Unifying Three Notions of Concepts*

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There isn't universal agreement as to what 'concept' means. Indeed, the present discussion might seem circular, since it may come across as an investigation into the concept of 'concept' and so presupposes an understanding of the technical term *concept*. But I'll try to avoid circularity in what follows by considering how certain primitive entities, or entities known under a different name, could reasonably be called 'concepts'.

I shall assume that concepts, whatever they are, (can be used to) *characterize* entities in some way. In particular, I'll discuss three notions of *concept*: Fregean *Begriffe*, the primitive concepts of Leibniz's 'calculus', and Fregean senses, all of which characterize entities. Though these kinds of concepts characterize objects in somewhat different ways, they can all be unified within object theory (Zalta 1983, 1988). For the purposes of this paper, I have to assume familiarity with that theory and, in particular, with its distinction between exemplification predication ($F^n x_1 \ldots x_n$) and encoding predication (xF).[1]

 forthcoming in *Theoria* (Special Issue: Context, Cognition and Communication), Tadeusz Ciecierski and Paweł Grabarczyk, guest editors. This paper comprises my keynote presentation at the 2nd Context, Cognition and Communication Conference ("Contexts, Concepts, and Objects"), held at the University of Warsaw, June 16–20, 2019. The goal of the presentation was to introduce object theory, by way of three connected applications, to an interdisciplinary audience consisting of philosophers, linguists, and cognitive scientists. So my focus was to show how the themes of the conference can be understood in object-theoretic terms rather than on presenting the latest developments of the theory. I'm indebted to Tadeusz Ciecierski and Paweł Grabarczyk for inviting me to speak at their conference and to Maciej Sendłak for our discussions and his kind assistance throughout the days I spent in Warsaw.

[1]For those who aren't familiar with this distinction, let me just give the main idea. Since abstract objects are not given in experience but rather *constituted* by the properties by which we conceive of them, the encoding mode of predication allows us to say exactly which properties are constitutive of an abstract object. As we shall see, a comprehension principle asserts that for any expressible condition on properties, there is an abstract object that encodes (i.e., that is constituted by) all and only the properties meeting the condition. In the end, encoding is a new primitive added to second-order logic; it is isn't defined but rather axiomatized (just as set membership is a primitive of set theory that isn't defined but rather axiomatized). We better understand encoding as we (a) derive more theorems and (b) apply the theory in new ways to philosophical problems and the analysis of natural language.

If the notion of *concept* used in cognitive science doesn't fall among the three notions just mentioned, then I believe it can be analyzed in some other way within object theory. For example, I think some cognitive scientists use 'concept' to refer to word *types* (as opposed of word *tokens*). If that is what they mean, then there is a way to analyze that notion in object theory, namely, by taking *types* (e.g., word types, sentence symbols and, in general, symbol types) to be the abstract objects that encode just the defining properties exemplified by the tokens of that type. At other times, cognitive scientists use 'concept' to mean certain mental tokens, possibly mental tokens in some language of thought. If that is what they mean, then object theory would analyze only the *content* of those tokens as Fregean senses, i.e., in terms the *second* notion of concept that arises from Frege's work. So I hope that the notion of concept used by the cognitive scientists will be covered in what follows.

1 Frege's Notion of *Begriff*

Frege (1892a) distinguished *concepts* (*Begriffe*) and *objects*; today we similarly distinguish *properties* and *individuals*. The former are *predicable* entities (or, as Frege would say, *unsaturated* entities), while the latter aren't predicable. Frege analyzed predication ('x *exemplifies* F' or 'Fx') by saying that x *falls under* the concept F. He explicitly says (1892a, 51) that the concepts under which an object falls are its properties.

But Frege *identifies* concepts as functions that map objects to truth values and defines: x *falls under* F just in case F maps x to the truth-value The True. He thereby analyzes predication as functional application. By contrast, the second-order predicate calculus takes relations and predication as basic: the sentence form '$F^n x_1 \ldots x_n$' is atomic and, in the 1-place case, 'Fx' asserts x exemplifies property F. Russell used such a calculus to identify functions as special relations, namely, as those relations R such that $\forall x\forall y\forall z(Rxy \,\&\, Rxz \rightarrow y\!=\!z)$.

So if *your* notion of *concept* is Frege's notion of *Begriff*, then I'll simply refer to such entities in what follows as properties (or relations). Since I take predication to be more fundamental than functional application, I'll start with second-order quantified (modal) logic, with quantification over properties and relations, as a background framework. I include the second-order comprehension principle for relations as part of this framework (restricted only to forestall the assertion of paradoxical relations constructed with encoding subformulas). I plan to steer clear of the debate about interpreting the second-order quantifiers in terms of plural quantification; I'm quite happy to suppose that logic requires a primitive domain of relations for the second-order quantifiers to range over. Indeed, object theory doesn't require 'full' second-order logic; second-order logic under general, Henkin models will suffice for the applications in the main part of this paper. And for the theory of Fregean senses discussed at the end, typed object theory under general, Henkin models will suffice.

Of course, many people are still influenced by Quine's reluctance to endorse properties (or other intensional entities like relations and propositions) on the grounds that they are 'creatures of darkness' (1956, 180), whose principle of individuation is 'obscure' (1956, 184). He also suggests that intensions are 'less economical' than extensions such as truth values, classes, relations-in-extension (1956, 184). But none of this applies to the object-theoretic conception of properties. Here is why.

Object theory not only includes a second mode of predication, but includes a comprehension principle for abstract ($A!$) object that asserts, for any formula φ with no free xs, that there exists an abstract object x that encodes all and only the properties F that satisfy the formula φ. We write this as:

$$\exists x(A!x \,\&\, \forall F(xF \equiv \varphi)), \text{ where } x \text{ doesn't occur free in } \varphi.$$

The quantifer $\forall F$ here ranges over (hyperintensional) properties, i.e., properties more fine-grained than Montague's intensions. The above principle not only guarantees that there is an abstract object that corresponds to every expressible condition on properties, but also, as a special case, that for every property P, there is an abstract object that encodes just P and nothing else. Hence, if identity were a primitive of the language, one could *prove* that $G\!=\!H \equiv \forall x(xG \equiv xH)$. To see this, we need only

examine the right-to-left direction, since the left-to-right direction is trivial, given the substitution of identicals. For the right-to-left direction we have to prove $\forall x(xG \equiv xH) \rightarrow G\!=\!H$. So suppose $\forall x(xG \equiv xH)$ and, for *reductio*, $G\!\neq\!H$. Now consider an abstract object, say a, that encodes all and only the properties identical to G. Then a encodes G but not H. But, by our hypothesis, a encodes G iff a encodes H. Hence a encodes H. Contradiction.

So, if identity were primitive, object theory would yield, as a theorem, that properties are identical whenever they are encoded by the same objects. But in the standard formulations of object theory, identity is *not* taken as a primitive. Instead we define property identity by saying:

$$F\!=\!G =_{df} \forall x(xF \equiv xG)$$

And in a modal setting, the definition becomes:

$$F\!=\!G =_{df} \Box\forall x(xF \equiv xG)$$

Given the modal logic of encoding ($\Diamond xF \rightarrow \Box xF$), one can then prove, in the modal setting, that $\forall x(xF \equiv xG)$ is sufficient for proving $F\!=\!G$.

Thus, the identity and individuation of properties in object theory isn't obscure.[2] The more one understands the richness of (the applications of) object theory, the better one understands why the definition of property identity gives us an insight into their nature. One may consistently assert, for some properties F and G, both that $\Box\forall x(Fx \equiv Gx)$ and $F\!\neq\!G$. So one can consistently assert that the following concepts are distinct despite being necessarily equivalent:

> Being a barber who shaves all and only those who don't shave themselves.
> $[\lambda x\, Bx \,\&\, \forall y(Sxy \equiv \neg Syy)]$
>
> Being a dog and not a dog.
> $[\lambda x\, Dx \,\&\, \neg Dx]$

[2]If we think semantically for the moment, and help ourselves to some set theory (which is strictly not a part of object theory), then we may suppose that properties have two extensions, rather than an intension and an extension. They have an extension among the objects that exemplify them (which may vary from world to world) and an extension among the objects that encode (which doesn't vary from world to world). Properties are to be identified whenever their encoding extensions are identical, *not* when their exemplification extensions are identical or even when their exemplication extensions are identical at every world.

These properties are exemplified by the same objects (namely, no objects) at every possible world and so they are necessarily equivalent. But they aren't identical. And the same applies to many other pairs of properties that are necessarily equivalent (in the classical sense), but intuitively distinct.

These results make the object-theoretic conception of properties *hyperintensional*, given the recent usage of that term. So if your preferred notion of *concept* is that of a hyperintensionally-conceived property, then object theory's second-order comprehension principle for properties and definition for the identity of properties jointly yield a precise theory of concepts.

Finally, as to the last of Quine's criticisms mentioned above, I think a theory that starts with objects, hyperintensionally-conceived relations, and axiomatizes two modes of predication is no less economical than a theory that starts with truth-values and classes, takes $x \in y$ as a primitive (of the form Rxy), and axiomatizes the membership relation. For Quine wasn't suggesting that we abandon the classical form of predication when asserting that intensions are less economical than extensions. Quine regarded the axioms of set theory as a first-order theory expressible in the predicate calculus, with its single form of predication $F^n x_1 \ldots x_n$. I don't see why an 'economy' that postulates a distinguished membership relation, a number of axioms (including comprehension conditions) for sets, and a principle for the identity of sets, is more economical than object theory. So, if your preferred notion of *concept* is that of Frege's *Begriff*, then economically speaking, you are no worse off by taking concepts to be properties conceived object-theoretically than you would be taking them to be extensional entities of some kind.

2 Leibniz's Notion of Concept

Leibniz had a more fine-grained view about concepts than Frege. His calculi for concepts treated them more like individualized collections of properties than like properties. If we use x, y, z to range over concepts, then we can say that, in his mature theory of 1690, Leibniz introduced the 'sum' operation on concepts: $x \oplus y$ ('the sum of x and y), is an idempotent, commutative, and associative operation (though Leibniz left the associativity axiom out of his list of axioms). Leibniz also introduced a binary relation of inclusion and its converse, containment, on concepts:

$x \preceq y$ ('x is included in y') and $x \succeq y$ ('x contains y'). These relations are provably reflexive, anti-symmetric, and transitive. One of Leibniz's central theorems relating the operation of concept summation and the relation of concept containment is $x \succeq y \equiv x \oplus y = y$. These and other theorems in Leibniz 1690 reveal that concepts are structured, at the very least, as a semi-lattice.

The way Leibniz speaks suggests that if x is the concept *red* and and y is the concept *bicycle*, then $x \oplus y$ is the concept *red bicycle*. But we can't interpret Leibniz's concepts as properties, for suppose we attempted to do so and modeled Leibniz's theory as follows:

$$\textit{The concept } F \;=_{df}\; F$$

$$F \textit{ contains } G \text{ ('}F \succeq G\text{')} \;=_{df}\; F \Rightarrow G \quad \text{(i.e., } \Box\forall x(Fx \rightarrow Gx))$$

$$\textit{The sum of concepts } F \textit{ and } G \text{ ('}F \oplus G\text{')} \;=_{df}\; [\lambda x\, Fx \,\&\, Gx]$$

Then the theorem mentioned above, $x \succeq y \equiv x \oplus y = y$, becomes:

$$F \Rightarrow G \;\equiv\; [\lambda x\, Fx \,\&\, Gx] = G$$

This seems clearly false as a principle about properties, if the latter are hyperintensionally conceived. For take the two properties we discussed earlier as being necessarily equivalent but distinct: $[\lambda x\, Bx \,\&\, \forall y(Sxy \equiv \neg Syy)]$ and $[\lambda x\, Dx \,\&\, \neg Dx]$. Each of these necessarily implies the other, but we wouldn't want to say that their conjunction is identical to one of the conjuncts. So Leibniz's notion of concepts can't really be modeled as properties.

In Zalta 2000a, I suggested that Leibnizian concepts be analyzed as abstract objects that encode properties:

$$\textit{Concept}(x) =_{df} A!x$$

Then concept summation, $x \oplus y$, can be defined as the abstract object that encodes not only every property x encodes but also every property y encodes:

$$x \oplus y \;=_{df}\; \imath z(\textit{Concept}(z) \,\&\, \forall F(zF \equiv xF \vee yF))$$

And concept inclusion, $x \preceq y$, holds whenever every property x encodes is one that y encodes:

$$x \preceq y \;=_{df}\; \forall F(xF \rightarrow yF)$$

This produces an interesting mereology of concepts. Indeed, Leibniz's axioms for $\oplus$ (idempotence, commutativity, and associativity), and his definition and theorems for $\preceq$ (reflexivity, anti-symmetry, and transitivity), all fall out as theorems (Zalta 2000a).

In addition, a natural analysis of Leibniz's containment theory of truth presents itself, at least if we restrict our attention to ordinary (i.e., non-abstract) objects. For suppose we let $u, v, \ldots$ range over ordinary individuals. Then we can define the concept of an ordinary individual u, written $\boldsymbol{c}_u$, as the concept that encodes just the properties u exemplifies:

$$\boldsymbol{c}_u \ =_{df} \ \imath x(\mathit{Concept}(x)\ \&\ \forall F(xF \equiv Fu))$$

To take an example, we might designate the concept of Alexander the Great as $\boldsymbol{c}_a$. Then from any premise of the form *Alexander exemplifies* F, it becomes provable that $\boldsymbol{c}_a$ encodes F.

Now we can extend our theory to analyze the concepts of properties: the concept of the property G is the abstract object (i.e., concept) that encodes all and only the properties necessarily implied by G, i.e.,

$$\boldsymbol{c}_G \ =_{df} \ \imath x(\mathit{Concept}(x)\ \&\ \forall F(xF \equiv G \Rightarrow F))$$

To take an exemple, we might designate the concept of *being a king* as $\boldsymbol{c}_K$. Then from any premise of the form, *being a king* necessarily implies G, it becomes provable that $\boldsymbol{c}_K$ encodes G.

Now since *concept containment* is the converse of *concept inclusion*, we may define:

$$x \succeq y \ =_{df} \ y \preceq x$$

Given the definition of $y \preceq x$, this implies that x contains y if and only if x encodes every property y encodes.

This series of definitions leaves us in a position to formalize Leibniz's containment theory of truth, and we do this by way of an analysis of the sentence 'Alexander is king'. According to Leibniz's containment theory, this sentence is true just in case:

The concept of Alexander contains the concept of being a king.

Given our analysis, we can represent this formally as:

$$\boldsymbol{c}_a \succeq \boldsymbol{c}_K$$

Indeed, the claim $\boldsymbol{c}_a \succeq \boldsymbol{c}_K$ is *derivable*, in object theory, from Ka:

Assume Ka, to show $\forall F(\boldsymbol{c}_K F \rightarrow \boldsymbol{c}_a F)$. By the rule of generalization, it suffices to show $\boldsymbol{c}_K F \rightarrow \boldsymbol{c}_a F$. So assume $\boldsymbol{c}_K F$. Then, $K \Rightarrow F$, i.e., $\Box\forall x(Kx \rightarrow Fx)$, by definition of $\boldsymbol{c}_K$. By the T schema of modal logic and universal elimination, it then follows that $Ka \rightarrow Fa$. Given our initial assumption, it follows that Fa and, hence, by the definition of $\boldsymbol{c}_a$, that $\boldsymbol{c}_a F$.

I shall not, at this point, take the analysis of Leibniz's notion of concepts further. For those who are interested, Zalta 2000a contains a discussion of how Leibniz's modal metaphysics of complete individual concepts can be formalized in object theory. It may be of interest that the precise, object-theoretic picture that results offers a way to reconcile a pair of opposing views about the truth of modal claims, namely, the Kripkean view that an '$\Diamond Fx$' is true in case there is a world where x itself exemplifies the property F, and the Lewisian view that '$\Diamond Fx$' is true in case there is a world where some counterpart of x, not x itself, exemplifies the property F. The Kripkean view is built into the semantics of object theory, whereas the Lewisian view becomes reconstructed at the level of concepts. This is something suggested by a number of Leibniz scholars, as a way to think about the modal metaphysics of (Leibnizian) complete individual concepts.[3]

I'll conclude this discussion by noting that mathematical concepts are to be found among the Leibnizian concepts. To sketch the point as briefly as possible, we may capture the conceptual (or inferential) role of mathematical constants and predicates if given, as data, the judgments made in mathematical practice. First, we treat mathematical judgments as relative to some theory that grounds the use of the terms. So, for example, we might point to mathematical judgments such as the following, where 'PA' stands for Peano Arithmetic and 'ZF' stands for Zermelo-Fraenkel set theory :

$$\text{PA} \vdash 2 < 3$$
$$\text{ZF} \vdash \emptyset \in \{\emptyset\}$$

Since mathematicians always avail themselves of property talk, the above judgments imply the following judgments:

$$\text{PA} \vdash [\lambda x\, x < 3]2$$
$$\text{ZF} \vdash [\lambda x\, x \in \{\emptyset\}]\emptyset$$

[3]See Mondadori 1973, 1975; Fitch 1979; Wilson 1979; and Vailati 1986.

The strategy for identifying mathematical objects, developed in previous work, is two-fold:[4]

(a) analyze each mathematical theory T as an abstract object that encodes propositions (namely, the theorems of T), so that object-theoretic statements of the form $T[\lambda y\, p]$ ('T encodes the property *being such that* p') can serve as the definiens for the definiendum $T \models p$ ('p is true in T').

(b) where Π is a metavariable ranging over predicates, and κ is a metavariable ranging over individual terms, import the unanalyzed judgment of the form $T \vdash \Pi\kappa$ as an object-theoretic, analytic truth of the form $T \models \Pi\kappa_T$, where κ_T is the expression κ indexed to T.

Consequently, we may import the specific judgments mentioned above as the following encoding claims:

$$\mathrm{PA} \models [\lambda x\, x < 3_{\mathrm{PA}}]2_{\mathrm{PA}}$$
$$\mathrm{ZF} \models [\lambda x\, x \in \{\emptyset\}_{\mathrm{ZF}}]\emptyset_{\mathrm{ZF}}$$

Then we may assert the following as a *principle*, not as a definition:

$$\kappa_T = \imath x(A!x \,\&\, \forall F(xF \equiv \ _T \models F\kappa_T))$$

That is, the object κ of theory T is the abstract object that encodes all and only the properties that κ_T exemplifies in T. This applies to any well-defined individual term of any mathematical theory. Thus, we have the following two examples, in which we identify the number Zero (0) of Peano Arithmetic and the null set ($\emptyset$) of Zermelo-Fraenkel set theory:

$$0_{\mathrm{PA}} = \imath x(A!x \,\&\, \forall F(xF \equiv \ \mathrm{PA} \models F0_{\mathrm{PA}}))$$

$$\emptyset_{\mathrm{ZF}} = \imath x(A!x \,\&\, \forall F(xF \equiv \ \mathrm{ZF} \models F\emptyset_{\mathrm{ZF}}))$$

In this way, mathematical objects become formally identified as Leibnizian concepts.[5]

[4]This strategy was developed and refined over many works, including Zalta 1983 (Chapter VI); Linsky & Zalta 1995; Zalta 2000b; Linsky & Zalta 2006, Zalta 2006, and Nodelman & Zalta 2014.

[5]For a recent criticism of this view and a reply, see Buijsman 2017 and Linsky & Zalta 2019.

3 Fregean Senses

A third notion of *concept* is that of a Fregean *sense*. Frege introduced senses (*Sinne*) in his 1892b, and Burge (1977) nicely summarizes the roles they play in Frege's philosophy of language, namely, as (a) modes of presentation (i.e., content associated with a term by which the person using the term cognizes the denotation of the term), (b) that which determines the denotation of the term, and (c) the entities denoted in belief and other intermediate contexts. I'll assume that these roles apply both to the senses of individual terms and the senses of terms that denote relations.

In what follows, I want to review the object-theoretic analysis of Fregean senses and show how this analysis explains why senses are modes of presentation. But I *don't* adopt all of Frege's views in the philosophy of language; I don't suppose that the sense of a term determines the denotation of that term — I allow that some terms have senses that underspecify the denotation and that some terms have senses what contain misinformation. Moreover, I don't accept Frege's view that sentences denote truth values; object theory takes the denotation of a sentence to be a proposition. Alternatively, if we reserve the term 'proposition' for the sense of a natural language sentence, then we might call the denotation of a sentence a *state of affairs*. I also don't require that the sense of a term be fixed for all speakers of a language; it seems to me that the sense of a term can vary from person to person. For simplicity and purposes of illustration, it proves useful to consider an ideal language and an ideal community of speakers, in which the sense of each term is fixed for all the speakers in that community.

Object theory provides an analysis of Fregean senses that explains how they can have the roles assigned to them in Frege's philosophy of language. The analysis is presupposes that the terms of a language, and the entities they denote, fall into logical types. Then we can summarize the analysis as follows: if a term τ has the logical type t, then the sense of τ is an abstract object of type t. For example, the sense of an individual term encodes properties of individuals, and the sense of a term denoting a property encodes properties of properties. By encoding properties, the sense of a term can *present* or *represent* the entity denoted by the term. If the encoded properties are sufficiently determinate, the sense of a term can individuate the entity denoted by the term. And the sense of a term is indeed an entity that can serve as the denotation of that term when the term is in propositional attitude and other intensional contexts, for on the above theory, the sense of τ has the same logical type as the denotation of τ.

3.1 Formalization of the Theory

To develop this theory of Fregean senses, we need to formulate *typed object theory*, i.e., object theory within the background of a relational type theory. We begin with a definition of the logical types:

> i is a type
> $\langle t_1, \ldots, t_n \rangle$ is a type whenever $t_1, \ldots, t_n$ are any types ($n \geq 0$)

So i is the type for individuals, and $\langle t_1, \ldots, t_n \rangle$ is the type for relations among entities having types $t_1, \ldots, t_n$. When $n = 0$, the empty type $\langle\, \rangle$ ('p') is the type for propositions (or states of affairs).

Using this definition, it is straightforward to type the language of object theory. We have two kinds of atomic formulas:

> *Exemplification* formulas of the form $Fx_1 \ldots x_n$, where F has any type of the form $\langle t_1, \ldots, t_n \rangle$ and $x_1, \ldots, x_n$ have, respectively, types $t_1, \ldots, t_n$
>
> *Encoding* formulas of the form xF where x is of any type t and F is of type $\langle t \rangle$.

Then we build up the language of typed object theory in the usual way. This means we can now assert a *typed* comprehension principle for abstract entities:

> Where t is any type, x^t is a variable of type t and φ is any condition on properties having type $\langle t \rangle$ in which x^t has no free occurrences, the instances of the following are axioms:
>
> $$\exists x^t (A!^{\langle t \rangle} x \,\&\, \forall F^{\langle t \rangle}(xF \equiv \varphi))$$

So, for example, when $t = i$, we obtain the comprehension principle for abstract individuals:

$$\exists x^i (A!^{\langle i \rangle} x \,\&\, \forall F^{\langle i \rangle}(xF \equiv \varphi)), \text{ provided } x^i \text{ isn't free in } \varphi$$

When $t = \langle i \rangle$, we obtain a comprehension principle for abstract properties of individuals:

$$\exists x^{\langle i \rangle} (A!^{\langle\langle i \rangle\rangle} x \,\&\, \forall F^{\langle\langle i \rangle\rangle}(xF \equiv \varphi)), \text{ provided } x^{\langle i \rangle} \text{ isn't free in } \varphi$$

And when $t = \langle i, i \rangle$, we obtain a comprehension principle for abstract relations among individuals:

$$\exists x^{\langle i,i \rangle} (A!^{\langle\langle i,i \rangle\rangle} x \,\&\, \forall F^{\langle\langle i,i \rangle\rangle}(xF \equiv \varphi)), \text{ provided } x^{\langle i,i \rangle} \text{ isn't free in } \varphi$$

It should be clear that the instances of these principles assert, respectively, the existence of abstract individuals, abstract properties, and abstract relations.

3.2 How Abstracta Serve as Fregean Senses

To see how the principles just asserted yield a theory of Fregean senses, let's start with the logical type i for individuals. I suggest that the sense of the name 'Samuel Clemens' for a person S is an abstract individual that encodes certain properties of individuals (namely, those that S cognitively associates with the name 'Samuel Clemens'). On this picture, if S doesn't know that Samuel Clemens is Mark Twain, then the sense of the name 'Mark Twain' for a person S is an abstract individual that encodes different properties of individuals (namely, those that S cognitively associates with the name 'Mark Twain'). If we think of these abstract objects as *concepts*, then we could say that S has two concepts of Samuel Clemens (i.e., two concepts of Mark Twain), the one that is the sense of 'Samuel Clemens' and the one that is the sense of 'Mark Twain'.

The very same ideas apply to expressions of type $\langle i \rangle$, which denote properties of individuals. Here we have to invoke the comprehension principle for abstract properties of individuals. An abstract property is a property of individuals that *encodes* (as well as *exemplifies*) properties of properties of individuals. For example, the sense of the predicate 'is a woodchuck' for a person S is an abstract property of individuals that encodes certain properties of properties of individuals (namely, those that S cognitively associates with the expression 'is a woodchuck'). On this picture, if S doesn't know that *being a woodchuck* is the same property as *being a groundhog*, then the sense of the expression 'is a groundhog' for a person S is an abstract property that encodes different properties of properties (namely, those that S cognitively associates with the expression 'is a groundhog'). If we think of these abstract properties as higher-order *concepts*, then we could say that S has two concepts of the property *being a woodchuck* (i.e., two concepts of the property *being a groundhog*), the one that is the sense of 'is a woodchuck' and the one that is the sense of 'is a groundhog'.

I hope it is now clear that, for an arbitrary logical type t, an abstract object of type t can serve as the mode of presentation (or 'cognitive significance') of a term of type t. It should be relatively easy to see that an abstract object of type t can, in principle, *determine reference* when it serves as the sense of a term of type t. For consider those abstract objects of type t that encode properties having type $\langle t \rangle$ that jointly individuate a unique entity of type t. That is, consider those abstract objects x^t for which there is a unique y^t that exemplifies all the properties with type $\langle t \rangle$ that x^t encodes. We may define senses, traditionally conceived, as abstract objects that meet this condition. Let x, y be variables of type t and F be a variable of type $\langle t \rangle$. Then we might define:

x *is a sense* if and only if $\exists! y \forall F(xF \rightarrow Fy)$

Clearly, if a term τ of type t was associated with a sense of type t, as just defined, then the sense of τ could determine the reference of τ, namely, as the witness to the unique existence claim. This would hold for terms that denote individuals, properties of individuals, relations among individuals, etc.

Though object theory allows us to analyze senses in this way, I don't think language works this way. I think that the sense of a term may vary from individual to individual, may vary from time to time, may fail to encode properties that jointly individuate a unique entity, and may encode properties that fail to characterize the object denoted by the term.[6] To see why, consider an example I've used in other works; it may be instructive to those encountering object theory for the first time.

Suppose, you pass a sign in front of a building which says "Dr. Gustav Lauben, General Practitioner, 8am–5pm" in large print and which also contains some fine print. Depending on one's interest and needs, a person may or may not read the fine print. Thus, the information one absorbs after reading the sign will differ from person to person. Suppose A and B both encounter the sign for the first time and only B reads the fine print. Person A's sense of 'Lauben' (i.e., concept of Lauben) may encode only the properties of *being a doctor*, *being a general practitioner*, *having a practice at such and such location*, etc., while person B's sense of 'Lauben' (or concept of Lauben) may encode more properties, gleaned from reading the fine print. Furthermore, suppose the sign presents misinformation; say that Lauben lost his license 2 days before, sold his office, and doesn't even practice in the building any more. Then the sense of 'Lauben' for both persons A and B would encode properties that Lauben doesn't exemplify. Such senses couldn't semantically determine Lauben as the reference of 'Lauben'. But A and B can nevertheless communicate about Lauben. If they both show up at the door shortly after 8am and no one answers the knock, they might both think that Lauben is late. And they would both, in some sense, have a belief about Lauben (since he is at the start of the causal chain involving the use of the name 'Lauben'), despite the fact that they are each conceiving Lauben in different ways.

[6] Indeed, the sense of a term may contain so much misinformation and be so misleading as to individuate an object distinct from the denotation of the term.

The next example shows how abstract objects can play the third role Fregean senses are supposed to play, namely, as the denotation of a term in intensional contexts. To set up the example, let us adopt the notation that when τ is a term of type t and κ is a name of a person using the term τ, then $\underline{\tau}_\kappa$ is a name of the abstract object that serves as the sense of the term τ for the person named by κ. The precise identity of $\underline{\tau}_\kappa$, of course, depends on the context. The example below also requires us to recognize that abstract individuals are, logically speaking, individuals just like ordinary individuals, and that abstract properties (or relations) are, logically speaking, properties (relations) just like ordinary properties (relations). So our theory asserts the existence of 0-place relations with abstract constituents at any position. Then where '$\underline{w}_j$' denotes the abstract individual that serves as the sense of the name 'Woodie' ('w') for John ('j'), '$\underline{c}_j$' denotes the abstract individual that serves as the sense of the name 'Chuckie' ('c') for John, '$\underline{W}_j$' denotes the abstract property that serves as the sense of the term 'woodchuck' for John, and '$\underline{G}_j$' denotes the abstract property that serves as the sense of the term 'groundhog' for John, we can give the a formal analysis of the following case:[7]

1. John believes that Woodie is a woodchuck.

    (a) $B(j, [\lambda\, Ww])$ (de re)

    (b) $B(j, [\lambda\, \underline{W}_j\, \underline{w}_j])$ (de dicto)

2. John doesn't believe that Chuckie is a woodchuck.

[7] I assume familiarity with terms of the form $[\lambda\varphi]$, which we read as '*that*-φ'. These denote entities of the empty type p, and so denote states of affairs.

(a) $\neg B(j, [\lambda\, Wc])$ (de re)

(b) $\neg B(j, [\lambda\, \underline{W}_j\, \underline{c}_j])$ (de dicto)

3. Woodie is Chuckie.

   (a) $w = c$

4. John doesn't believe that Woodie is a groundhog.

   (a) $\neg B(j, [\lambda\, Gw])$ (de re)

   (b) $\neg B(j, [\lambda\, \underline{G}_j\, \underline{w}_j])$ (de dicto)

5. Being a woodchuck just is being a groundhog.

   (a) $W = G$

If we assume that a simple logic governs these formal representations, then the reading on which (1), (2) and (3) are jointly inconsistent is captured by the fact that the formal representations (1a), (2a), and (3a) are inconsistent, by the substitution of identicals. In addition, the reading on which (1), (4) and (5) are jointly inconsistent is captured by the fact that the formal representations (1a), (4a), and (5a) are inconsistent, by the substitution of identicals.

However, the above offers a reading on which there is a failure of substitutivity. (1b), (2b), and (3a) are not inconsistent; we can't infer the negation of (1b) from (2b) and (3a). And (1b), (4b) and (5a) are not inconsistent; we can't infer the negation of (1b) from (4b) and (5a). On the *de dicto* readings of (1), (2) and (4), the senses of the English expressions 'Woodie', 'Chuckie', 'woodchuck', and 'groundhog' are are constituents of the states of affairs that play a role in the truth conditions of the belief report. The truth of the reports (1), (2), and (4), as represented by (1b), (2b) and (4b), respectively, depends on whether (or not) John is belief-related to a state of affairs containing abstract constituents. The identity claims $w = c$ and $W = G$ have no bearing on such states of affairs.

Of course, if we distinguish the *truth of the belief reported* from the *truth of the belief report*, then we can define the former so that the truth of both the *de re* and the *de dicto* belief is tied to the truth of the *de re* state of affairs. Where φ^* is the result of removing all the underlines and subscripts from the terms in φ, we may define:

x *truly believes* that φ if and only if both x believes that φ and φ^*

Formally:

$$TB(x, [\lambda\, \varphi]) \equiv B(x, [\lambda\, \varphi])\ \&\ \varphi^*$$

This yields that the truth of the beliefs represented in both (1a) and (1b) is tied to the truth of the state of affairs *that Woodie is a woodchuck*, i.e., $[\lambda\, Ww]$. So, although the truth of the report (1), as represented by (1b), relates John to an intermediate state of affairs that represents $[\lambda\, Ww]$ to John, the truth of belief reported by (1), as represented by (1b), depends on the truth of $[\lambda\, Ww]$. And analogous remarks apply to (4b).

3.3 Other Features of the Theory

Now that we've discussed the ways in which abstract objects of type t have the right features to serve as the Fregean senses of terms of type t, note that we now have a clear solution to the problem that has puzzled a number of recent philosophers, namely, the problem of precisely identifying what Fregean senses are. For example, in D. Kaplan (1969, 119), we find:

> My own view is that Frege's explanation, by way of ambiguity, of what appears to be the logically deviant behavior of terms in intermediate contexts is so theoretically satisfying that if we have not yet discovered or satisfactorily grasped the peculiar intermediate objects in question, then we should simply continue looking.

and in G. Forbes (1987, 31), we find:

> My overall conclusion is that a Fregean theory of the semantics of attitude contexts is from the structural point of view the best that is available. Its ultimate viability depends of course on how successful the efforts to develop a detailed theory of the nature of modes of presentation will be.

The model of Fregean senses as abstract entities gives us a detailed theory of the nature of these objects.

It is important to mention again that in object theory, unlike most other intensional logics or type theories, both the denotation and the sense of a term are of the same logical type as the term itself. This works throughout the type hierarchy. If we use senses as the intensions of expressions, then we don't need the technique of 'type-raising' when semantically interpreting contexts that are sensitive to the intensions. We've seen an example: we've represented both *de re* and *de dicto* beliefs as relations of type $\langle i, p\rangle$. We don't have to type-raise for the *de dicto* readings; type-raising isn't needed to handle failures of substitution. See Zalta (forthcoming) for a number of examples where we can avoid the technique of type-raising to give an analysis of natural language.

Finally, let's think further about the suggestion that the sense of a term can vary from person to person and, indeed, that the sense of a term for a single person can vary from time to time. Indeed, this is just one way in which the sense of an expression can vary with the context. One might wonder, how can there be communication if persons A and B use the term τ but the sense of τ differs for A and B? And how can my own beliefs remain stable over time if my sense of τ changes over time?

The answer to this question lies first in the recognition that communication is a matter of degree. All things being equal, persons A and B communicate to a greater degree the more overlap there is among the properties encoded by τ_A and τ_B. In the case where τ is the name 'Frege', communication can break down whenever the τ_A and τ_B encode properties that Frege doesn't exemplify. A and B miscommunicate to some extent if τ_A encodes *being German* and *being a logician* while τ_B encodes *being German* and *being a historian*, while A and B miscommunicate a bit differently if τ_A encodes *being German* and *being a logician* and while τ_B encodes *being Austrian* and *being a logician*. But what is holding communication together is the fact that A and B both acquired the name by way of causal communication chains that trace back to the 'christening' of Gottlob Frege as 'Gottlob Frege'. So even B can truly believe that Frege was a logician despite the fact that B's sense of 'Frege' (i.e., $\underline{\text{Frege}}_B$) encodes *being an Austrian*. That's because the truth of the belief is tied to the *de re* state of affairs (which obtains just in case Frege was a logician); the truth of the belief is not tied to the proposition that B uses to represent that state of affairs, a proposition in which $\underline{\text{Frege}}_B$ is a constituent and represents Frege to B. Here we distinguish the truth of the (*de dicto*) belief report (in which $\underline{\text{Frege}}_B$ plays a role) and the truth of the (*de re*) belief being reported (in which Frege plays a role).

These same facts allow us to say that even if my sense of 'Frege' varies over time or with the context, then although the truth of *de dicto* belief reports about me will involve different abstract objects at different times and contexts, the truth of the *de re* belief being reported only involve

Frege, even as my sense of 'Frege' changes over time and between contexts. Thus, we can say that whereas my concept of Frege has changed over time or with the context, my belief that *Frege was a logician* remains true under the same conditions across times and contexts. This is the perspective that object theory brings to bear on context, cognition, and communication in general, and on concepts and objects in particular.

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