worlds' as well as impossible worlds, I eschew such talk. An 'incomplete world' is simply a situation which is not maximal.[12]

To complete the definition of impossible world, we define the notion of actual and possible situations:

$$Actual(s) =_{df} \forall p(s \models p \ \rightarrow\ p)$$

$$Possible(s) =_{df} \Diamond Actual(s)$$

[11]In order to disambiguate formulas containing '$\models$', we adopt the following convention: '$\models$' shall be dominated by all the other connectives in a formula. For example, a formula of the form '$s \models p \rightarrow p$' shall be short for '$(s \models p) \rightarrow p$.' We write '$s \models (p \rightarrow p)$' to assert that the complex state of affairs $p \rightarrow p$ holds true in s.

[12]Similarly, I eschew talk of 'the world of fiction f'. There is no unique world where everything true in a given (consistent) fiction f is true; there are far too many worlds compatible with such fictions. So 'fictional worlds' are simply situations in which there are objects that exemplify *simpliciter* all of the properties that the characters of the fiction exemplify according to the fiction.

So if s is an actual situation, every state of affairs that holds true in s does hold true. If s is a possible situation, then it could be the case that every state of affairs which holds true in s holds true.

At last we come to the definition which expresses our theory of impossible worlds:

$$ImpossibleWorld(s) =_{df} Maximal(s) \ \&\ \neg Possible(s)$$

Thus, an impossible world is a maximal situation s such that it is impossible for all of the states of affairs true in s to be true.

Note that we could have defined a 'possible' world as a maximal situation s such that it is possible that every state of affairs true in s is true. Such a definition is equivalent to the one given several paragraphs back. Thus we could have given an alternative yet equivalent definition of an 'impossible world', namely, as a maximal situation that is not a possible world. Moreover, if we define a 'world' (with no modal modifier) to be any maximal situation, then an impossible world is simply any world which is not a possible world.

In what follows, we use 'w' as a variable ranging just over *possible* worlds. Also, in what follows, we shall sometimes need to distinguish pairs of situations, possible worlds, and impossible worlds. The identity conditions governing these objects are rather simple. Since situations are defined as abstract objects, situations are identical whenever they encode the same properties. Moreover, since situations encode *only* properties of the form $[\lambda y\, p]$, situations are identical whenever they encode the same states of affairs. In formal terms:

$$s_1 = s_2 \equiv \forall p(s_1 \models p \ \equiv\ s_2 \models p)$$

Since possible worlds and impossible worlds are species of situation, this fact about situations will prove useful in distinguishing possible worlds and impossible worlds as well.

3. Consequences of the Theory

A proof of the existence of impossible worlds is now a straightforward matter. Just consider the following object, which we know is well-defined (by the comprehension and identity principles for abstract objects):

$$\imath x(A!x \ \&\ \forall F(xF \equiv \exists p(F = [\lambda y\, p])))$$

This is the abstract object that encodes all and only those properties F of the form *being such that* p (for some state of affairs p). In other words, this object encodes every state of affairs (and that is all that it encodes). Clearly, this object is a situation and is maximal, in the senses defined above. Since every state of affairs (and its negation) holds true in this situation, it is impossible that every state of affairs true in this situation be true. So this is an impossible world. In what follows we call this object the 'the universal situation' s_u. It is, in some sense, uninteresting and trivial.[13]

Another example of an impossible world is the situation s which makes true all and only the states of affairs which fail to be true:

$$\imath x(A!x \,\&\, \forall F(xF \equiv \exists p(\neg p \,\&\, F = [\lambda y\, p])))$$

Call this object '$\bot$'. Clearly $\bot$ is a situation. To see that it satisfies the definition of maximality, consider an arbitrary state of affairs q_0 (to show that $\bot \models q_0 \vee \ \bot \models \neg q_0$). By the laws of classical logic, we know that $q_0 \ \vee \ \neg q_0$. If q_0 holds true, then so does $\neg\neg q_0$ and so by the definition of $\bot$, $\bot \models \neg q_0$. So $\bot \models q_0 \vee \ \bot \models \neg q_0$. If $\neg q_0$ holds true, then again by definition of $\bot$, it follows that $\bot \models q_0$. So $\bot \models q_0 \vee \ \bot \models \neg q_0$. So by our syllogism, it follows that $\bot$ is maximal. Now to see that $\bot$ is not a possible situation, pick an arbitrary state, say q_1. We know from classical logic that $\neg(q_1 \,\&\, \neg q_1)$. So by the definition of $\bot$, it follows that $\bot \models (q_1 \,\&\, \neg q_1)$. Clearly, then, $\bot$ is not a possible situation, since it is not possible for all the states of affairs true in $\bot$ to be true.[14]

We next prove that, if given any *possible* world w and state of affairs p false at w, then there is an impossible world which is just like w but where both p and its negation obtain. To prove this, we define *the p-extension* of w ('w^{+p}') as follows:

$$w^{+p} =_{df} \imath x(A!x \,\&\, \forall F(xF \equiv \exists q(w \models q \,\&\, F = [\lambda y\, q]) \ \vee \ F = [\lambda y\, p]))$$

In other words, w^{+p} both encodes the states which are true at w and encodes p. Now if we are given a possible world w_1 and a state that doesn't obtain in w_1, say p_1, then ${w_1}^{+p_1}$ is a world (i.e., maximal situation) where both p_1 and $\neg p_1$ hold. It is, therefore, an impossible world. To see this, note that if p_1 fails to hold in w_1 (i.e., $w_1 \not\models p_1$), then by the maximality of worlds, $w_1 \models \neg p_1$. So, by definition of ${w_1}^{+p_1}$, it follows both that ${w_1}^{+p_1} \models \neg p_1$ and that ${w_1}^{+p_1} \models p_1$. Thus, ${w_1}^{+p_1}$ is an impossible world, since it is not possible for all of the states of affairs true there to obtain.

We now prove the fundamental theorem of impossible worlds, namely, that for every way things can't possibly be, there is an impossible world (other than s_u) where things are that way, or more precisely:

$$\neg\Diamond p \rightarrow \exists s(\mathit{ImpossibleWorld}(s) \ \&\ s \neq s_u \ \&\ s \models p)$$

To prove this claim, assume that $\neg\Diamond p_1$, for an arbitrarily chosen state of affairs p_1 (the state p_1 is, intuitively, a 'way' that things couldn't possibly be). Now consider the actual world w_α. In previous work, it has been established that there is a unique possible world w_α which is such that $\mathit{Actual}(w_\alpha)$. I will leave it as an exercise for the reader unfamiliar with this work to show that from the following definition of w_α, it can be proven that w_α satisfies the definitions of possible world and actual situation, and that it does so uniquely:

$$w_\alpha =_{df} \imath x(A!x \,\&\, \forall F(xF \equiv \exists p(p \,\&\, F = [\lambda y\, p])))$$

To conclude the proof of the fundamental theorem, then, we simply now show that the p_1-extension of w_α (i.e., ${w_\alpha}^{+p_1}$) is an impossible world in which p_1 holds true. Note that since $\neg\Diamond p_1$, we know that $\neg p_1$. Now it is a fact about w_α that $w_\alpha \models p \ \equiv \ p$. So $w_\alpha \not\models p_1$, and by the maximality of worlds, it follows that $w_\alpha \models \neg p_1$. Thus, the conditions of our previous theorem apply: we have a possible world w_α and state of affairs p_1 false at w_α. So ${w_\alpha}^{+p_1}$ is an impossible world (it is just like the possible world w_α except for the fact that p_1 also holds true). It remains simply to show that ${w_\alpha}^{+p_1}$ is not identical to s_u. But this is easily established, for we need only find a state of affairs which obtains in s_u which doesn't obtain at ${w_\alpha}^{+p_1}$. But note that the conjunctive state of affairs $p_1 \,\&\, \neg p_1$ holds true in s_u (since every state of affairs obtains in s_u), but fails to hold true in ${w_\alpha}^{+p_1}$ (the only states of affairs true at ${w_\alpha}^{+p_1}$ are the states true at w_α and p_1; so since $p_1 \,\&\, \neg p_1$ isn't true at w_α, it fails to be true at ${w_\alpha}^{+p_1}$). Thus, s_u is distinct from ${w_\alpha}^{+p_1}$.

So for every way things can't possibly be, there is an impossible world where things are that way. Now there are several other facts about impossible worlds which should prove to be of interest to paraconsistent

[13] Note that this consequence predicts and confirms the existence of the trivial world Priest describes in [1987] (p. 110), where 'every propositional parameter takes the value $\{0, 1\}$'.

[14] This constitutes a classically based proof of the existence of an impossible world which Priest semantically 'constructs' in [1992] (p. 300) using a non-classical system interpreted with Dunn's four-valued semantics.

logicians. The first is that from the fact that a contradiction $p \& \neg p$ holds at an impossible world s, it does not follow that every state of affairs q also holds at s. For suppose that the states $p_1 \& \neg p_1$ and q_1 are distinct states, where q_1 is some arbitrarily chosen false state of affairs. Then consider the $(p_1 \& \neg p_1)$-extension of the actual world. It is provable that the only states holding true at this impossible world are: (1) the states of affairs which in fact obtain and (2) the conjunctive state of affairs $p_1 \& \neg p_1$. In particular, q_1 doesn't obtain at this world. So not everything obtains at an impossible world where a contradiction holds. Let us formulate the law *ex contradictione quodlibet* as follows:

$$s \models (p \& \neg p) \ \vdash \ s \models q$$

In other words, the law asserts that the truth of q in s is derivable from the truth of $p \& \neg p$ in s, for any situation s. On this formulation, the law does not govern impossible worlds.[15]

Second, it follows immediately that there are impossible worlds that are not 'modally closed'. Let us define p *necessarily implies* q ('$p \Rightarrow q$') and the notion of a modally closed situation as follows:

$$p \Rightarrow q =_{df} \Box(p \rightarrow q)$$

$$\mathit{ModallyClosed}(s) =_{df} s \models p \ \& \ p \Rightarrow q \ \rightarrow \ s \models q$$

So a situation s is modally closed just in case every state necessarily implied by a state of affairs holding true at s is also true at s. Now it is easy to establish that *possible* worlds are examples of modally closed situations.[16] But to see that there are impossible worlds which are not modally closed, pick distinct states p_1 and q_1 and compare the contradictory state $p_1 \ \& \ \neg p_1$ with q_1. The state q_1 is necessarily implied by the contradictory state $p_1 \ \& \ \neg p_1$. But whereas the latter is true at the $(p_1 \& \neg p_1)$-extension of the actual world, the former is not. So not all the states necessarily implied by states holding at the $(p_1 \& \neg p_1)$-extension of the actual world also hold at that world. It is, therefore, an impossible world which is not modally closed. There are, however, impossible worlds which are modally closed. The universal situation s_u is an example.

Third, from the fact that $\neg p$ and $p \lor q$ both hold at an impossible world, it does not follow that q holds at that world. For pick any distinct states p_1 and q_1 such that p_1 obtains and q_1 doesn't. Then the following object is, provably, an impossible world where $\neg p_1$ and $p_1 \lor q_1$ are both true, but in which q_1 is not true:

$$\imath x(A!x \ \& \ \forall F(xF \equiv \exists p(w_\alpha \models p \ \& \ F = [\lambda y \, p]) \ \lor \ F = [\lambda y \, \neg p_1] \ \lor \ F = [\lambda y \, p_1 \lor q_1]))$$

This is the world which is just like the actual world but where the complex states $\neg p_1$ and $p_1 \lor q_1$ are also true. Since q_1 is not true, it is not true at the actual world and so not true at the impossible world just defined. Now we may formulate the law of disjunctive syllogism as follows:

$$s \models \neg p, \ s \models (p \lor q) \ \vdash \ s \models q$$

In other words, the law of disjunctive syllogism asserts that the truth of q in s is derivable from the truth of $\neg p$ in s and the truth of $p \lor q$ in s. With this formulation, we have established that there are impossible worlds in which the law of disjunctive syllogism fails.[17]

It is now important to point out (and it might come as a surprise) that not every impossible world is a world where some contradiction is true. To see why, let us contrast the definition of a possible situation with that of a consistent situation:

[15] Nor does the law govern impossible situations. An 'impossible situation' is simply any situation s such that for some state of affairs p it is not possible for all the states of affairs holding true in s to be true. Then, it is easily provable that there are impossible situations in which a contradiction holds, but in which *no* other states of affairs are true. This is enough to demonstrate that the law in question fails in impossible situations. We shall have more to say about 'possible' and 'impossible' situations in what follows.

[16] Here is the proof. Assume $w \models p_1$ and that $p_1 \Rightarrow q$ (to show: $w \models q$). Let ψ abbreviate the formula $w \models q$, and let us first establish $\Diamond\psi$. We establish $\Diamond\psi$ by using the following modal law: $\Box(\phi \rightarrow \psi) \rightarrow (\Diamond\phi \rightarrow \Diamond\psi)$. Now since w is a possible world, we know by the definition of a possible world $\Diamond\forall p(w \models p \equiv p)$. So let $\phi = \forall p(w \models p \equiv p)$. Thus we know $\Diamond\phi$. So all that remains to show $\Diamond\psi$ is that $\Box(\phi \rightarrow \psi)$. So we prove $\phi \rightarrow \psi$ and then apply the Rule of Necessitation. So assume ϕ; i.e., that $\forall p(w \models p \equiv p)$. Then it follows that $w \models p_1 \equiv p_1$. Now one of our assumptions is $w \models p_1$. (Note that this is necessary, by the rigidity of encoding.) So it follows that p_1. However, our other assumption is that $p_1 \rightarrow q$ is necessarily true. Since p_1 and $p_1 \rightarrow q$ both hold, it follows that q. But since we have assumed that $\forall p(w \models p \equiv p)$, it follows that $w \models q$. Since we have produced a proof of ψ from ϕ which appeals only to necessary truths, we apply the Rule of Necessitation to yield that $\Box(\phi \rightarrow \psi)$. So by our modal law, we have established that $\Diamond w \models q$. But, given the rigidity of encoding, possibly true encoding claims are necessarily true, and hence just true. So $w \models q$.

[17] Indeed, the law of disjunctive syllogism fails for situations in general, for they are often 'incomplete' (i.e., fail to be maximal). It is easy to prove the existence of incomplete situations in which $\neg p$ and $p \lor q$ are true but in which q is not true (and neither is $\neg q$). There is further discussion of non-maximal situations below.

$$Possible(s) =_{df} \Diamond\forall p(s \models p \rightarrow p)$$

$$Consistent(s) =_{df} \neg\exists p(s \models (p \,\&\, \neg p))$$

(Note that this definition of consistency is not equivalent to the definition which takes consistent situations to be such that $\neg\exists p(s \models p \,\&\, s \models \neg p)$, but we will come to this in a minute.) Now it is straightforward to establish that if a situation is possible, then it is consistent, in the senses just defined. So, clearly, inconsistent situations are not possible ones. However, one cannot establish that a situation that is not possible is not consistent. The reason is that a situation may be impossible for metaphysical rather than logical reasons. There may be laws of metaphysics which preclude the possibility of certain states of affairs. Those metaphysically impossible states of affairs may be true in an impossible world without there being a logical contradiction true at that world.

This suggests that the nonclassical worlds studied by the paraconsistent logician form just a subspecies of the impossible worlds. Not only do they study those impossible worlds other than s_u where some contradiction is true, but moreover, it seems that they study those impossible worlds which are 'coherent' with respect to inconsistency, namely, those impossible worlds s which satisfy the following condition:

$$s \models (p \,\&\, \neg p) \;\equiv\; s \models p \,\&\, s \models \neg p$$

In other words, we may define a 'coherent inconsistent world' to be an impossible world s other than s_u such that some contradiction is true at s and such that whenever some contradiction is true at s, the states involved in the contradiction are also true at s. Coherent inconsistent worlds seem to have genuine 'truth value gluts'—in such situations, not only there states p such that both p and its negation are true (i.e., p is both true and false), but the conjunction of p and its negation is true as well. It may be that there are other conditions that impossible worlds must satisfy in order to identify them as the subject matter of paraconsistent logic (and this might depend on the particular logic).

I conclude this section by employing the above definitions to define a few other concepts that seem to have played a role in the study of alternative logics recently. First, it seems worth pointing out that we don't always have to appeal to impossible worlds to find a subject matter for 'non-classical' logics. We need only appeal to the more general notion of a situation which is both closed under an entailment relation and closed under conjunction. Let $\stackrel{R}{\Rightarrow}$ be your favorite axiomatized (relevant) entailment relation on states of affairs. We may carve out a class of *situations* which are both closed under $\stackrel{R}{\Rightarrow}$ and closed under conjunction as follows:

$$\stackrel{R}{\Rightarrow}\text{-}Closed(s) =_{df} s \models p \,\&\, p \stackrel{R}{\Rightarrow} q \;\rightarrow\; s \models q$$

$$\&\text{-}Closed(s) =_{df} s \models p \,\&\, s \models q \;\rightarrow\; s \models (p \,\&\, q)$$

The first definition tells us that if q is any state of affairs which is $\stackrel{R}{\Rightarrow}$-entailed by a state of affairs p and p is true at an $\stackrel{R}{\Rightarrow}$-closed situation s, then q is also true at s. The second definition tells us that conjunction-closed situations always make a conjunction true whenever they make the individual conjuncts true. Situations which are both $\stackrel{R}{\Rightarrow}$-closed and conjunction-closed need not be possible worlds nor impossible worlds. Relevant logic, from the present point of view, is the study of such situations which are closed under some relevant entailment relation and closed under conjunction.

Finally, note that situations are not necessarily maximal. There are 'incomplete' situations s and states of affairs p such that neither $s \models p$ nor $s \models \neg p$. Such situations offer us a classical, two-valued conception of truth value gaps! If a situation s is indeterminate with respect to p in this way, we may say that p has no truth value at s. It is a matter of debate whether the actual world is a situation having such truth-value gaps (or gluts, for that matter). But whether it does or not, we have a classically conceived subject matter for the logic of truth-value gaps.

4. This is No Ersatz Conception of Impossible Worlds

Someone might object at this point that we have only produced an 'ersatz' theory of impossible worlds. Here are two examples of ersatz theories of impossible worlds: (1) an impossible world is a maximal but inconsistent set of states of affairs, (2) an impossible world is a maximal, but inconsistent set of sentences. In its simplest form, the complaint about such theories is that such entities are the wrong kind of thing to be worlds. Whatever worlds are, they are not sets of states of affairs or sets of sentences. Though there are numerous reasons that one can give for thinking that worlds are not sets, one philosophically cogent reason concerns the notion truth at a world. On the ersatz conception, truth at a world amounts to set membership. But there is much more to the truth of a state of affairs at world w than the membership of that state in some set.

The state of affairs must not simply classify the world in question but instead characterize it. If states of affairs are ways that a world can (or couldn't possibly) be, then when a state of affairs is true in a possible or impossible world, the world must *be* that way, in some important sense. Set membership is not even close to being the right relation. A maximal (in)consistent set of states of affairs or sentences might serve as a logically useful *model* of (im)possible worlds, but it is no philosophical theory of such entities.

But worlds defined in terms of encoded properties are not subject to such a complaint. The properties that an abstract object encodes *characterize* that object. If x encodes F, there is a sense in which x 'has' or is characterized by F, for encoding is a mode of predication. It is an auxiliary hypothesis of our theory that there is an ambiguity in the predicative copula 'is' of ordinary language statements like 'x is F'. On certain uses of the predicative copula, the ordinary copula is best analyzed in terms of exemplification, while on others, it is best analyzed in terms of encoding. The fact that F characterizes those objects that encode it plays an important role in the explanatory power of the theory. For example, though nothing exemplifies the property of being a monster, the theory of abstract objects asserts that there are numerous objects that encode this property. These objects are used in the analysis of reports about nightmares involving monsters. When someone has a dream which caused them to wake up screaming, we can explain the fear of the person involved by pointing to the fact that the dream was about an object which, in some sense, 'is' a monster. Things which are *characterized* by the property of being a monster are things which can engender fear.

It is for this reason that abstract objects can not be understood as sets of properties. A set of properties simply contains properties as elements. The properties which are the elements of such a set do not characterize the set in any way. So although the theory of abstract objects I have proposed can be proved consistent by modeling the theory in the theory of Zermelo-Fraenkel set theory, we can not eliminate the encoding mode of predication in favor of set membership, for we would lose the explanatory power that encoding affords us, such as that described at the end of the previous paragraph.

Now the fact that situations, possible worlds, and impossible worlds encode properties of the form *being such that p* is extremely important. For these properties characterize the situations in question. The formula

'$s \models p$' can be read in ordinary English as "situation s is such that p". Recall that it is a theorem of our world-theory that p is possible iff there is a possible world where p is true ($\Diamond p \equiv \exists w(w \models p)$). So given that it is possible that Clinton not be president ($\Diamond\neg Pc$), there is a possible world where it is true that Clinton fails to be president ($\exists w(w \models \neg Pc)$). Let w_1 be such a world. Then the theoretical sentence '$w_1 \models \neg Pc$' can be read in ordinary English as "w_1 *is* such that Clinton fails to be President". This is, therefore, no ersatz theory of possible worlds.

Similarly for impossible worlds. By a previous theorem, we know that for every way things can't be, there is an impossible world which is that way. So given that it is not possible that Clinton be president and fail to be president ($\neg\Diamond(Pc \,\&\, \neg Pc)$), there is an impossible world where it is true that Clinton both is president and fails to be president ($\exists w(w \models (Pc \,\&\, \neg Pc))$). Any such world *is* such that Clinton is president and fails to be President. So our theory of impossible worlds is no more ersatz than our theory of possible worlds. The logic of encoding gives us a logic of impossibility which seriously attempts to account for the way in which an impossible object can 'have' impossible properties. When a contradictory state of affairs is true at an impossible world, the world in question is characterized by a contradiction. But we need not give up any ordinary pretheoretic intuitions about the classical mode of predication. No contradiction is true; it is not possible for there to be an object x and property F such that both x instantiates or exemplifies F and x fails to exemplify F.

A second objection to the present account might run as follows: worlds are concrete objects, but we have defined them to be abstract objects; so our 'worlds' are not real worlds but only ersatz. This objection, however, simply presupposes something that is controversial, namely, that worlds are correctly conceived as (maximally large, spatiotemporally connected) concrete objects. But, of course, there is an alternative conception of worlds, namely, the (Tractarian) Wittgensteinian conception on which worlds are all that is the case and not just a totality of things. The present conception of worlds is a Wittgensteinian one. Our worlds encode all that is the case. Moreover, our conception is consistent with the existence of maximally large (mereological sums of) spatiotemporally-connected concrete objects. If one wants to call these latter objects 'worlds' that is fine. But on such a conception, a 'world' is not defined in terms of the notion of truth or in terms of the states of affairs which are true at that world. By contrast, our Wittgensteinian conception of a world describes an intrinsic connection between a world and that which goes on there. This is no ersatz conception of worlds.

5. Are Impossible Worlds Philosophically Useful?

We have now developed a metaphysically grounded and essentially classical theory of impossible worlds. Since the theory asserts the existence of impossible worlds and impossible situations, it can be justified, if for no other reason, by the fact that such entities constitute the domain of application for alternative logics. Now various philosophers have developed other reasons for asserting the existence of impossible worlds. But given the analytic power of the logic of encoding, it turns out that few of those reasons are cogent ones. In this section, we examine the philosophical claims about other alleged theoretical benefits of accepting impossible worlds and show how most of those claims can be undermined. In the process, we discover exactly if and where else impossible worlds prove to be useful.

One claim made on behalf of impossible worlds is that they can help us solve problems with a certain classic (extensional) analysis of propositions as functions from worlds to truth-values.[18] It is well known that such analyses yield unintuitive results, probably the worst being that the analysis implies that there is only one necessarily true proposition and one necessarily false proposition. Such a result is unintuitive because it seems that the proposition that there is a barber who shaves all and only those who don't shave themselves is *not* identical with the proposition that there is a dog and which fails to be a dog (someone could believe the first without believing the second). Neither proposition could possibly be true, yet the classic analysis of propositions identifies the two propositions, for they are the same function: they both map every possible world to the truth-value False. However, the claim is that if propositions are identified with functions defined on both possible and impossible worlds, then one can distinguish the above-mentioned propositions by supposing that although they have the same truth value at all possible worlds, there are impossible worlds where the two propositions have distinct truth values. Thus, by adding impossible worlds to the picture, it is claimed that we can distinguish propositions that are intuitively distinct.

[18] See, for example, Cresswell [1972] and Yagisawa [1988].

Though such a claim on behalf of impossible worlds demonstrates their usefulness for a metaphysics which includes sets (functions) and worlds as *basic* entities, the present metaphysics does not require impossible worlds in order to distinguish necessarily equivalent propositions. To see why, we need only analyze our pretheoretical notion of proposition in terms of our theoretical notion of a states of affairs. Suppose, for example, that we analyze the proposition:

> There is a barber who shaves all and only those who don't shave themselves

in terms of the state of affairs:

p_1: $\exists x(Bx \,\&\, \forall y(Sxy \equiv \neg Syy))$

And suppose we analyze the proposition:

> There is an object which is both a dog and which fails to be a dog

in terms of the state of affairs:

q_1: $\exists x(Dx \,\&\, \neg Dx)$

The logic of encoding allows us to distinguish the two propositions without quantifying over *primitive* sets or worlds. We may consistently add to our theory the claim:

$$p_1 \neq q_1$$

The states of affairs p_1 and q_1 are good examples of states which are distinct but necessarily equivalent in the sense that $\Box(p_1 \equiv q_1)$. Given that they are distinct, there will be impossible worlds where p_1 is true but not q_1 and vice versa. But we don't have to appeal to those impossible worlds to distinguish p_1 and q_1 or to distinguish other such necessarily equivalent, but intuitively distinct, states of affairs. The logic of encoding yields theoretical consequences when we intuitively judge that states of affairs p and q are distinct, namely, that the property *being such that* p is distinct from the property *being such that* q (where this, in turn, means that it is possible for there to be an abstract object that encodes the one without encoding the latter).

Nor are impossible worlds required to distinguish necessarily equivalent, but distinct, properties and relations. The theory of abstract objects and logic of encoding offer precise identity conditions for properties and relations. These conditions do not identify properties and relations which are necessarily equivalent in the sense that have the same exemplification extension at all possible worlds. So we need not appeal to impossible worlds to distinguish such properties and relations. We may regard properties and relations as structured entities subject to a theory (with precise comprehension and identity conditions) that is consistent with our pretheoretic intuitions.[19]

[19]In Yagisawa [1988], we find certain objections to structured properties. He concludes that the structuralist view of properties 'does not seem to be attractive to a Lewisian modal realist.' This may be true, but if one is not a Lewisian modal realist, one can nevertheless defend the structured view of properties by rejecting Yagisawa's identity condition, namely, that structured properties are identical whenever they have have the same constituent properties and the same structure. Such a definition does indeed fall prey to the counterexample of *being a vixen* and *being a female fox* (intuitively, these are the same property; but most theories of structured properties are forced to distinguish the properties corresponding to the predicates 'is a vixen' and 'is a female fox'—the semantic representations offered by such theories have neither the same constituent properties nor the same structure). By contrast, the present theory of structured properties gives them identity conditions in terms of encoding. The (structured) properties *being a vixen* and *being a female fox* are identified whenever they are necessarily encoded by the same objects.

I believe that the other objections that Yagisawa levels against structured universals can be met. But I will not take the opportunity of replying here. Instead, let me briefly discuss one other passage from Yagisawa [1988]:

> Alan McMichael claims that an actualist does need abstracta to explain the actual world, but that everyone is stuck with abstracta anyway. It is not unfair to say that an actualist is stuck with intensional abstracta. But it is not true to say that everyone is stuck with them. A modal extensionalist is not. Perhaps he is stuck with mereological entities and sets, but nothing intensional. ... An actualist needs not just abstracta—say, extensional abstracta, e.g., sets—but a specific kind of abstracta, namely, intensional abstacta. We may say that McMichael is a Platonistic actualist in believing in intensional abstracta. Modal extensionalism avoids Platonism in this sense. (p. 178)

Here, by 'intensional abstracta', Yagisawa is referring to properties and relations. I would like to point out that it is not true to say that an actualist needs intensional abstracta in the sense of 'intensional' that Yagisawa finds objectionable (i.e., having nonextensional identity conditions). On the present theory, though properties and relations are conceived intensionally (i.e., necessarily equivalent ones are distinguishable), they still have extensional identity conditions. It was noted in the first section of the paper that it is a consequence of our theory that when $\forall x(xF \equiv xG)$, then F and G are identical. These are clearly extensional identity conditions. Moreover, the abstract objects themselves, though hyperintensional entities (since they encode intensionally conceived properties), also have extensionally defined identity conditions—it is a consequence of our theory that whenever $\forall F(xF \equiv yF)$, then abstract objects x and y are identical.

So I think Yagisawa has no complaint against the present version of Platonism—it offers a precise, extensional account of intensional entities.

Note that with the exception of 'hyperintensional' contexts, the problem of distinguishing *beliefs* involving such necessarily equivalent propositions is therefore solved without an appeal to impossible worlds. In Yagisawa [1988], we find the suggestion that by appealing to impossible worlds and distinguishing necessarily equivalent propositions such as p_1 and q_1 (above), we need not infer 'S believes q_1' from 'S believes that p_1'. But our logic forces no such inference and does so without invoking primitive impossible worlds. Moreover, as Perszyk pointed out in [1993], the move to impossible worlds doesn't offer a general solution to such problematic intensional contexts, for there are hyperintensional contexts in which ordinary inferences appear to fail. From the fact that 'S believes that x is a woodchuck' it does not follow that 'S believes that x is a groundhog', even though the property of being a woodchuck is the same property as the property of being a groundhog.[20] It should be pointed out that the theory of abstract objects and the logic of encoding offers a straightforward and general account of such hyperintensional contexts. On this account, a typed version of the theory of abstract objects is developed and applied.[21] True claims of the form '$F = G$' are given the following analysis: the properties denoted by the two predicates are identical, but the senses of the predicates are distinct *abstract properties* (i.e., properties that encode properties of properties). Abstract properties and ordinary properties are entities of the same logical type, in the same way that the abstract individuals and ordinary individuals discussed throughout this paper are entities of the same logical type. So abstract properties can occupy the same constituent role in a state of affairs as ordinary properties. In the above cases where substitution fails, the explanation appeals to the distinct abstract properties serving as the senses of the predicates. The details of this analysis can be pursued at much greater length in Zalta [1988a] and [1988b].[22]

[20]Perszyk uses an example involving the predicates 'is a female fox' and 'is a vixen'—they denote the same property but aren't substitutable for one another in certain belief contexts.

[21]The typed theory of abstract objects was first developed and applied in Zalta [1982] and in [1983], Chapters 5 and 6.

[22]See especially Chapters 9 - 12 of [1988a]. The analysis of hyperintensional contexts involving predicates is simply a generalization of the hyperintensional cases involving

The next claim made on behalf of impossible worlds is that they are needed for the analysis of impossible objects such as the round square, the even prime number greater than 2, the Russell set, the barber who shaves all and only those who don't shave themselves, etc. We often seem to have mental states directed towards such objects, such as when we think about them, conclude that they are impossible, etc. One might think that the theory of impossible objects would be grounded in the theory of impossible worlds.

However, given the view developed here, we don't need the theory of impossible worlds to ground the theory of impossible objects. Instead, we simply need the logic of encoding and theory of abstract objects to ground the general theory of impossibilia. An 'impossible object' can be regarded as any abstract object x such that it is not possible that there be some object that exemplifies all the properties x encodes:

$$\mathit{ImpossibleObject}(x) =_{df} \neg\Diamond\exists y\forall F(xF \rightarrow Fy)$$

Note that this definition classifies impossible worlds as a species of impossible object. So the theory of impossibilia can be unified, not by starting with primitive impossible worlds, but rather by defining the various impossibilia in terms of the logic of encoding. This logic gives us a genuine sense in which an impossible object can 'have' incompatible properties.

There are two final claims about the theoretical benefits of accepting impossible worlds I want to discuss. Both are put forward by Yagisawa in [1988]. I think the second claim is the more convincing of the two. The first involves certain puzzles which Yagisawa produces for Lewis's view about possible worlds. In the attempt to force one to accept the existence of 'alternative logical spaces', Yagisawa considers such statements as 'w could have been inaccessible from w'' and 'there could be more worlds than there actually are in our logical space'. I don't find myself persuaded by these examples; they strike me as illegitimate for the following reason. The project Lewis is engaged in is the attempt to systematize our modal beliefs. Our modal beliefs are expressed in ordinary language; they involve no theoretical notion of 'possible world', but rather ordinary modal notions such as what is possible, necessary, etc. Lewis employs a theoretical language and offers a systematic way to render our modal beliefs in the theoretical language. It strikes me as illegitimate for someone to take as data to be explained sentences which employ both our pretheoretic modal notions and our theoretical notions. Talk of 'what worlds there might have been' strikes me, therefore, as a kind of confusion of formal mode and material mode. The sentences that Yagisawa produces in the attempt to force acceptance of impossible worlds are of this kind. It is unclear to me that such sentences constitute new data to be explained.

proper names; for example, from the fact that 'S believes that Mark Twain wrote x' it does not follow that 'S believes that Samuel Clemens wrote x'. In such cases, the denotation of 'Mark Twain' and 'Samuel Clemens' is the same individual, but the senses of the names are distinct abstract individuals (which encode distinct properties of individuals). It should be mentioned that this understanding of Fregean senses is not one on which it is supposed that sense determines denotation.

I find the counterfactuals with impossible antecedents that Yagisawa considers much more convincing. Here we seem to require the resources of worlds other than possible worlds to find a proper subject matter and semantics for these counterfactuals. Though Yagisawa attempts to convict Lewis out of his own mouth by citing Lewis's own technical use of counterfactuals with impossible antecedents, I think the more convincing cases concern ordinary language, such as 'if I had been born to different parents, ... ', 'if I had been (identical to) y, ... ', 'if there had been a round square, ... ', 'if there had been a barber who shaves all and only those who don't shave themselves, ...', etc. Examples such as these may in fact require an appeal to impossible worlds if we are to avoid the result that all such sentences come out vacuously true. The defense Perszyk offers on Lewis's behalf in [1993] is limited, since his defense disarms only a specific technical counterfactual. The defense carries no weight against someone appealing to impossible worlds to systematize (preserving truth value) the counterfactuals with impossible antecedents which we utter in everyday, ordinary language.

To summarize the results of this section: most of the claims about the philosophical benefits of impossible worlds are not sustainable. Impossible worlds are not needed to distinguish necessarily equivalent propositions and properties; they are not needed for the analysis of hyperintensional contexts; they are not needed for the analysis of impossible objects; they may, however, prove useful for the analysis of counterfactual conditionals of ordinary language having impossible antecedents.

6. Philosophical Observations

In the foregoing, we have developed a metaphysical theory, not a semantic model, of impossible worlds. Our impossible worlds are not primitive elements of some set-theoretic model which are stipulated to obey certain constraints. Although our possible worlds are not concrete objects, they are not 'ersatz impossible worlds', but rather conform to the Wittgensteinian notion of a world (i.e., defined in terms of 'all that is the case'). Impossible worlds are abstract objects which have an intrinsic nature as maximal situations that are individuated by the incompatible states of affairs that obtain there. They are a species of 'impossible' object, for they 'have', in a precise theoretical sense, incompatible properties.

The logic and theory of impossibilia developed here preserves our pretheoretic understanding of the traditional mode of predication, namely, exemplification. Pretheoretically, we know what it is, say, for a piece of cloth to be colored or to have a certain shape, or what it is for someone to be wearing clothes, or what it is for someone to be located in a certain place. It is a part of that pretheoretic understanding that if something really instantiates the property of being colored or having a certain shape, then it can not be the case that it fails to instantiate that property. Our understanding of what it is for something to instantiate the property of wearing clothes or being located in a certain place excludes the failure to wear clothes or to be located in that place. Finally, part of our pretheoretic understanding is that if two things stand in, or exemplify, a relation R, then they do not fail to stand in that relation. The logic of encoding preserves this understanding by preserving classical exemplification logic. But it also includes a mode of predication which has the capacity to take impossibilia seriously.

On the present view, logic is not just about inferences, but rather about (modes of) predication. Logic is about the modes in which objects are characterized by properties and relations. These include not just the exemplification and encoding modes of predication and their molecular and quantificational forms, but also the alethic modalities, the logic of actuality, predications involving complex properties and/or objects described in complex ways. This is a conception on which logic is about the ways of characterizing things in the world, and not just about the consequences of such characterizations. Whereas classical logic is based on a single mode of characterization that excludes the existence of objects with incompatible properties, the logic of encoding extends classical logic by including a mode of predication that does not exclude the existence of such objects.

I think this logic offers an alternative analysis of the 'dialethic' modalities. For the most part, I am sympathetic to the work of the paraconsistent logicians, for they have also been exploring ways to modify classical logic to better represent and systematize philosophically interesting truths and inferences of ordinary language. We have seen that paraconsistent logic does have a subject matter, namely, certain species of impossible worlds and impossible situations. The law of noncontradiction, when formulated in terms of truth in a situation, does indeed fail in inconsistent worlds and inconsistent situations. We may therefore think of dialethism, in general, as the study of 'impossible objects', in the sense defined above. The present theory offers a classically based analysis of cases that seem to involve 'true contradictions'. In these cases, we can analyze the apparent contradiction in terms of objects that encode the contradictory properties. The objects do then 'have', in an important sense, the contradictory properties. Though I shall not take the time to argue for this here, I believe that the appeal to such objects constitutes a very general method of dealing with some of the famous paradoxes that might lead one to accept that there are true contradictions.

At present, the wide variety of philosophical theorems and applications of the logic of encoding convince me that the core principles of classical logic and the theory of abstract objects should be preserved intact.[23] The present piece establishes that a classically-based logic has the capacity to represent the dialethic phenomena as well as a lot more. Until the dialethic logician comes up with a similar *systematically applied* formal system having consequences of the same magnitude and firepower, I plan to stick to my classical guns.

Bibliography

Cresswell, M., 1967, 'The Interpretation of Some Lewis Systems of Modal Logic', *Australasian Journal of Philosophy* 45: 198-206

Cresswell, M., 1972, 'Intensional Logics and Logical Truth', *Journal of Philosophical Logic* **1**: 2-15

Kripke, S., 1965, 'Semantic Analysis of Modal Logic II: Non-normal

[23]A complete listing of all theorems of the theory of abstract objects have been compiled in the unpublished monograph *Principia Metaphysica* (see Zalta [unpublished] in the Bibliography). This document hasn't been published because when new results are discovered, such as the ones described in the present paper, the manuscript is updated. However, the 'current' version of the document can be found online on the World Wide Web. For the URL, see the Bibliographic citation.

Modal Propositional Calculi', in *The Theory of Models*, J. Addison *et al.* (eds.), North Holland Publishing

Linsky, B., and Zalta, E., 1995, 'Naturalized Platonism vs. Platonized Naturalism' (coauthor: Bernard Linsky), *The Journal of Philosophy*, XCII/10 (October): 525-555

Mares, E., 1997, 'Who's Afraid of Impossible Worlds', *Notre Dame Journal of Formal Logic*, **38**/4: 516–526.

Morgan, C., 1973, 'Systems of Modal Logic for Impossible Worlds', *Inquiry* **16**: 280-289

Paśniczek, J., 1994, 'Non-standard Possible Worlds, Generalised Quantifiers, and Modal Logic', in *Philosophical Logic in Poland*, J. Wolenski (ed.), Dordrecht: Kluwer, pp. 187-198

Perszyk, K., 1993, 'Against Extended Modal Realism', *Journal of Philosophical Logic* **22**/2: 205-214

Priest, G., 1992, 'What is a Non-Normal World?' *Logique et Analyse* **35**: 291-302

Priest, G., 1987, *In Contradiction*, Dordrecht: Nijhoff

Priest, G. and Sylvan, R., 1992, 'Simplified Semantics for Basic Relevant Logics', *Journal of Philosophical Logic* **21**: 217-232

Rantala, V., 1982, 'Impossible World Semantics and Logical Omniscience', *Acta Philosophica Fennica* **35**: 106-115

Rescher, N. and Brandom, R., 1980, *The Logic of Inconsistency: A Study in Non-Standard Possible World Semantics and Ontology*, Oxford: Basil Blackwell

Restall, G., 1997, 'Ways Things Can't Be', *Notre Dame Journal of Formal Logic*, **38**/4: 583–596.

Routley, R., 1980, *Exploring Meinong's Jungle and Beyond*, Departmental Monograph #3, Philosophy Department, Research School of Social Sciences, Australian National University, Canberra

Yagisawa, T., 1988, 'Beyond Possible Worlds', *Philosophical Studies* **53**: 175-204

Zalta, E., unpublished, *Principia Metaphysica*, available on the World Wide Web at the URL = https://mally.stanford.edu/theory.html

———, 1997, 'The Modal Object Calculus and its Interpretation', in *Advances in Intensional Logic*, M. de Rijke (ed.), Dordrecht: Kluwer, pp. 249–279

———, 1993, 'Twenty-Five Basic Theorems in Situation and World Theory', *Journal of Philosophical Logic*, **22**: 385–428

———, 1988a, *Intensional Logic and the Metaphysics of Intentionality*, Cambridge, MA: Bradford Books/The MIT Press

———, 1988b, 'A Comparison of Two Intensional Logics', *Linguistics and Philosophy*, **11** (February): 59–89

———, 1983, *Abstract Objects: An Introduction to Axiomatic Metaphysics*, Dordrecht: D. Reidel Publishing Co.

———, 1982, 'Meinongian Type Theory and Its Applications', *Studia Logica*, **41**/2-3: 297–307