Avoiding Russell-Kaplan Paradoxes: Worlds and Propositions Set Free* (DRAFT)

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Abstract

The authors first address two paradoxes in the theory of possible worlds and propositions stemming from Russell and Kaplan and show that these paradoxes don't affect the object-theoretic analysis of worlds and propositions. However, Kit Fine has formulated an object theoretic version of Kaplan's paradox that threatens to show that object theory is, after all, no better off. The initial, most straightforward version of the paradox is blocked by theoretical restrictions specific to object theory, but the paradox can be revised so as to comport with these restrictions by redefining one of the terms in an essential premise. The authors then argue that the premise that results given the new definition is entirely implausible if propositions are understood, as they are in object theory, to be fine-grained intensional entities rather than sets of possible worlds. Object theory, therefore, can block the revised paradox as well.

Introduction

Paradoxes deriving from Russell (1903) and Kaplan (1995) show that serious logical problems can arise quickly in an ontology that countenances propositions, sets, and possible worlds. Unsurprisingly, then, particularly potent versions of these paradoxes arise for theories that bring all three together by *identifying* worlds as sets of propositions or vice versa. There are, of course, several such theories (Adams 1981, Plantinga 1974, and Lycan and Shapiro 1986) and, as will become clear, all of them face difficulties.

In sorting through these issues, an important question arises, namely, whether one should take possible worlds as primitive and define propositions or take propositions as primitive and define worlds. Stalnaker (1976) comes down in favor of the former. As part of his argument for this claim, Stalnaker notes:

> Whatever propositions are, if there are propositions at all then there are sets of them, and for any set of propositions, it is something determinately true or false that all the members of the set are true. (*ibid.*, 73)

This passage is preserved in his (1984, 55) and (2003, 36). Our argument in what follows, if correct, shows that the first sentence is in error. We develop a metaphysical theory that can do the work one expects of a foundational theory but that asserts the existence of propositions without requiring that there are sets of them. Furthermore, we try to show that it is better to take propositions as primitive 0-place relations, axiomatize them (as part of a general theory of n-place relations), and then define worlds in terms of a background theory of objects. At least, we will show that one can do so in a way that is paradox-free and which provides a metaphysical foundation for using possible worlds in possible world semantics.

The argument in what follows also undermines a claim of Sider's (2002, 307) concerning world theories and cardinality problems:

> *Every* theory of worlds encounters trouble in this area. The linguistic ersatzist, for example, may admit arbitrarily large worlds, but cannot admit a world with so many individuals that they cannot all be members of a set (except in special cases where the objects display symmetries allowing simpler

> description); for a linguistic-ersatz world is a maximal consistent set of sentences, and sentences themselves are also sets. That is a limitation on possibility, although a bit less severe than an upper bound on world size. Similar problems confront other views that identify possible worlds with abstract entities other than sets of sentences.

We shall be defending a view that identifies possible worlds as abstract entities, but we shall establish that no obvious size-based argument that places limits on possibility affects our theory. Moreover the view we shall defend avoids a similar problem that arises for Lewis (1986, 101–104) that forces him to restrict his principle of recombination in such a way that there must be a cardinal upper bound on the number of worlds and possibilia.

In Section 1 below, we examine the Russell- and Kaplan-style paradoxes and note important similarities between them. In Section 2, we show that the object-theoretic analysis of worlds and propositions provides a principled and unified strategy to block them. In Section 3, we spell out a suggestion of Kit Fine's for reconstructing a similar paradox in object theory. In Section 4, we argue that there are principled reasons for rejecting the central premise of Fine's paradox and conclude that the paradox doesn't present a concern for object theory. Finally, in Section 5 we provide some concluding thoughts. Our moral: sets should not be used in the theory of propositions and worlds; Russell-Kaplan paradoxes show that neither propositions nor worlds should be identified with sets. Object theory provides an alternative framework that allows for the formulation of the relevant concepts without invoking sets. Worlds and propositions, finally, can be set free.

1 The Paradoxes

1.1 Worlds as Sets of Propositions

Not long after Russell showed that the famous set-theoretic paradox that now bears his name infected Frege's (1893) *Grundgesetze der Arithmetik*, Russell (1903, §500) discovered another paradox involving propositions

that he published in an appendix to *The Principles of Mathematics* (*PoM*).[1] The premises are as follows:

(1) There is a set P of all propositions.

(2) For every set S of propositions, there is a uniquely identifiable proposition p_S.[2]

(3) For sets S, S' of propositions, if $p_S = p_{S'}$, then $S = S'$.

Now, for any set S of propositions, since p_S is a proposition, we have either $p_S \in S$ or $p_S \notin S$. Let $R \subseteq P$ consist of exactly those of the latter sort, that is:

R $x \in R \leftrightarrow \exists S \subseteq P(x = p_S \wedge x \notin S)$.

Since $R \subseteq P$, by (2) we have a corresponding proposition p_R and the obvious question: $p_R \in R$? Suppose so. Then by **R**, for some $S \subseteq P$, $p_R = p_S$ and $p_R \notin S$. By (3), $R = S$. Hence, $p_R \notin R$. So suppose not, i.e., that $p_R \notin R$. Then by **R** once again, for any $S \subseteq P$, if $p_R = p_S$, then $p_R \in S$. So, in particular, since $R \subseteq P$ and, obviously, $p_R = p_R$, it follows that $p_R \in R$. Contradiction. Call this Russell's *PoM* paradox.

Say that a set of propositions is *maximal* if it contains, for every proposition p, either p or its complement $\neg p$. The set P of all propositions, if it exists, is of course maximal. It is, moreover, obviously inconsistent, in the sense that it is impossible that its members be simultaneously true. Interestingly, however, replacing 'proposition' with 'truth' uniformly throughout the above argument yields a corresponding paradox for the set of all *truths* which, of course, if it exists, is just as obviously consistent as P is inconsistent.

This is significant, of course, as it shows that there are serious problems with the otherwise intuitive idea of defining possible worlds as maximal, consistent sets of propositions — a definition proposed initially by Robert Adams (1974, 225) that persists in modern expositions, e.g., Stalnaker and Oderberg (2009, 48). The argument above shows

[1] Today the latter paradox is usually taken to show that there can be no set of all propositions although, at the time, prior to Zermelo's axiomatization of set theory and his own ramified type theory, Russell did not fix upon a single premise as particularly problematic.

[2] Russell identifies p_S with S's "logical product", i.e., the proposition *that every member of S is true*, but this is entirely incidental to the proof. All that matters is that the propositions p_S satisfy (3).

that, given minimal assumptions, postulating a particularly significant instance of one such world — the *actual* world, the world that contains all and only true propositions — is incoherent.

The *PoM* paradox cannot be generalized to arbitrary possible worlds without introducing irreducibly modal notions into the proof.[3] However, with a bit of additional set theory and a somewhat stronger account of propositions, a variation on the paradox can be formulated that applies to any maximal set of propositions and hence, in particular, to any possible world in the sense at hand. The premises of this paradox are as follows:

(4) There is a maximal set of propositions.

(5) Every proposition q has a complement $\neg q$.

(6) For every set S of propositions, there exists a uniquely identifiable proposition p_S.

(7) If S and S' are distinct sets of propositions, then p_S, $p_{S'}$, and their complements are pairwise distinct.

(5)–(7) seem reasonable on any "fine-grained" understanding of propositions. Indeed (6) is rather weak, for all that is required for its truth is that every set be correlated with only one proposition, say, the proposition *that S has at least one member*. Intuitively, this proposition is specifically about S and, hence, is distinct from the proposition *that S′ has at least one member*, when $S \neq S'$. However, assuming some basic set theory, these premises are inconsistent.

By (4), let S^* be a maximal set of propositions. By (5), (6), and the definition of maximality, for every member S of the power set $\wp(S^*)$ of S^*, either $q_S \in S^*$ or $\neg q_S \in S^*$ (perhaps both). By the axiom of Separation, let R consist of all such propositions, that is, let

$$R = \{p \in S^* : \exists S \in \wp(S^*)(p = q_S \vee p = \neg q_S)\}.$$

Since $R \subseteq S^*$, it follows that R is no larger than S^*. However, R contains, for every $S \in \wp(S^*)$, q_s or $\neg q_S$ and, by (7), all such propositions are pairwise distinct. Hence, we can map $\wp(S^*)$ one-to-one[4] into R and, hence, $\wp(S^*)$ is no larger than R. It follows that $\wp(S^*)$ is no larger than S^*, contradicting Cantor's theorem that, for all sets S, $\wp(S)$ is strictly larger than S.[5]

This paradox challenges a number of other theories of possible worlds. Plantinga (1974) defines worlds to be *states of affairs* of a certain sort, but also postulates that, corresponding to each possible world w is the *book* on w, that is, the set of all propositions true at w (*ibid*., 44–46).[6] It is easy to demonstrate that every such book is a world in exactly the sense of Adams (1974) and, hence, that it is subject to the paradox above.[7]

Adams (1981) provides a more sophisticated version of his earlier theory. In this account, he first defines a possible world (or "world story") as above to be a maximal consistent set of propositions (*ibid*., 21ff). He then proceeds to qualify the definition so as to reflect his "existentialism", that is, his view that singular propositions are ontologically dependent on the individuals they are about.[8] Specifically, worlds in which an individual a fails to exist (that is, worlds lacking the proposition *that a exists*) contain no propositions involving a as a "constituent". Given this qualification, some possible worlds turn out not to be maximal in the sense above. However, it has no effect on the nature of worlds containing the same individuals as the actual world — notably, of course, the actual world itself, that is, the set of true propositions. Such worlds are still maximal in the sense above and, hence, the paradox above still applies to Adams' account.

Another theory that also faces the paradox has been developed by William Lycan and Stewart Shapiro (1986). The theory explicitly identifies worlds with sets of propositions (*ibid*., 345). There are constraints, however, on the language in which the theory is formulated so that the

[3]For example, one can alter (2) to stipulate that, for every $S \subseteq P$, there is a *necessarily true* proposition p_S.

[4]The preceding fact guarantees a one-to-many mapping from $\wp(S^*)$ into R. To derive a one-to-one mapping f it is necessary only that, for $S \in \wp(S^*)$, f "choose" q_S or $\neg q_S$ in those cases where R contains both. For example, by the Powerset and Separation axioms, we can define the mapping $f : \wp(S^*) \longrightarrow R$ such that $f(S) = q_S$ if $q_S \in R$ and $f(S) = \neg q_s$, otherwise. This mapping "chooses" q_S when both it and $\neg q_S$ are in R.

[5]This is a somewhat tighter and more general version of the paradox in Bringsjord (1985). See also follow-up discussions by Menzel (1986a), Grim (1986), and Menzel (2012).

[6]Where a proposition p is true at a world w just in case, had w obtained, p would have been true.

[7]See also Chihara (1998, 126-7), who reconstructs basically the paradox here directly in terms of Plantinga's states of affairs.

[8]See Menzel (2008, §4.2.2) for a detailed exposition of Adams' account.

identification of worlds with sets of propositions cannot be made in the object language. No such constraints are found at the metalanguage, and the paradox immediately arises. Alternatively, at the object level, by extending the language so that sets are included, the paradox can be immediately formulated. Additional restrictions would then need to be included, but it is unclear how that could be done in a principled way. The paradox would emerge again.

1.2 Propositions as Sets of Worlds

Paradoxes deriving from Kaplan (1995) concern those theories — notably, David Lewis's theory of concrete worlds — that follow possible world semantics in defining propositions to be sets of worlds.[9] Kaplan expresses the paradox in terms of the unsatisfiability of a certain intuitively possible sentence in a modal language L^+ with propositional variables and nonlogical sentence operators, viz.:

K $\forall p \Diamond \forall q (Qq \leftrightarrow q = p)$.

Intuitively, **K** asserts that there is a property Q of propositions such that, for every proposition p, it is possible that only p has Q. The problem is that, if (as on standard possible worlds semantics) there is a proposition for every set of worlds, there are (by Cantor's Theorem) not enough worlds to satisfy **K**.

Following the sketch in Davies (1981, 262), Lewis (1986, 104-5) expresses the paradox in more informal terms with, in particular, a definite example of the property Q. The premises (which we express a bit more generally) are as follows:

(8) There is a set W of all possible worlds.

(9) p is a proposition $=_{df}$ $p \in \wp(W)$.

(10) Let t be a given time. For every proposition p, there is the proposition q_p *that p is the only proposition entertained (by anyone) at t*.

(11) For every proposition p, the proposition q_p is possible.

[9] The 1995 publication date is much later than Kaplan's actual discovery of the paradox, which he had communicated to a number of philosophers in the late 1970s. See, e.g., Davies (1981, 262).

(12) A proposition p is possible if and only if there is a possible world w in which p is true, i.e., a world w such that $w \in p$.

As with the Russellian paradoxes, given some set theory, these premises are inconsistent. By (8), (10), and the axiom of Separation, let K be the set of all the propositions q_p, i.e., let $K = \{r \in \wp(W) : \exists p(p \in \wp(W) \wedge r = q_p)\}$. By (10)–(12), if $p \neq p'$, then $q_p \neq q_{p'}$. For suppose to the contrary that $p \neq p'$ but that $q_p = q_{p'}$. By (11), q_p is possible and, hence, by (12), q_p is true in some world w. So, by (10), it is true in w that p is the only proposition entertained at t. But $q_p = q_{p'}$, so $q_{p'}$ is also true in w, i.e., it is also true in w that p' is the only proposition entertained at t. Hence, it must be that $p = p'$, contradiction. So $p \neq p'$ implies $q_p \neq q_{p'}$, and so the mapping $f(p) = q_p$ from $\wp(W)$ into K is one-to-one.[10] Thus, the set of all propositions $\wp(W)$ is no larger than K.[11] But K, in turn, is no larger than W. For, by (11), every q_p is possible and, hence, by (12), nonempty. Moreover, the preceding reasoning shows that there is no world in which distinct members of K are true, that is, for all distinct p, p', $q_p \cap q_{p'} = \varnothing$. So K is a set of nonempty, pairwise disjoint sets. Accordingly, let c be a choice function on K. c obviously maps K one-to-one into W. Hence, K is no larger than W. It follows that $\wp(W)$ is no larger than W, once again in violation of Cantor's Theorem.[12]

2 Diagnosis and Solution

Our diagnosis of this problem is that (a) possible worlds should not be identified with sets of propositions, and (b) propositions should not be identified with sets of possible worlds. Rather, possible worlds and propositions should be analyzed without the use of notions from set theory. Indeed, our view is that set theory is not required for a correct theory of worlds or propositions. Though the model of propositions as sets of possible worlds and the model of possible worlds as maximal consistent

[10] $f \subseteq \wp(W) \times K$ and hence exists by the Powerset, Union, and Pairing axioms.

[11] Indeed, since K is a set of propositions, $K \subseteq \wp(W)$, so K is no larger than $\wp(W)$ and, hence, they are the same size.

[12] The foregoing speaks to the point Stalnaker raises in (1986), when he says that "Second, many people, including some distinguished mathematicians such as Dana Scott and Jon Barwise, have suggested that there is or may be a mathematical problem — a threat of paradox — at the foundation of possible worlds semantics. I have never seen a hard argument here." The above constitutes a hard argument.

sets of propositions have proved useful for a variety of purposes, these models are no substitute for theories of propositions or worlds. We plan to show, in what follows, that the formal theory of objects, propositions, and worlds described in Zalta 1983, 1993, and elsewhere, which is constructed without taking set-theoretic notions as basic, is immune to the paradoxes.

The following presentation of object theory is included only so that the paper is self-contained. Readers familiar with the basic theory may skip ahead to the next subsection.

2.1 Review of Object Theory

The theory of objects, propositions and worlds (henceforth 'object theory') that we shall be discussing is couched in a syntactically second-order modal language.[13] Thus, it uses primitive variables ranging over *objects* ($x, y, z, \ldots$) and primitive variables ranging over n-place *relations* ($F^n, G^n, H^n, \ldots$, for $n \geq 0$), where *properties* are identified as 1-place relations and *propositions* are identified as 0-place relations. A simultaneous definition of *term* and *formula* is given so that the language includes two atomic forms of predication ($F^n x_1 \ldots x_n$ and xF^1), complex predicates ($[\lambda x_1 \ldots x_n\, \varphi]$, for $n \geq 0$), and rigid definite descriptions ($\imath x\varphi$). The second atomic formula, xF^1 (hereafter xF) is to be read: *x encodes F*. These encoding formulas express a second mode of predication that has been motivated and applied in a variety of other publications. As we shall see, such formulas, and the axioms stated in terms of them, will make up for the loss of the notion of set membership and the axioms of set theory. For reasons to be discussed below, the complex predicates (λ-expressions) of the language, may not contain any encoding subformulas in φ. Finally, the language of object theory includes a distinguished predicate $E!x$ ('*x is concrete*').

Before we describe the sentences expressible in the above language that shall serve as the logical and proper axioms, it is important to lay out a few of the important definitions that use the language. One important distinction is between ordinary objects ($O!x$), defined as possibly concrete objects ($\Diamond E!x$), and abstract objects ($A!x$), defined as objects that couldn't possibly be concrete ($\neg\Diamond E!x$). Identity is not a primitive but rather defined: x and y are identical iff they are both ordinary objects and necessarily exemplify the same properties, or they are both abstract objects and necessarily encode the same properties. In other words, we may define:

Id $x = y =_{df} (O!x \wedge O!y \wedge \Box\forall F(Fx \leftrightarrow Fy) \vee (A!x \wedge A!y \wedge \Box\forall F(xF \leftrightarrow yF))$.

Moreover, identity is defined for properties, relations, and propositions as well. We present here only the definition of identity for properties and propositions, where F, G range over properties and p, q range over propositions:

Id₀ $p = q =_{df} [\lambda x\, p] = [\lambda x\, q]$

Id₁ $F = G =_{df} \Box\forall x(xF \leftrightarrow xG)$.

In other words, properties F and G are identical whenever they are necessarily encoded by the same objects, and propositions p and q are identical whenever the property of *being such that p* is identical to the property of *being such that q*. This reduces proposition identity to that of property identity. Identity for n-place relations ($n \geq 2$) is also reducible to identity for properties, but we shall not present the details here.[14] We annex to the above modal language a classical second-order S5 quantified modal logic, modified only so as to account for the presence of rigid definite descriptions.[15] Thus, both the first- and second-order Barcan formulas are theorems. In addition to the usual axioms of S5 quantified modal logic, objct theory includes an modal axiom governing the logic of encoding:

Enc $\Diamond xF \rightarrow \Box xF$.

This guarantees that encoded properties are rigidly encoded; the properties that an abstract object encodes are not relative to any circumstance. Though the above definitions allow us to prove $\alpha = \alpha$, for any object variable or relation variable α, object theory takes the traditional principle of

[13] We say 'syntactically second-order' because the theory doesn't require full second-order logic. Although we won't pursue the matter here, it should suffice to mention that the theory can be interpreted using *general* models, in which the domain of properties is *not* the full power set of the domain of individuals.

[14] The definition of relations asserts that F^n and G^n are identical ($n \geq 2$) iff all of the pairwise 1-place relational properties that result from the various ways of 'plugging' $n-1$ arbitrarily chosen objects into F^n and G^n are identical.

[15] The usual Russell axiom governing definite descriptions becomes a contingent logical truth when the descriptions are interpreted rigidly, and so the Rule of Necessitation must be restricted: it may not be applied to any line that depends on the Russell axiom.

the substitution of identicals as an axiom. Finally, the usual logical axioms governing λ-expressions and rigid definite descriptions apply. We review here only the λ-conversion principle:[16]

LC $[\lambda y_1 \ldots y_n\, \varphi]x_1 \ldots x_n \leftrightarrow \varphi^{x_1,\ldots,x_n}_{y_1,\ldots,y_n}$.

This principle yields comprehension principles for properties, relations, and propositions.[17] Note that we have now identified the most important principles of our theory of properties, relations and propositions, namely, their existence and identity conditions. This theory asserts the existence of a wide variety of complex properties, relations, and propositions, including complex propositional properties of the form $[\lambda y\, p]$ (*being such that p*) and complex propositions of the form $[\lambda\, p]$ (*'that p'*), where p is a proposition. These latter are governed by special instances of the 0-place case of λ-conversion, namely, $[\lambda\, p] \leftrightarrow p$, which assert: the proposition that-p is true iff p.

The two proper axioms of object theory are:

O! $O!x \rightarrow \Box\neg\exists F\, xF$

OC $\exists x(A!x \wedge \forall F(xF \leftrightarrow \varphi))$, where x is not free in φ.

The first tells us that ordinary objects necessarily fail to encode properties. The second asserts: for every condition on properties, there is an abstract individual that encodes just the properties satisfying the condition. This second axiom is a comprehension principle that tells us the conditions under which abstract objects exist. Given the principle of identity

[16]The other usual axioms for λ-expressions will be assumed: namely that interchange of bound variable makes no difference to the denotation of the λ-expression ($[\lambda y_1 \ldots y_n\, \varphi] = [\lambda y'_1 \ldots y'_n\, \varphi']$) and elementary λ-expressions denote the *governing* relation ($[\lambda y_1 \ldots y_n\, F^n y_1 \ldots y_n] = F^n$).

[17]For example, after n applications of universal generalization, an application of the rule of necessitation, and an application of existential generalization, we derive the following comprehension principle for relations:

$\exists F^n \Box \forall x_1 \ldots \forall x_n (F^n x_1 \ldots x_n \leftrightarrow \varphi)$, where φ has no free occurrences of F^n and no encoding subformulas

As special cases, we have comprehension for properties and propositions:

$\exists F \Box \forall x (Fx \leftrightarrow \varphi)$, where φ has no free occurrences of F and no encoding subformulas

$\exists p \Box (p \leftrightarrow \varphi)$, where φ has no free occurrences of p and no encoding subformulas

for objects described earlier, we can derive a strengthened version of this principle as a theorem:

(13) $\exists! x(A!x \wedge \forall F(xF \leftrightarrow \varphi))$, where x is not free in φ.

That is, for any condition φ, there is a *unique* abstract object that encodes exactly the properties satisfying φ. There couldn't be two distinct abstract objects encoding exactly the properties satisfying φ since distinct abstract objects must differ by one of their encoded properties.

This theorem yields a class of canonical descriptions of the form:

(14) $\iota x(A!x \wedge \forall F(xF \leftrightarrow \varphi))$, where x is not free in φ.

Object theory guarantees that these descriptions have denotations and so are well-defined.[18]

2.2 How Object Theory Avoids Classical Paradoxes

It is important for our discussion in the final section of the paper to appreciate why restrictions were placed on the formation of λ-expressions, restrictions that were subsequently inherited in the derived principles of relation comprehension. Recall that λ-expressions and instances of relation comprehension may not be formed when the matrix φ has encoding subformulas. The reason for this restriction concerns a Russell-

[18]For those encountering object theory for the first time, some examples may help. The following identifications of abstract objects involving such descriptions are grounded by the above principles. Let b denote Alexander. Then the individual concept of Alexander c_b is defined as:

$c_b =_{df} \iota x(A!x \wedge \forall F(xF \leftrightarrow Fb))$

(see Zalta 2000a). Let T denote the property of *being a triangle*. Then the Form of the Triangle (Φ_T) is defined as:

$\Phi_T =_{df} \iota x(A!x \wedge \forall F(xF \leftrightarrow \Box\forall y(Ty \rightarrow Fy)))$

(see Pelletier & Zalta 2000). Consider any property G. Then the extension of the property G (ϵG) is defined as:

$\epsilon G =_{df} \iota x(A!x \wedge \forall F(xF \leftrightarrow \forall y(Fy \leftrightarrow Gy)))$

(see Anderson & Zalta 2004). Let $G \approx_E F$ abbreviate the claim that there is a one-one correspondence between the ordinary objects exemplifying G and exemplifying F. Then, the natural cardinal of G (#G) is defined as:

$\#G =_{df} \iota x(A!x \wedge \forall F(xF \leftrightarrow F \approx_E G))$

(see Zalta 1999). These examples of objects demonstrate a variety of ways in which object theory has been applied.

style paradox (the "Clark paradox") that arises in the foundations of object theory.[19] This paradox is unrelated to the Russell-Kaplan paradoxes with which we began our paper, and so it would serve well to rehearse it briefly here.

Suppose we could form the λ-expression '$[\lambda x\, \exists G(xG \wedge \neg Gx)]$' expressing, intuitively, the property *being an object that encodes a property it does not exemplify*. Then we could then formulate the following instance of the comprehension principle for abstract objects:

$$\textbf{C}\quad \exists z(A!z \wedge \forall F(zF \leftrightarrow \forall y(Fy \leftrightarrow [\lambda x\, \exists G(xG \wedge \neg Gx)]y))).$$

With a little bit of reasoning, a contradiction follows.[20] So by banishing encoding formulas from the formation of λ-expressions and relation comprehension, we forestall the Clark paradox.

A related paradox was discovered by Alan McMichael.[21] Suppose that identity ('=') were taken as a primitive. Then one could formulate the λ-expression '$[\lambda y\, y = z]$' expressing, intuitively, the property *being identical with z*. Call such a property a 'haecceity of z'. If such λ-expressions were legitimate, then so would be the following instance of comprehension for abstract objects:

$$\textbf{M}\quad \exists x \forall F(xF \leftrightarrow \exists z(F = [\lambda y\, y = z] \wedge \neg zF)).$$

M asserts the existence of an object that encodes exactly those haecceities that are not encoded by their instances. But as with **C**, from **M** a contradiction quickly ensues.[22]

[19]This paradox was first reported in the literature by Clark (1978, 184), rehearsed in Rapaport (1978), and formulated more precisely in object theory in Zalta (1983). It was developed independently in Boolos (1987, 17).

[20]Let K be the property $[\lambda x\, \exists G(xG \wedge \neg Gx)]$ and let b be a witness to the existential claim **C**. Then b encodes all and only the properties F which are materially equivalent to K. Either Kb or $\neg Kb$. Suppose the former. Then by λ-conversion, $\exists G(bG \wedge \neg Gb)$, i.e., there is some property, say Q, such that bQ and $\neg Qb$. But from the former, it follows that $\forall y(Qy \leftrightarrow Ky)$, by definition of b. But from $\neg Qb$, it then follows that $\neg Kb$, contrary to hypothesis. So suppose $\neg Kb$. Then by λ-conversion once again it follows that $\forall G(bG \rightarrow Gb)$, and in particular, $bK \rightarrow Kb$. But, by the definition of b, we know that $bK \leftrightarrow \forall y(Ky \leftrightarrow Ky)$. Since the right-hand side of the biconditional is derivable from logic alone, it follows that bK. Hence, Kb. Contradiction.

[21]This paradox was first reported in McMichael and Zalta (1980). It was discovered independently and reported in Boolos (1987, 17).

[22]Let b be a witness to this existential claim **M**. By definition of b, we know:

$$\textbf{M}'\quad \forall F(bF \leftrightarrow \exists z(F = [\lambda y\, y = z] \wedge \neg zF))$$

Now, the same restriction provided earlier in the context of the Clark paradox, namely, the banishment of encoding subformulas from λ-expressions and comprehension, also solves the McMichael paradox. For in object theory, in the above definition of object identity, the defined notation '$x = y$' is given in terms of a definiens containing encoding subformulas. Thus, $[\lambda xy\, x = y]$, $[\lambda y\, y = z]$, and $[\lambda y\, z = y]$ are all ill-formed, as are the corresponding instances of comprehension.[23] The paradox does not get off the ground.[24]

The fact that banishing encoding formulas from λ-expressions avoids the Clark paradox and the McMichael paradox provides a strong, practical justification for the proscription. However, we believe there are powerful *theoretical* justifications for the move as well.

Now consider the property $[\lambda y\, y = b]$ and suppose $b[\lambda y\, y = b]$. Then by $\textbf{M}'$, it follows that $\exists z([\lambda y\, y = b] = [\lambda y\, y = z] \wedge \neg z[\lambda y\, y = b])$. Call such an object c. So, $[\lambda y\, y = b] = [\lambda y\, y = c] \wedge \neg c[\lambda y\, y = b]$. Note independently that $b = b$ by the laws of identity, from which it follows by λ-conversion that $[\lambda y\, y = b]b$. Since $[\lambda y\, y = b] = [\lambda y\, y = c]$, it follows that $[\lambda y\, y = c]b$. So by λ-conversion, it follows that $b = c$. But since $\neg c[\lambda y\, y = b]$, it follows that $\neg b[\lambda y\, y = b]$, contrary to hypothesis. So suppose instead $\neg b[\lambda y\, y = b]$. Then, by $\textbf{M}'$, it follows that $\neg \exists z([\lambda y\, y = b] = [\lambda y\, y = z] \wedge \neg z[\lambda y\, y = b])$, i.e., $\forall z([\lambda y\, y = b] = [\lambda y\, y = z] \rightarrow z[\lambda y\, y = b])$. By instantiating the universal claim to b, we get $[\lambda y\, y = b] = [\lambda y\, y = b] \rightarrow b[\lambda y\, y = b]$. And since the antecedent is true by the laws of identity, it follows that $b[\lambda y\, y = b]$. Contradiction.

[23]Note, however, that the classical, Leibnizian definition of indiscernibility can still be treated as a *bona fide* relation. For the λ-expression '$[\lambda xy\, \forall F(Fx \leftrightarrow Fy)]$' is well-formed. Moreover, in previous work in object theory, indiscernibility plays a role in the definiens of the notion of identity for ordinary objects, $x =_E y$, which is defined as: $O!x \wedge O!y \wedge \Box\forall F(Fx \leftrightarrow Fy)$. Thus, the λ-expression '$[\lambda xy\, x =_E y]$' is also well-formed. It denotes a relation that is well-behaved on the ordinary objects — it is provably an equivalence relation on the ordinary objects. So object theory does allow for a relation of identity, but as with all other relations, one can prove that there are abstract objects that are indiscernible with respect to $x =_E y$, as explained in the next footnote.

[24]It is interesting note that object theory yields the theorem that some distinct abstract objects can't be distinguished by the traditional notion of exemplification. It is provable that for any relation R, there are at least two distinct abstract objects a, b such that $[\lambda x\, Rxa] = [\lambda x\, Rxb]$. Consider the following instance of comprehension:

$$\exists x(A!x \wedge \forall F(xF \leftrightarrow \exists y(F = [\lambda z\, Rzy] \wedge \neg yF)))$$

Call such an object k. From the assumption that k doesn't encode the property $[\lambda z\, Rzk]$, one can prove that k does encode that property. Then, from the definition of k, the fact that k does encode this property yields that there is a distinct object, j, such that $[\lambda z\, Rzk] = [\lambda z\, Rzj]$.

Now from this result, by letting R be $[\lambda xy\, \forall F(Fx \leftrightarrow Fy)]$, one can prove that there are distinct abstract objects a, b such that $\forall F(Fa \leftrightarrow Fb)$. This establishes that there are too many abstract objects for the traditional notion of exemplification to distinguish. Readers who wish to see the reasoning spelled out in detail should consult Zalta (1999, 626 and footnote 16).

First, the fact that proscribing the occurrence of encoding subformulas provides a *unified* solution to both the Clark and the McMichael paradoxes itself offers theoretical warrant. Object theory thereby helps to illuminate and unify two paradoxes that may otherwise seem to be importantly different. The paradoxes differ in their formulation but are fundamentally similar in their solution. And by categorizing paradoxes in terms of the way in which they can be solved, object theory offers an understanding of what these paradoxes challenge.

Second, a solution to a theoretical problem accrues some theoretical justification if it has no detrimental impact on the power and applicability of the theory. An illustrative case is set theory itself. With set theory beset by paradox, Zermelo (1908) endeavored to axiomatize the theory in a way that preserved its already impressive array of extraordinary results (see, e.g., Kanamori 1996, 2004). Although it would be some twenty years before it was sufficiently understood how the fine structure of the cumulative hierarchy explained the effectiveness of the restrictions on naive comprehension that Zermelo had introduced in the axiom schema of Separation, the power of the theory even with those restrictions provided some warrant for believing that they reflected deeper structural facts about sets. The proscription on encoding subformulas in object theory is analogous — it avoids the paradoxes without compromising the breadth and power of the theory.

However, this analogy is not perfect, as we believe that we can already identify a deeper structural justification for the proscription on encoding subformulas in λ-predicates. We begin with an informal observation about the basic philosophical idea of object theory, namely, that there is a domain of abstract objects that encode properties with which we are already familiar. The underlying intention is simply to assert the existence of new *objects* that are *constituted* by familiar properties; it is not to assert the existence of new and unfamiliar "encoding properties" — properties that, if anything, emerge solely as artifacts of the increased expressive power that encoding predication brings to the language. On this basis alone one is led to disallow encoding formulas from λ-expressions and comprehension.

However, there is an even deeper theoretical justification underlying this basic idea. Object theory introduces a special type of connection — encoding — between properties and objects of a special sort. This connection is indeed often considered akin to exemplification — most fruitfully, perhaps, in applications where a philosophical problem can be solved by appealing to an ambiguity in the copula between encoding and exemplification.[25] Parallels aside, however, encoding is completely distinct from exemplification. Encoding is a unique connection between objects (of a special sort) and properties (not n-place relations generally) whose theoretical role is to provide a mechanism whereby properties satisfying certain conditions are unified into a single conceptual object. There is no independent reason to think this metaphysical connection plays a role in the logical structure of fine-grained relations. There is, in fact, reason to think it does not.

On object theory's fine-grained conception of relations, to *be* a relation is to be either a structurally simple n-place universal or to be built up from such by means a series of natural logical operations: predication, negation, conjunction, quantification, etc. The effect of these operations is reflected in the syntactic structure of λ-predicates (though not necessarily exactly reflected). Structurally simple n-place relations correspond to primitive n-place predicates such as 'R'. Now the relation R that this predicate expresses just *is* the property *exemplifying* R (or *standing in* R) and this is reflected in an object theoretic axiom that governs elementary λ-expressions, namely, $[\lambda x_1...x_n\, Rx_1...x_n] = R$. The argument places in the predicate reveal, as Frege (1891) put it, the unsaturated (*ungesättigt*) or "gappy" nature of relations, which indicates their predicability of any n or fewer objects. No separate operation is needed for a property to be suitably "prepared" for combining logically with objects or other relations: Given F, i.e., $[\lambda x\, Fx]$, we have its negation $[\lambda x\, \neg Fx]$; given $[\lambda y\, Gy]$ we then have the conjunctive relation $[\lambda xy\, \neg Fx \wedge Gy]$; and given an object b, a (2-place) predication operation yields the proposition $[\lambda\, \neg Fb \wedge Gb]$ (the proposition that b is not F but G); and so on.

By contrast, by virtue of what would an encoding property be suitably prepared to combine logically in the same sort of way with objects and other relations? In order to provide a general semantics for λ-predicates containing encoding formulas involving free variables, one must initially have, for properties F, atomic encoding properties $[\lambda x\, xF]$, i.e., properties of the form *encoding* F.[26] But unlike F itself, as $[\lambda x\, xF] \neq$

[25]For example, in the analysis of fiction, object theory analyzes ordinary natural language claim such as "Holmes is a detective" as ambiguous between an exemplification reading (Dh), which is false, and an encoding reading (hD), which is true. See Zalta 2000b.

[26]Thus, in particular, if there *were* such a property as *encoding* F, $[\lambda x\, xF]$, the predication

F, $[\lambda x\, xF]$ must be a complex property; that is, its existence must be due to some sort of transformation on F — specifically, a primitive transformation that takes F as input and yields the encoding property $[\lambda x\, xF]$ as output. But unlike all the other operations that yield complex properties, the proposed transformation is *not* a logical operation; unlike predication, negation, conjunction, etc, it is a purely metaphysical transformation corresponding to no logical intuition whatsoever.

To be clear, then, what is being denied here is that there is a logical operation that *transforms* the property F — that is, the property *exemplifying* F — into the property *encoding* F, and it is that transformation, we claim, that corresponds to no natural logical operation. There is, therefore, simply no philosophical warrant for postulating such a transformation and, hence, no syntactic warrant for permitting encoding formulas to occur in the λ-predicates of object theory. Indeed, the reflections here suggest that permitting them so to occur, and thereby enabling encoding to bleed into the logical structure of relations, would be a type of category mistake.

Far from an *ad hoc* maneuver introduced to avoid paradox, then, the proscription on encoding subformulas in λ-predicates is independently and antecedently motivated by object theory's account of fine-grained relations.

2.3 How Object Theory Solves the Russell-Kaplan Paradoxes

To show that object theory is immune to the paradoxes with which we began, we have to review its theory of possible worlds. In previous work (Zalta 1993), worlds are defined as situations, where these in turn are defined as abstract objects that encode only propositional properties:

Sit $Situation(x) =_{df} A!x \wedge \forall F(xF \rightarrow \exists p(F = [\lambda y\, p]))$.

If we let s range over situations, then to say that *p is true in s* ('$s \models p$') is to say that s encodes the propositional property *being such that p*:

TI $s \models p =_{df} s[\lambda y\, p]$.

operation applied to that property and an object b would reasonably be thought to return a proposition, viz., *that b encodes F*.

Finally, possible worlds are defined as situations s that might be such that all and only true propositions are true in s:[27]

PW $PossibleWorld(s) =_{df} \Diamond\forall p(s \models p \leftrightarrow p)$.

In what follows, when s is a world w, we read $w \models p$ as p is *true at* w.

These definitions, plus the definitions of *maximal*, *consistent*, *actual*, etc., allow one to derive the basic principles of world theory as theorems (Zalta 1983, 1993). It is provable that worlds are maximal, consistent, modally-closed, that there is a unique actual world *at* which all and only the truths are true, that a proposition is necessarily true iff true at all possible worlds, and that a proposition is possible iff it is true at some world. The latter theorem constitutes a conditional existence principle for worlds: whenever we add to the theory a proposition p such that $\Diamond p$ and $\neg p$, the principle ensures that there exists a possible world distinct from the actual world at which p is true.[28] Indeed, the definition of identity here is crucial to this latter fact, for it is provable that if there is a proposition, say p, that is true at w and not at w', then $w \neq w'$.[29]

Moreover, in contrast to those that would model worlds as sets of propositions, the worlds of object theory are not *ersatz* worlds. The propositions true at a world are the ones they encode. But encoding is a mode of predication, and as such, the predicate in encoding formulas *characterizes* the subject. Given that object theory treats both encoding and exemplification forms of predication as ways of disambiguating the copula 'is', the definiens of 'p is true in w' (namely, $w[\lambda y\, p]$), can be understood as asserting that *w is such that p*. So the propositions true at a world do in fact *characterize* those worlds.

Given this analysis of possible worlds, and the underlying theory of propositions, we can see why object theory is immune to the Russell paradox. First, object theory doesn't employ any set theory in its theory of propositions. In particular, it is not committed to the existence of

[27]In the definiens of the following definition, the symbol $\leftrightarrow$ dominates $\models$. So the definiens is to be parsed $\forall p((s \models p) \leftrightarrow p)$, not as $\forall p(s \models (p \leftrightarrow p))$.

[28]See Menzel and Zalta (under review) for a study of the smallest models that are required to make this principle, and the axioms used to derive it, true.

[29]Suppose $w \models p$ and $w' \not\models p$ (to show that $w \neq w'$). Then by the definition of $\models$, w encodes a property, namely, $[\lambda y\, p]$, that w' doesn't encode. Since worlds are situations, they are both abstract. So by the definition of identity for abstract objects, w and w' are distinct.

maximal consistent *sets* of propositions.[30] So neither version of the Russell paradox can even get started. Second, object theory is not committed to either of the following claims:

- There is a set of all possible worlds.
- Propositions are sets of worlds.

So, it is not committed to the idea that there is a power set of the set of possible worlds or that this power set constitutes the set of all propositions. So one can't, *per impossibile*, correlate each element of the power set of possible worlds one-to-one with a subset of the set of possible worlds. Thus, the Kaplan paradox falls by the wayside.

By (a) not having primitive sets, primitive worlds, and primitive sets of worlds, and (b) not reconstructing propositions as sets of worlds, object theory is immune to the reasoning in the Russell and Kaplan paradoxes. Propositions are instead axiomatized, as a subtheory of the theory of relations, and worlds are defined as abstract objects. No paradoxical set-theoretic correlations between worlds and propositions can be established.

[30] One might worry about the set theory that is used in the metatheory for object theory. Does the paradox arise in the metatheory for object theory? No, it doesn't. The metatheory uses only the minimal amount of set theory needed to define satisfaction and truth. The metatheory does postulate a domain of propositions, but object theory doesn't require that in this domain there are *sets* of propositions. If nothing requires that there are sets of propositions, then nothing requires the existence of a maximal consistent set of propositions. *A fortiori*, there is no one-to-one correlation between a power set of any maximal consistent set of propositions and one of its subsets. Thus, the paradoxical sets are not required to be in the metatheory. Indeed, even if one were to suppose that in the metatheory, worlds are correlated with maximal, consistent sets of propositions, nothing in object theory correlates the power set of maximal consistent sets of propositions with subsets of those sets. Finally, the object theorist can put this worry to rest by pointing out that object theory is not committed to *any* of the set theory used in the metatheory. The set theory used in the metatheory of object theory has only one function, namely, to establish that object theory is consistent relative to theories like ZF which are thought to be consistent. Thus, set-theoretic models establishing the consistency of object theory have been formulated: a model by Dana Scott was presented in Zalta 1983; a model by Peter Aczel was described in Zalta 1999, and another model was developed in Menzel & Zalta (under review).

3 A Kaplan Paradox for Object Theory?

In conversation, however, Kit Fine has suggested that a Kaplan-style paradox can be formulated in object theory.[31] We will present the paradox and examine whether object theory has the resources to block it in a principled way.

3.1 Fine's Paradox

Let 'Ep' express the claim that p is entertained, and let the claim that proposition *p is uniquely entertained* be defined as follows:

UE $UE(p) =_{df} Ep \wedge \forall q(Eq \rightarrow q = p)$.

Fine then suggests that a paradox arises if one endorses the principle:

F $\forall p \Diamond UE(p)$.

F asserts that, for every proposition p, it is possible that p is uniquely entertained. The principle is not implausible — intuitively, for any given proposition, it seems that there should be a world with, say, a single highly-focused agent who entertains only that proposition. It should therefore prove embarrassing for any theory wishing to preserve a robust and intuitive sense of possibility if the principle must for any reason be denied, especially as a matter of *logic*. But this is precisely the specter Fine has raised for certain theories meeting rather minimal conditions.

Define the proposition q_0 as follows:

(15) $q_0 = [\lambda\, \exists p(UE(p) \wedge \neg p)]$.

So q_0 is the proposition that there exists a false proposition p that is uniquely entertained. But, so long as one's language permits (15), from **F** a contradiction quickly ensues assuming only basic predicate logic with identity, λ-conversion, and the principle of Necessitation. The argument in a nutshell is this: The proposition

(16) $\neg UE(q_0)$

is a theorem of this basic logic. By Necessitation, so is $\Box\neg UE(q_0)$, i.e., $\neg\Diamond UE(q_0)$. But the latter proposition is obviously inconsistent with **F**.

[31] Personal communication, at the Australasian Association of Philosophy Conference in Melbourne, Australia, in July 2009.

To show that (16) is indeed a theorem, let us note first that, by the definition (15) of q_0 and λ-conversion, we have

(17) $q_0 \leftrightarrow \exists p(UE(p) \wedge \neg p)$.

Now, assume $UE(q_0)$. Then, by unpacking the definition **UE** of the predicate UE, we have:

(18) $Eq_0 \wedge \forall q(Eq \rightarrow q = q_0)$.

Either q_0 or $\neg q_0$. Suppose the former. Then from (17) it follows that $\exists p(UE(p) \wedge \neg p)$. Let s be such a p; so we have

(19) $UE(s) \wedge \neg s$.

From this, by unpacking $UE(s)$, we have $\forall q(Eq \rightarrow q = s)$ and so, in particular, $Eq_0 \rightarrow q_0 = s$. But from (18) we have Eq_0 and, hence, $q_0 = s$. But by (19) once again, $\neg s$. Hence, $\neg q_0$, contradicting our assumption that q_0.

So suppose instead that $\neg q_0$. By (17), we have $\forall p(UE(p) \rightarrow p)$. But we assumed at the outset that $UE(q_0)$, so it follows in particular that q_0, contradicting our assumption $\neg q_0$. Either way, we get a contradiction from the assumption that $UE(q_0)$. So our theorem (16), and with it the logical falsity of **F**, is established.

Call this *Fine's Paradox*. Note, however, that this paradox is *not* formulable in object theory. It is blocked because the λ-expression used in (15) is ill-formed. As noted in the previous section, in object theory, identity for propositions is a defined notion, and when a formula involving identity is expressed in terms of the primitive notation, we find encoding formulas. Thus, in particular, the definition **UE** of the predicate 'UE' unpacks as:

(20) $UE(q) =_{df} Eq \wedge \forall r(Er \rightarrow [\lambda y\, r] = [\lambda y\, q])$,

where, by $\mathbf{Id_0}$,

(21) $[\lambda y\, r] = [\lambda y\, q] =_{df} \Box\forall x(x[\lambda y\, r] \leftrightarrow x[\lambda y\, q])$.

However, formulas with encoding subformulas are banished from λ-predicates, and so $UE(p)$ may not appear in λ-expressions. Thus, precisely the same considerations that prevent the usual Russell-style paradoxes from emerging in object theory also prevent Fine's paradox from getting off the ground.

3.2 Fine's Revised Paradox

End of the story? Not quite. Upon taking note of this response, Fine suggested that one should redefine 'UE' as follows:

UE* $UE(p) =_{df} Ep \wedge \forall q(Eq \rightarrow \Box(q \leftrightarrow p))$

Under **UE***, '$UE(p)$' now expresses, as we might put it, that p is uniquely entertained *up to necessary equivalence* (similar to the way that the standard model of Peano Arithmetic is unique up to isomorphism). Accordingly, let us now say that '$UE(p)$' expresses that p is *uniquely* entertained*, for short. Under **UE*** rather than **UE**, the definition (15) of q_0 is legitimate in object theory. So we may instantiate q_0 into **F** to get $\Diamond UE(q_0)$. But now, by reasoning more or less as above, albeit with a bit more modal logic, from **F** we can derive a contradiction.[32]

Call this Fine's *Revised Paradox*.

4 Solution to Fine's Revised Paradox

By defining the 'UE' predicate as in **UE*** and understanding **F** accordingly, Fine's revised paradox avoids the use of a grammatically unacceptable λ-predicate. However, understood in terms of **UE***, **F** is objectionable because in object theory, propositions (and, indeed, n-place relations generally) are fine-grained intensional entities.

Two facts about object theory's conception of fine-grainedness should be clear from the overview of object theory in §2.1 and the discussion of the logical structure of relations in §2.2: first, that properties, relations and propositions are not to be identified with sets and truth-values, nor with functions from worlds to sets and functions from worlds to truth-values, respectively; and second, that properties, relations and proposi-

[32]The proof is identical through (17) and continues as follows: Now, assume $UE(q_0)$. By unpacking the definition **UE*** of 'UE' instead of **UE**, we have

(18′) $Eq_0 \wedge \forall q(Eq \rightarrow \Box(q \leftrightarrow q_0))$

instead of (18). Now, either q_0 or $\neg q_0$. Suppose the former. Then from (17) it follows that $\exists p(UE(p) \wedge \neg p)$. Let s be such a p; so again we have (19). By unpacking $UE(s)$ (according to **UE***), from (19) it follows that $\forall q(Eq \rightarrow \Box(q \leftrightarrow s))$ and so, in particular, $Eq_0 \rightarrow \Box(q_0 \leftrightarrow s)$. But from (18′) we have Eq_0 and, hence, $\Box(q_0 \leftrightarrow s)$ and so, by the T-schema ($\Box\varphi \rightarrow \varphi$), we have $q_0 \leftrightarrow s$. But by (19) once again, $\neg s$. Hence, $\neg q_0$, contradicting our assumption that q_0. The proof now continues and concludes as in the original paradox.

tons are not, as a matter of logic, to be considered identical when they are necessarily equivalent.

To expand on the latter point, recall from §2.1 ($\mathbf{Id_0}$ and $\mathbf{Id_1}$) that in object theory $p = q$ is defined as $[\lambda x\, p] = [\lambda x\, q]$ and $F = G$ is defined as $\Box\forall x(xF \leftrightarrow xG)$. So we may consistently assert:

(22) $\Box(p \leftrightarrow q) \wedge p \neq q$

(23) $\Box\forall x(Fx \leftrightarrow Gx) \wedge F \neq G$.

For example, though the properties of *being red and not red* ($[\lambda x\, Rx \wedge \neg Rx]$) and *being blue and not blue* ($[\lambda x\, Bx \wedge \neg Bx]$) are provably necessarily equivalent, we may assert that they are distinct. Similarly, for distinct propositions p and q, the necessary equivalence of $[\lambda\, p \wedge \neg p]$ and $[\lambda\, q \wedge \neg q]$ is provable, but we may consistently assert that they are distinct. So properties and propositions are, respectively, more fine-grained than functions from possible worlds to sets of individuals and functions from possible worlds to truth-values.[33]

[33]Note that the definitions of identity for properties and propositions do preserve a sense in which properties and propositions are *extensional* entities: our definitions stipulate that F and G are identical whenever they are necessarily encoded by the same objects. (If we allow ourselves to think model-theoretically about this, then this definition requires that F and G be identified whenever they have the same 'encoding extension' at every possible world, without requiring that they be identified when they have the same exemplification extension at each possible world.) Similarly, p and q are identical whenever the properties $[\lambda x\, p]$ and $[\lambda x\, q]$ are necessarily encoded by the same objects. (Since n-place relation identity, $n \geq 2$, is also defined in terms of property identity, there is also a sense in which they are extensional. But we omit discussion of the general case.) This sense in which properties and propositions are extensional entities should address the concerns of philosophers who, like Quine, regard them as creatures of darkness. But, for the purposes of the present discussion, we are focusing on the fact that they are, respectively, more fine-grained than functions from possible worlds to sets of individuals and functions from possible worlds to truth-values.

Finally, it is interesting to note that the definitions of property and proposition identity in object theory don't *require* that these entities have a logical structure indicated by the syntactic structure of the predicates and formulas that denote them. For example, one may consistently assert that the fine-grained propositions $[\lambda\, p \wedge q]$ and $[\lambda\, q \wedge p]$ are identical. One may add such auxiliary hypotheses if one is convinced there is good reason. In general, if one is convinced that there couldn't possibly be a pure object of thought that encodes property P without thereby encoding property Q, one may consistently add the claim that the two properties in question are identical. So although we might allow that the two necessarily empty properties *being red and not red* and *being blue and not blue* are distinct, we may neverthess assert the identity of *being red and not red* and *being not red and red*. In this latter case, the syntactic structure doesn't strictly indicate the logical structure of the property.

Now, given that such a conception yields properties and propositions that are more fine-grained than conceptions that collapse necessarily equivalent entities, it is logically impossible that every proposition be uniquely* entertained; it is logically impossible that every proposition is such that it and (zero or more) propositions logically equivalent to it are the only propositions that are entertained. Consider, for example, any conjunctive proposition $p = [\lambda\, q \wedge r]$, where q and r are contingent and logically independent. Given these assumptions, p is logically equivalent to neither q nor r. However, it seems quite reasonable to suggest that it is not possible to entertain p without simultaneously entertaining both q and r; for p just *is* the proposition *that q and r*. It follows that $\neg\Diamond UE(p)$ and, hence, that **F** is false. Fine's paradox can only get off the ground in a setting that, as in possible world semantics, takes propositions to be logically unstructured and, hence, a setting wherein the idea of a logically complex proposition and its structurally simpler parts has no significant purchase.

Being entertained is meant to be a *prima facie* plausible example of a particular property Q such that, for any proposition p, it is possible that only p, and perhaps some propositions logically equivalent to p, have Q. The *logic* of Fine's paradox, however, depends only on the existence of *some* such property. The paradox, that is, does not depend on any particular "intended" interpretation of the predicate 'E' in **UE***. One might, therefore, object that, insofar as we only address one particular interpretation, our answer to the paradox is not sufficiently general. However, it is very difficult to imagine any other interpretation of 'E' would not be subject to essentially the same objection. For it seems that the possible candidates would involve intentionality, on the ability of an (ideal) agent to selectively adopt some sort of intentional stance toward propositions. Insofar as this stance can *only* be toward logically equivalent propositions if we are to validate **F**, it seems that any purported example of a property satisfying **F** will be subject to the same sort of objection as the one given in the preceding paragraph, viz., that there will always be fine-grained propositions that cannot possibly have the property in question. We therefore conclude that, on a fine-grained conception of propositions, there are no properties that render **F** plausible.

In sum, then: Object theory is subject to a Kaplan-style paradox only if there is some good reason for thinking object theory should be committed to **F**, the proposition $\forall p\Diamond UEp$ (under some interpretation of 'E').

There is, however, no such reason. When formulated in terms of **UE**, **F** is rejected in object theory due to the use of identity in the '*UE*' predicate, thereby violating the proscription on the occurrence of encoding subformulas in λ-predicates justified in §2.2. When formulated in terms of **UE***, **F** is similarly rejected given that, in object theory, propositions are fine-grained intensional entities. In both cases, the paradox is then blocked in a principled way.

One final point of significance can now be mentioned. While other theories of fine-grained propositions in the literature could of course offer this same response to Fine's revised paradox,[34] the question arises: how do they block Fine's *original* paradox? For that paradox (as developed in Section 3.1) arises for any fine-grained theory of propositions that (i) accepts **F**, (ii) allows for the formulation of the proposition q_0, (iii) includes a notion of proposition identity and substitution of identicals, and (iv) includes full λ-conversion. The distinguishing feature of object theory's conception of fine-grained propositions is that it includes a *theory* of identity for propositions that blocks Fine's original paradox by rejecting (ii) on principled grounds.

5 Conclusion

The examination of the various paradoxes above indicates that object theory offers a framework in which worlds and propositions can be properly characterized without invoking sets. First, existence and identity conditions for worlds and propositions are explicitly formulated. Second, object theory (through its theorems) captures the relevant prior data about worlds and propositions. With regard to worlds, as noted above, the central principles governing possible worlds are derivable as theorems of object theory. With regard to propositions, object theory properly represents the data of how we can believe a necessarily true proposition p without it validly following that for every other necessarily true proposition q, we believe q as well. And all of this is done in a framework in which the paradoxes that plagued earlier theories of worlds and propositions don't arise.

The pernicious effects of hitching one's modal wagon to set theory should now be clear. Without invoking sets, object theory offers an integrated account of worlds and propositions that assigns them clear existence and identity conditions and captures a broad array of modal and intensional intuitions. In the end, propositions and worlds can be set free.

[34] See Bealer (1982), Salmon (1986), Menzel (1986b, 1993), and King (1996).

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