

# Deriving and Validating Kripkean Claims Using the Theory of Abstract Objects\*

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Saul Kripke has advanced interesting metaphysical and semantic claims that have strong appeal and engender conviction. In some cases, Kripke suggests that these claims constitute only a ‘picture’ rather than a theory, while in others, it is clear that the claims in question constitute a (formally) precise theory. In the former case, it is important to determine whether one can turn the picture into a precise theory and what the consequences are when this is done. In the latter case, it is important to determine whether Kripke’s claims are to be construed as proper (i.e., non-logical) axioms of metaphysics or whether they can be derived as theorems from a more general theory. Moreover, it would be of interest to learn that claims Kripke has put forward on a variety of different topics can be unified within the context of a single, precisely formulated theory.

In what follows, I show that a variety of Kripke’s most important metaphysical and semantical claims can be derived or validated within the theory of abstract objects (Zalta [1983], [1988]). Hereafter, we refer

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to this theory more simply as ‘object theory’. To derive one of Kripke’s metaphysical claims in object theory, one must prove an accurate representation of that claim from the axioms of object theory. (Some of these proofs can be found in the Appendix.) To validate (within object theory) one of Kripke’s semantical claims concerning natural language sentences of a certain kind, one has to show how Kripke’s analysis of those sentences is preserved or predicted by the formal representations of those sentences in object theory, given the semantic interpretation of that theory.

Our results should therefore prove interesting not only because they *systematize* claims that Kripke presupposes, asserts, or argues for on a variety of topics, but also because they *unify* those claims within the framework of a single, axiomatic theory. The results may become even more striking once it is recognized that the language, logic, semantics, and proper axioms of object theory were not specifically designed to derive or model Kripkean claims, but were rather put forward independently, from within a rather different philosophical tradition. The present investigation may therefore show that independent lines of research have converged, and this may come as a surprise. It would be significant if Kripke’s assumptions and formal principles were shown to be theorems of a more general formal theory.

In what follows, basic familiarity with object theory will often be presupposed. At the very least, readers should be familiar with the fact that object theory is formulated in a ‘syntactically second-order’ modal predicate calculus modified only so as to admit a second kind of atomic formula ( $\langle xF \rangle$ ), which asserts that object  $x$  *encodes* property  $F$ .<sup>1</sup> Thus, in the case of 1-place predications, the theory distinguishes between an object encoding a property and an object exemplifying a property ( $\langle Fx \rangle$ ), where this latter is just the 1-place case of the more general and familiar form of predication  $F^n x_1 \dots x_n$ .<sup>2</sup> The theory also takes the 1-place the-

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<sup>1</sup>We say ‘syntactically second-order’ because although second-order language is the appropriate language for formulating the theory’s quantification over properties, models of the theory show that the second-order quantifier ‘ $\forall F$ ’ doesn’t range over a domain (of properties) which is as large as the power set of the set of the domain of individuals. Thus, the theory doesn’t require *full* second-order logic. See Zalta 1999, 626–628.

<sup>2</sup>This distinction traces back to E. Mally’s [1912] distinction between an object satisfying (*erfüllen*) a property and an object being determined by (*determiniert sein*) a property. It will be discussed more thoroughly below. Whereas the exemplification (or instantiation) mode of predication can be assumed as understood, the intuition underlying the encoding mode of predication is this: whereas ordinary objects have a locus at which one can discover the properties they exemplify, there is no such locus

oretical relation ‘ $E!x$ ’ (‘ $x$  is concrete’) as primitive and defines *ordinary* objects (‘ $O!x$ ’) as possibly concrete objects (‘ $\diamond E!x$ ’) and *abstract* objects (‘ $A!x$ ’) as objects which couldn’t be concrete (‘ $\neg \diamond E!x$ ’). The two most important proper (non-logical) axioms of the theory are:

$$O!x \rightarrow \Box \neg \exists Fx F$$

$$\exists x(A!x \ \& \ \forall F(xF \equiv \varphi)), \text{ where } \varphi \text{ has no free } xs$$

The first asserts that ordinary objects necessarily fail to encode properties. The second is a comprehension principle for abstract objects, the instances of which assert, for a given formula  $\varphi$ , that there exists an abstract object that encodes all and only the properties satisfying  $\varphi$ .<sup>3</sup> The theory also defines a relation of identity,  $x =_E y$ , on ordinary objects;  $x =_E y$  obtains whenever  $x$  and  $y$  are both ordinary objects and necessarily exemplify the same properties. The general notion of identify,  $x = y$ , is then defined as: either  $x =_E y$  or  $x$  and  $y$  are both abstract objects that necessarily encode the same properties.

Finally, object theory includes a theory of relations, namely, comprehension and identity principles for  $n$ -place relations ( $n \geq 0$ ), where 1-place relations are properties and 0-place relations are propositions. For the present purposes, only familiarity with the comprehension and identity principles for properties is needed:

$$\exists F \forall x(Fx \equiv \varphi), \text{ where } \varphi \text{ has no free } Fs \text{ or encoding subformulas}$$

$$F = G =_{df} \Box \forall x(xF \equiv xG)$$

Instances of the first principle assert, when given a formula  $\varphi$  with no free  $F$ s and no encoding subformulas, that there exists a property  $F$

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for abstract objects; instead, abstract objects are *constituted* or *determined* by the properties by which we conceive them. I use the technical term ‘ $x$  encodes  $F$ ’ for this idea. However, abstract objects will also exemplify properties, though these will not typically be their defining properties.

<sup>3</sup>Although there are an infinite number of instances of comprehension, here is a simple example:

$$\exists x(A!x \ \& \ \forall F(xF \equiv Fa))$$

This asserts that there exists an abstract object that encodes exactly the properties  $F$  that object  $a$  exemplifies. In what follows, we shall see other instances of comprehension.

which is exemplified by all and only the objects satisfying  $\varphi$ .<sup>4</sup> The second principle tells us that  $F$  and  $G$  are identical if necessarily encoded by the same objects; it is consistent with the idea that necessarily equivalent properties (i.e., properties  $F$  and  $G$  such that  $\Box \forall x(Fx \equiv Gx)$ ) may be distinct.

These and other basic facts of object theory will sometimes be made more explicit in what follows. Readers who wish to learn more than the basics may consult the publications on object theory cited in the paper.

## 1. Possible Worlds

In this section, we summarize Kripke’s work on possible worlds and relate it to the theory of possible worlds formulable in object theory. In his work on the semantics of modal logic in the late 50s and early 60s, Kripke employs a metalanguage that has quantifiers over possible worlds. Moreover, the metatheory Kripke uses to develop his modal semantics presupposes certain facts about worlds, for example, that there are some, that they are maximal and consistent, that there is a unique actual world, and that worlds are coherent (i.e., if a proposition is true at a world, then the negation of that proposition is not true at that world; if a conjunction is true at a world, then both conjuncts are true at that world, etc.). Finally, Kripke stipulates truth conditions for statements of the form  $\Box \varphi$  in his target modal object language. If we simplify the stipulation by omitting the accessibility relation, then the truth conditions for  $\Box \varphi$  are that  $\varphi$  is true in all possible worlds. This fact has numerous consequences, such as that  $\diamond \varphi$  is true iff  $\varphi$  is true in some possible world, and that if  $\Box(\varphi \rightarrow \psi)$  and  $\Box \varphi$  are both true, then so is  $\Box \psi$ .

The presuppositions of Kripke’s metalanguage underlying these latter stipulations have all been shown to be *derivable* as theorems in object theory. This is confirmed by the series of definitions and theorems described below, part of which was first constructed in Zalta [1983] and expanded in Zalta [1993] and part of which is new material.

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<sup>4</sup>Thus, for example, the following instance asserts that arbitrarily chosen property  $G$  has a negation:

$$\exists F \forall x(Fx \equiv \neg Gx)$$

It should be clear that this yields a rich variety of properties: there are conjunctive and disjunctive properties, properties projected from two-place relations, and vacuous properties constructed out of propositions.

To understand the definitions that follows, note that in object theory, worlds are not analyzed as concrete objects (as in Lewis [1986]), but rather as abstract objects (as in Wittgenstein [1922]). On this analysis, the identity of a world is intimately bound up with the propositions true at it. If we say informally that the propositional property *being such that*  $p$  ( $[\lambda y p]$ ) is constructed out of the proposition  $p$  then, we may define a *possible world* to be any object that *might* be such that it encodes all and only those propositional properties constructed out of true propositions. In formal terms:

$$\text{PossibleWorld}(x) =_{df} \diamond \forall p(x[\lambda y p] \equiv p)$$

And, using ‘ $w$ ’ as a restricted variable ranging over possible worlds, we define:  $p$  is *true at world*  $w$  (‘ $w \models p$ ’) iff  $w$  encodes *being such that*  $p$ :

$$w \models p =_{df} x[\lambda y p]$$

Finally, we say that a world  $w$  is *actual* just in case all and only true propositions are true at  $w$ :

$$\text{Actual}(w) =_{df} \forall p(w \models p \equiv p)$$

From these latter three, simple definitions, object theory yields theorems that systematize the assumptions of Kripke’s metatheory. The following theorems assert that possible worlds are provably maximal, consistent, and possibly actual:<sup>5</sup>

$$\vdash \forall w \forall p(w \models p \vee w \models \neg p)$$

$$\vdash \neg \exists w \exists p(w \models (p \& \neg p))$$

$$\vdash \forall w(\diamond \text{Actual}(w))$$

From these theorems, little work is needed to establish that worlds are coherent. That is, the following are straightforwardly provable from the foregoing (suppressing the universal quantifiers at the beginning):<sup>6</sup>

$$\vdash (w \models p) \equiv (w \not\models \neg p)$$

$$\vdash (w \models p \& q) \equiv (w \models p \& w \models q)$$

<sup>5</sup>See Zalta [1993] for the proofs of the first and third, and the Appendix for the proof of the second.

<sup>6</sup>We omit the proofs here, though the Appendix to Zalta [1993] should provide a clue as to how they would go.

It is especially important to note that the definition of ‘possible world’ is fine-grained enough to yield the existence of a unique actual world. The claim that there is a unique actual world is formalizable in just the way you would expect, given the definitions:<sup>7</sup>

$$\vdash \exists! w \text{Actual}(w)$$

Here, we use ‘ $\exists! x \varphi$ ’ as the usual abbreviation of ‘ $\exists x \forall y (\varphi_x^y \equiv y = x)$ ’. Note that the derivation of this theorem proceeds by considering the abstract object that encodes all and only true propositions, which is asserted by the following instance of object comprehension:

$$\exists x(A!x \& \forall F(xF \equiv \exists p(p \& F = [\lambda y p])))$$

One can establish that any such object is a world, that it is actual, and that any other object which is an actual world is identical to it. I suggest that this theorem of axiomatic metaphysics is what *justifies* Kripke’s use of a distinguished actual world in Kripke models.

It is also derivable in object theory that a proposition is necessarily true iff it is true in all possible worlds, and that a proposition is possibly true iff it is true in some possible world:<sup>8</sup>

$$\vdash \Box p \equiv \forall w(w \models p)$$

$$\vdash \Diamond p \equiv \exists w(w \models p)$$

From the latter theorem, it follows that for every false proposition which is possibly true, there exists a non-actual possible world where it is true. That is, the following is a theorem of object theory:

$$\vdash \forall p[(\neg p \& \Diamond p) \rightarrow \exists w(\neg \text{Actual}(w) \& w \models p)]$$

(The derivation is in the Appendix.) This theorem provides further metaphysical justification for Kripke’s constructing the semantics of modal logic so that whenever a sentence of the form ‘ $\neg \varphi \& \Diamond \varphi$ ’ is true, there is a non-actual possible world in the domain of worlds in which  $\varphi$  is true. Clearly, then, given this modal metaphysics, all we have to do to prove the existence of non-actual possible worlds is to assert the existence of at least one false proposition which is possibly true.

<sup>7</sup>See Zalta [1983] or Zalta [1993], for the proof.

<sup>8</sup>Again, see Zalta [1983] or Zalta [1993], for the proof. It is a corollary to these claims that any proposition  $q$  necessarily implied by a proposition  $p$  true at a world is also true at that world; i.e.,  $(w \models p \& p \Rightarrow q) \rightarrow w \models q$ .

The foregoing theorems, then, establish that the metaphysical presuppositions of Kripke's metalanguage in his early papers on modal logic are derivable in object theory.

## 2. Identity and Necessity

In this section, we show that the analysis of identity developed in object theory preserves and extends (the theorems of) modal logic which ground Kripke's claims about rigid designators. In [1971], Kripke argues that certain identity statements of natural language, namely, those having rigid designators on both sides of the identity sign, are necessary if true (p. 71). Kripke's argument relies on a certain general modal fact, namely,  $x = y \rightarrow \Box(x = y)$ . Kripke had noted earlier in his paper (p. 67) that this modal fact is a theorem of quantified modal logic, derivable from the following premises (from which the initial universal quantifiers have been removed):

- (1)  $x = y \rightarrow (Fx \rightarrow Fy)$
- (2)  $\Box(x = x)$

According to Kripke, the argument for the necessity of identity now goes by way of the fact that the following is a substitution instance of (1):<sup>9</sup>

- (3)  $x = y \rightarrow [\Box(x = x) \rightarrow \Box(x = y)]$

Then, given (2), we can simplify (3) to (4):

- (4)  $x = y \rightarrow \Box(x = y)$

It is interesting to note that Kripke interprets (1) as follows ([1971], 67):

... the law of the substitutivity of identity says that, for any objects  $x$  and  $y$ , if  $x$  is identical to  $y$ , then if  $x$  has a certain property  $F$ , so does  $y$ .

But it is not clear why (3) is supposed to be a direct *instance* of (1), since the consequent of (1), namely, ' $Fx \rightarrow Fy$ ', contains atomic formulas in the antecedent and consequent, whereas the conditional consequent of (3) contains modal (i.e., complex) formulas in the antecedent and consequent. Of course, there is no deep problem here, since Kripke could reformulate (1) as the following schema:

<sup>9</sup>I believe that the following formulation eliminates a typographical error in item (3) in Kripke [1971], p. 67.

- (5)  $x = y \rightarrow [\varphi(x, x) \rightarrow \varphi(x, y)]$ , where  $\varphi(x, x)$  is any formula in which  $x$  may or may not be free, and where both  $y$  is substitutable for  $x$  and  $\varphi(x, y)$  is the formula which results by replacing one or more occurrences of  $x$  by  $y$  in  $\varphi(x, x)$

Clearly, then, (3) is a direct instance of (5), since ' $\Box x = x$ ' is a formula of the form  $\varphi(x, x)$  and ' $\Box x = y$ ' is the corresponding formula of the form  $\varphi(x, y)$ .

In any case, there is a direct argument for the necessity of identity from (5) and (2) to (4).<sup>10</sup> The premises have the following status: (5) is a logical axiom (as part of the logic of identity) and (2) is a logical theorem derivable from the logical axiom ' $x = x$ ' using the Rule of Necessitation. These facts then form part of the background framework for Kripke's views on rigid designation and the necessity of true identity statements of natural language involving rigid designators.

By way of comparison, it is important to emphasize that identity is *not* taken as a primitive in object theory, but rather defined. Object theory thus offers a *analysis* of identity, and the theorems which result from that analysis offer a non-trivial, non-logical (i.e., proper) metaphysical theory of identity. The interesting results concerning object theory are as follows: (a) the reflexivity of identity can be derived as a proper theorem rather than taken as logical axiom; (b) sentence (4) above becomes a proper theorem of metaphysics rather than a theorem of logic; and (c) the following, corresponding claims about properties, relations, and propositions are also proper theorems:

$$\begin{aligned} &\vdash F^1 = G^1 \rightarrow \Box(F^1 = G^1) \\ &\vdash F^n = G^n \rightarrow \Box(F^n = G^n) \quad (n \geq 2) \\ &\vdash p = q \rightarrow \Box(p = q) \end{aligned}$$

<sup>10</sup>Alternatively, Kripke could have formulated an 'indirect' argument for the necessity of identity. Instead of reformulating (1) as (5), Kripke could have instantiated (1) using the property-denoting term ' $[\lambda z \Box x = z]$ ', which denotes the property of being something to which  $x$  is necessarily identical. Then, substituting this term for the variable ' $F$ ' in (1), we would get:

$$x = y \rightarrow ([\lambda z \Box x = z]x \rightarrow [\lambda z \Box x = z]y)$$

Then, we could derive (3) from this truth by appealing to  $\lambda$ -conversion, since  $\lambda$ -conversion tells us that  $[\lambda z \Box x = z]x$  is simply equivalent to  $\Box x = x$  and that  $[\lambda z \Box x = z]y$  is simply equivalent to  $\Box x = y$ . So, on this indirect argument for (4), (3) is derivable as a consequence of (1) (and the principle of  $\lambda$ -conversion); it is not a direct substitution instance of (1).

To see why (a) – (c) are true, we first have to understand why the substitutivity of identity, i.e., (5), has a different status in object theory.

Recall, from our brief sketch of the theory in the introduction, that ordinary objects are objects which are possibly concrete ( $\Diamond E!x$ ) and that a relation of identity on ordinary objects,  $x =_E y$ , is defined as follows:

$$x =_E y \text{ =}_{df} O!x \ \& \ O!y \ \& \ \Box \forall F (Fx \equiv Fy)$$

By contrast, abstract objects are those which couldn't possibly be concrete ( $\neg \Diamond E!x$ ), and since the distinction between ordinary and abstract objects constitutes a partition of the domain of objects, the following disjunctive definition constitutes a general definition of identity:

$$(6) \ x = y \text{ =}_{df} (x =_E y) \ \vee \ (A!x \ \& \ A!y \ \& \ \Box \forall F (xF \equiv yF))$$

If we eliminate the defined notation, (6) becomes (6'):

$$(6') \ x = y \text{ =}_{df}$$

$$[O!x \ \& \ O!y \ \& \ \Box \forall F (Fx \equiv Fy)] \ \vee \ [A!x \ \& \ A!y \ \& \ \Box \forall F (xF \equiv yF)]$$

Given that the general notion of identity is *defined*, the substitutivity of identity principle (5) now constitutes a *proper* axiom schema and not a logical axiom schema.<sup>11</sup> (5) not only governs (6) and (6'), but also governs  $x =_E y$ , since one can immediately infer  $x = y$  from  $x =_E y$ .

Now with this understanding of the substitutivity of identity in object theory, we can turn to result (a), namely, that the reflexivity of identity is a proper theorem. This is a simple consequence of the fact that every object is either ordinary or abstract ( $O!x \ \vee \ A!x$ ). From each disjunct, we can easily derive the one of the disjuncts that results from substituting ' $x$ ' for ' $y$ ' in (6'). Thus, we obtain (7):

$$(7) \ \vdash \ x = x$$

<sup>11</sup>The reason (5) is a proper and not a logical axiom schema is that there are non-standard interpretations of the language of object theory which don't preserve the substitutivity of  $x$  and  $y$  whenever the conditions defining ' $x =_E y$ ' or ' $x = y$ ' obtain. Consequently, in such interpretations, (5) would not be true. Since (5) is not logically true (i.e., true in all interpretations), it would not be taken as proper axiom schema. Of course, on the other hand, one could simply constrain interpretations of the language of object theory so as to make (5) a logical truth. But we won't explore this option here.

(The derivation which establishes (7) is in the Appendix.) With (5) and (7), one can easily prove that our defined notion of identity is an equivalence condition — it is symmetrical and transitive as well as reflexive.

With our metaphysical theory of identity in place, it is straightforward to show result (b), namely, that the necessity of identity is a proper theorem of metaphysics. Using the Rule of Necessitation, we can infer (2) (i.e.,  $\Box x = x$ ) from (7). And (3), as we noted, is an instance of (5). Thus, object theory, with its more fine-grained analysis of identity, preserves the direct argument from (5) and (2) to (4).<sup>12</sup>

So the necessity of identity holds for all objects. Moreover, recall that we introduced a special notion of identity for ordinary objects, namely,  $=_E$ . It turns out that the necessity of identity provably holds for this notion of identity as well:

$$(4') \ \vdash \ x =_E y \rightarrow \Box x =_E y$$

Although we leave the full proof for the Appendix, it is worth noting that (4') can't be derived by starting with  $x =_E y$ , inferring  $x = y$  from (6), deriving  $\Box(x = y)$  from (4), and then inferring  $\Box(x =_E y)$ . The last step is invalid. Instead, (4') may be derived by an argument that appeals to (1) (which is a simple consequence of (5) in object theory). The proof of (4') employs the property  $[\lambda z \Box x =_E z]$ .

Now for result (c), concerning the necessity of identity for properties, relations, and propositions. Recall that not only does Kripke subscribe to the idea that identity claims of natural language having the form  $a = b$  (where ' $a$ ' and ' $b$ ' are rigid designators) are necessary if true, he also makes a similar claim for natural kind terms. When ' $P$ ' and ' $Q$ ' are natural kind terms, Kripke holds that ' $P = Q$ ' is necessary if true ([1972], 127-134). In arguing for this claim, Kripke presupposes a modal fact

<sup>12</sup>Some readers might find it interesting to note that the derivation of (4) in object theory must go directly through (5) rather than indirectly through (1). Although (1) itself is a theorem of object theory (it is an instance of (5)), it can't be used in the derivation of (4) if the expression ' $F$ ' counts as a variable which ranges over properties. The reason is that the  $\lambda$ -expression ' $[\lambda z \Box x = z]$ ' does not count as a well-formed property-denoting expression in object theory. No such expression can be used to instantiate the variable ' $F$ '. As we saw above, the definition of '=' contains encoding subformulas (' $xF$ ') and these are not allowed either in instances of property comprehension or in  $\lambda$ -expressions. This banishment of encoding formulas from  $\lambda$ -expressions and instances of property comprehension prevents the paradoxes of encoding (Zalta [1983], Appendix A).

concerning natural kinds, namely, that if natural kinds are identical, they are necessarily identical.

If we suppose that natural kind terms are best represented formally as terms that denote properties, then the presupposition of Kripke's argument about identity statements involving natural kind terms can be represented as the following modal fact, in which ' $F$ ' and ' $G$ ' are variables ranging over properties:

(8) If  $F$  and  $G$  are identical, they are necessarily identical.

(8')  $F = G \rightarrow \Box(F = G)$

Indeed, this turns out to be a theorem of object theory as well. It follows straightforwardly from the following definition of property identity we mentioned at the outset:

(9) Properties  $F$  and  $G$  are *identical* iff necessarily, they are encoded by the same objects.

(9')  $F = G =_{df} \Box \forall x (x F \equiv x G)$

The derivation of (8) from (9) is immediate from the S4 schema, which is implied by the S5 modal logic adopted in object theory.

Finally, it is worth noting that, in object theory, (9) can be generalized to yield identity conditions for propositions (considered as 0-place relations) and  $n$ -place relations (for  $n \geq 2$ ).<sup>13</sup> From these definitions of identity, one can derive the necessity of identity with respect to both propositions and relations. That is, the following are theorems of object theory:

(10) If  $p$  and  $q$  are identical, they are necessarily identical.

(10')  $p = q \rightarrow \Box(p = q)$

<sup>13</sup>In Zalta [1983], [1988], [1993], and elsewhere, we defined:

Propositions  $p$  and  $q$  are *identical* iff the propositional properties *being such that*  $p$  and *being such that*  $q$  are identical.  $p = q =_{df} [\lambda y p] = [\lambda y q]$

Relations  $F^n$  and  $G^n$  are *identical* just in case, for each way of "plugging them up" with  $n - 1$  arbitrarily chosen objects (plugging up  $F^n$  and  $G^n$  the same way), the resulting 1-place properties are identical (where property identity is as defined above).

We omit the technical definition of relation identity for simplicity, but the reader may find it in the works cited at the beginning of this note.

(11) If  $F^n$  and  $G^n$  are identical, they are necessarily identical.

(11')  $F^n = G^n \rightarrow \Box(F^n = G^n)$

The derivation of (10) is in the Appendix, but we leave the derivation of (11) for another occasion. (It requires the definition of relation identity, which we have not reproduced here.)

### 3. Fictions

In the Addenda to [1972], Kripke made several noteworthy claims about the nature of fiction. In [1973], however, he developed an informal analysis of fictional names and sentences about fictions. In this section, we show that certain consequences of object theory capture the claims of [1972] when these are interpreted in light of Kripke's analysis of [1973]. In particular, we plan to derive, as theorems of object theory: (a) that a fictional individual is not identical with any possible individual, and (b) that a fictional species is not identical with any possible species. Once Sherlock Holmes is identified as a fictional detective, and the property of being a unicorn is identified as a fictional species, then the following instances of (a) and (b) will be derivable: (a') Sherlock Holmes is not identical with any possible detective, and (b') being a unicorn is not identical with any possible species.

Claims (a') and (b'), and their generalizations (a) and (b), capture Kripke's remarks on pp. 156-158 of [1972] when these are interpreted in light of his analysis of [1973]. Although Kripke gives various arguments for these claims, the argument that is most persuasive from the point of view of object theory is that there seems to be no non-arbitrary way of identifying a denotation for 'Sherlock Holmes' within the domain of possible objects—there are too many different possible objects which are all consistent with the Conan Doyle novels. Holmes, after all, is only incompletely specified by the story. Similarly, the property of being a unicorn is a particular fictional species only incompletely specified by the myth; there are too many candidate possible species consistent with the myth for us to identify one of them as the species of being a unicorn.

To see that claims (a) and (b) are theorems of object theory, recall the definitions from the theory of fiction (formalized in Zalta [1983] and extended in Zalta [2000]):  $x$  is a *story* iff both (i)  $x$  encodes only propositions

and (ii)  $x$  is authored ( $'Axy'$ ) by some concrete object.<sup>14</sup> A proposition  $p$  is *true in story  $s$*  ( $'In\ story\ s,\ p'$ ) just in case  $s$  encodes the propositional property of being such that  $p$ .<sup>15</sup> An individual  $x$  is *a character of  $s$*  just in case there is some property  $F$  such that, in story  $s$ ,  $x$  exemplifies  $F$ .<sup>16</sup> An individual  $x$  *originates in  $s$*  just in case  $x$  is an abstract object that is a character of  $s$  and  $x$  is not a character of any earlier story.<sup>17</sup> An individual  $x$  is *fictional* just in case  $x$  originates in some story.<sup>18</sup> In previous work on the theory of fiction, we've used these definitions to analyze natural language sentences that contain names and descriptions of fictional objects.<sup>19</sup>

Note that a simple consequence of the above definitions is that if  $x$  is fictional, then  $x$  is abstract:

$$(12) \text{ Fictional}(x) \rightarrow A!x$$

(For if  $x$  is fictional, then  $x$  originates in some story  $s$ , and if the latter, then  $x$  is abstract.) (12) is actually central to the theory of fiction described in Kripke's [1973]. We'll return to this fact in the final section, when we examine how object theory validates Kripke's view that natural language quantifies over a realm of abstract objects.

To complete the analysis of fictional individuals, let us introduce one more definition. We say that an individual  $x$  is a *fictional- $G$*  just in case  $x$  originates in a story in which  $x$  exemplifies  $G$ .<sup>20</sup> This defines the precise sense in which Holmes is a fictional detective. It now follows, from the fact that Holmes is a fictional detective, that Holmes is not identical with any possibly concrete object. For since Holmes is a fictional detective, there is a story  $s$  in which he originates and exemplifies the property of

<sup>14</sup>In formal terms:

$$\text{Story}(x) =_{df} \forall F[xF \rightarrow \exists p(F=[\lambda z p]) \ \& \ \exists y(E!y \ \& \ Ayx)]$$

<sup>15</sup>In formal terms:

$$s \models p =_{df} s[\lambda x p]$$

<sup>16</sup>In formal terms:

$$\text{Character}(x, s) =_{df} \exists F(s \models Fx)$$

<sup>17</sup>Using ' $<$ ' for the relation of 'earlier than', the formal definition is:

$$\begin{aligned} \text{Originates}(x, s) &=_{df} A!x \ \& \ \text{Character}(x, s) \ \& \\ &\forall y \forall z \forall s' ((Azs' < Ays) \rightarrow \neg \text{Character}(x, s')) \end{aligned}$$

<sup>18</sup>In formal terms:

$$\text{Fictional}(x) =_{df} \exists s \text{Originates}(x, s)$$

<sup>19</sup>See Zalta [1983], Chapter IV; [1988], Chapter 7; and Zalta [2000].

<sup>20</sup>In formal terms:

$$\text{Fictional-}G(x) =_{df} \exists s[\text{Originates}(x, s) \ \& \ s \models Gx]$$

being a detective. So call such a story  $s_1$ . Since Holmes originates in  $s_1$ , he is fictional, and so, by (12), he is an abstract object. But in object theory, it is a theorem that if  $x$  is abstract ( $'A!x'$ ),  $x$  is not identical with any object  $y$  which is possibly concrete:

$$(13) A!x \rightarrow \neg \exists y(\diamond E!y \ \& \ y=x)$$

(13) is a consequence of the definition of being 'abstract' ( $'A!x'$ ). For if  $x$  is abstract, then, by definition,  $x$  couldn't possibly be concrete ( $'\neg \diamond E!x'$ ), and this latter is simply equivalent to the claim that  $x$  is not identical to any possibly concrete object. So our argument concerning Holmes can now be summarized: we have established that it *follows* from the fact that he is a fictional detective that he is not identical with any possibly concrete object.

In general, then, we have established the two following theorems, both of which help us to capture claim (a) described at the outset of this section:

$$(14) \text{ If } x \text{ is fictional, then } x \text{ is not identical with any possible (concrete) object.} \quad \text{Fictional}(x) \rightarrow \neg \exists y(\diamond E!y \ \& \ y=x)$$

$$(15) \text{ If } x \text{ is a fictional-}G, \text{ then } x \text{ is not identical with any possibly concrete } G. \quad \text{Fictional-}G(x) \rightarrow \neg \exists y[\diamond(E!y \ \& \ Gy) \ \& \ y=x]$$

(The derivation of (14) is a consequence of (12) and (13), and the derivation of (15) is a consequence of the definitions of 'abstract' and 'fictional- $G$ '.) (14) is just the claim (a) we mentioned at the outset. So given that Holmes is fictional, there is a clear and derivable sense in which he is not a possible object. And, in particular, given that Holmes is a fictional detective, there is a clear and derivable sense in which he is not a possible detective. Paraphrasing Kripke, there is no possible detective of which we could say, "That is Holmes and the stories are about him".

These ideas extend naturally to fictional properties and fictional species, once we generalize the above definitions in the context of higher-order object theory. In previous work, we've developed the definitions needed to generalize the theory of abstract objects with respect to a simple theory of types. Instead of presenting all of the technical details here, we offer a semi-technical description of how the theory goes. Basically, the idea is that the theory of abstract objects reiterates at each type. Here is how.

Let us suppose we have a type  $i$  for the type of individuals and complex types  $\langle t_1, \dots, t_n \rangle$  for relations among objects having types  $t_1, \dots, t_n$

( $n \geq 0$ ). So, properties of individuals have type  $\langle i \rangle$ , properties of properties of individuals have type  $\langle\langle i \rangle\rangle$ , relations among individuals have type  $\langle i, i \rangle$ , properties of relations among individuals have type  $\langle\langle i, i \rangle\rangle$ , etc. Using this type scheme, one can type the language, definitions, and axioms of object theory.<sup>21</sup> Now the important idea to grasp is simply this: at each type, object theory requires that there are two subdomains of entities having that type—ordinary objects of that type and abstract objects of that type. The ordinary objects of a given type are defined as the ‘possibly concrete’ objects of that type, while the abstract objects of that type are defined as the objects of that type which couldn’t be concrete.<sup>22</sup>

Now to make this more vivid, consider properties of individuals, having type  $\langle i \rangle$ . Object theory treats properties such as the property of having mass, having spin, having a shape, being yellow, being red and round, etc., as concrete properties of type  $\langle i \rangle$ ; it treats relations such as to the left of, loves, meets, etc., as concrete relations of type  $\langle i, i \rangle$ , and so forth. These concrete properties and relations will therefore be ordinary, since their concreteness implies their possible concreteness. By contrast, abstract properties and relations are entities that couldn’t possibly be concrete.

Analogously, the objects in the domain of an arbitrary type  $t$  exhaustively and exclusively divide into the ordinary objects of that type and abstract objects of that type. It is axiomatic that the ordinary objects of a given type (necessarily) do not encode properties.<sup>23</sup> So not only do ordinary individuals fail to encode properties, but ordinary properties (i.e., the concrete properties mentioned above and others like them which are concrete in some world) also fail to encode properties, as do ordinary relations, and so on.

The typed comprehension schema for abstract objects now comes into play. It asserts the existence of abstract entities at each type. Consider again, for example, properties of individuals, which have type  $\langle i \rangle$ . For

<sup>21</sup>For example, to type the two basic kinds of atomic formula, we say: where  $t$  is any type,  $x^t F^{(t)}$  is a simple encoding formula; and where  $t_1, \dots, t_n$  are any types,  $F^{(t_1, \dots, t_n)} x^{t_1} \dots x^{t_n}$  is a simple exemplification formula.

<sup>22</sup>For each type  $t$ , we suppose that there is a distinguished 1-place predicate ‘ $E!^{(t)}$ ’, which applies to entities of type  $t$ . ‘ $E!^{(t)}$ ’ is the (typically ambiguous) ‘concreteness’ predicate.

<sup>23</sup>Formally:

$$O!^{(t)} x^t \rightarrow \Box \neg \exists F^{(t)} x F$$

(Here we adopt the convention of suppressing type indications on the reoccurrences of a term whose type has already been specified in the formula.)

each formula  $\varphi$  which places a condition on properties of such properties, the comprehension schema will have an instance that asserts that there is an abstract property of individuals that encodes all and only the properties of properties which satisfy  $\varphi$ .<sup>24</sup> We’ll soon see how this axiom schema gets applied, but for now, it is important to recognize that just as abstract individuals encode, and are individuated by, properties of individuals, abstract properties encode, and are individuated by, properties of properties. Similarly, abstract relations encode, and are individuated by, properties of relations.

With this understanding, one can type all of the definitions relating to fiction which were introduced above. In these typed definitions, we take propositions to be entities of the empty type  $\langle \rangle$  (we use ‘ $p$ ’, ‘ $q$ ’, ... as variables ranging over propositions), and we take stories to be abstract objects of type  $i$  (which encode propositional properties of the form  $[\lambda x^i p]$ , where  $p$  is a proposition).<sup>25</sup>

But before we turn to the identification of fictional species (e.g., unicorns and hobbits) as abstract properties, consider again the domain of properties of individuals and note that abstract properties are distinct even from those ordinary properties which aren’t or couldn’t be exemplified. For example, the ordinary property of being a giraffe in the Arctic Circle is not exemplified in this world. And the ordinary, conjunctive property of being round and square is not exemplified in any possible world. We cannot conclude that a property is abstract just because it necessarily fails to be exemplified, though we may assert it as axiomatic that abstract properties necessarily fail to be exemplified. This reflects

<sup>24</sup>Thus, if  $\varphi$  is a condition which can be satisfied by objects of type  $\langle\langle i \rangle\rangle$ , there is an instance of comprehension which asserts that there is an abstract property (i.e., an abstract object  $x$  of type  $\langle i \rangle$ ) which encodes all and only those properties of properties satisfying the condition  $\varphi$ .

<sup>25</sup>In formal terms, the definitions become:

$$\text{Story}(x^i) =_{df} \forall F^{(i)} [x F \rightarrow \exists p (F = [\lambda z^i p])] \ \& \ \exists y^i (E!^{(i)} y \ \& \ A^{(i, i)} y x)$$

$$s^i \models p =_{df} s [\lambda y^i p]$$

$$\text{Character}(x^t, s^i) =_{df} \exists F^{(t)} (s \models F x)$$

$$\text{Originates}(x^t, s^i) =_{df} A^{(t)}! x \ \& \ \text{Character}(x, s) \ \& \ \forall y^i \forall z^i \forall s' ((A^{(i, i)} z s' < A y s) \rightarrow \neg \text{Character}(x, s'))$$

$$\text{Fictional}(x^t) =_{df} \exists s^i \text{Originates}(x, s)$$

$$\text{Fictional-}G^{(t)}(x^t) =_{df} \exists s^i [\text{Originates}(x, s) \ \& \ s \models G x]$$



the fact that abstract properties are fundamentally different in kind from ordinary properties. (We've already seen reasons to think this; while the former encode properties of properties and are often incomplete with respect to the properties they encode, the latter couldn't possibly encode properties of properties.)

Now if we identify fictional species as abstract properties, then we will be able to *derive* the claim that fictional species aren't identical with any possible species. Kripke says ([1972, 157]):

If we suppose, as I do, that the unicorns of the myth were supposed to be a particular species, but that the myth provides insufficient information about their internal structure to determine a unique species, then there is no actual or possible species of which we can say that it would have been the species of unicorns.

So if we think of the possible species that Kripke is referring to here as our ordinary (i.e., possibly concrete) properties, and treat mythical species as abstract properties, we will be able to derive that no mythical species is a possible species.

Letting  $F$  be a variable ranging over properties (i.e., ranging over entities of type  $\langle i \rangle$ ), then our first theorem for higher-order fictions is that fictional properties are abstract properties.

- (16) If a property (of individuals)  $F$  is fictional,  $F$  is abstract  

$$Fictional(F) \rightarrow A!F$$

In (16), the predicate ' $A!$ ' has the type  $\langle\langle i \rangle\rangle$  and ' $Fictional$ ' is a condition defined so that it applies to properties of type  $\langle i \rangle$ . So (16) tells us that fictional properties (of individuals) are abstract objects of type  $\langle i \rangle$ . (16) is just the higher-order counterpart of (12). Before we derive (16), consider how it might be applied. From the premise that the property of being a unicorn (' $U$ ') is fictional, an instance of (16) would imply that  $U$  is an abstract property. Note that the derivation of (16) just follows the derivation of (12). From the definition of 'fictional' and the fact that  $F$  is fictional, we may infer that  $F$  originated in some story  $s$ . If so, then again by definition,  $F$  is abstract, is a character of  $s$ , and is not a character of any earlier story. Given this derivation of (16), it is straightforward to derive the following higher-order counterpart to (14), where ' $E!$ ' is a property of properties (having type  $\langle\langle i \rangle\rangle$ ):

- (17) A fictional property is not identical with any possible (i.e., possibly concrete) property.  

$$Fictional(F) \rightarrow \neg \exists G(\diamond E!G \ \& \ G = F)$$

If fictional properties are abstract, then clearly, they cannot be (identical to) ordinary properties. This is a plausible representation of Kripke's claim (b) mentioned at the outset of this section.

We can get even more specific by employing the following instance of the typed definition of the defined condition ' $Fictional-G$ ', where  $G$  is a variable ranging over properties of properties (i.e., ranging over entities with type  $\langle\langle i \rangle\rangle$ ):

- (18)  $F$  is a fictional  $G$  iff  $F$  originates in a story  $s$  such that in  $s$ ,  $F$  exemplifies  $G$ .  

$$Fictional-G(F) =_{df} \exists s[Originates(F, s) \ \& \ s \models GF]$$

Now to apply this definition, let ' $U$ ' again denote the property of being a unicorn (which is a property of individuals) and let ' $S$ ' be the property of being a species (which is a property of properties). So ' $U$ ' is of type  $\langle i \rangle$  and ' $S$ ' is of type  $\langle\langle i \rangle\rangle$ ; we may therefore substitute these terms for ' $F$ ' and ' $G$ ', respectively, in (18). Thus, the following instance of (18) tells us that the property of being a unicorn is a fictional species if and only if it originates in a story  $s$  according to which it exemplifies the higher-order property of being a species:

$$Fictional-S(U) \equiv \exists s[Originates(U, s) \ \& \ s \models SU]$$

So, given our work above, we now know how to derive the Kripkean thesis that if the property of being a unicorn is a fictional species, then it is not identical with any possible species.<sup>26</sup>

(One note of caution is in order, to avoid an ambiguity which arises from the disanalogy between the phrases 'fictional species' and 'fictional property' as they are used above. A fictional species such as being a unicorn does not exemplify the property of being a species, but only encodes it, just as the fictional detective Holmes encodes rather than exemplifies the property of being a detective. However, a fictional property (e.g., of type  $\langle i \rangle$ ) does exemplify the higher-order, categorial property of being a property, for it does fall into the domain of objects of type  $\langle i \rangle$ . The analogy to a case of lower type is that Holmes, as a fictional individual,

<sup>26</sup>In formal terms, the following is now derivable:

$$Fictional(F^{(i)}) \rightarrow \neg \exists G^{(i)}[\diamond(E!^{(i)}G \ \& \ SG) \ \& \ G = F]$$

This is an immediate consequence of (17).

has the logical type of an individual and so is in the domain of type  $i$ . Now if we assume that, in the stories, Holmes is attributed the categorial property of being an individual, our theory ensures that he encodes this property as well. Similarly, if in the myth, the property of being a unicorn is attributed the higher-order categorial property of being a property, then the fictional property encodes the higher-order property as well.)

Let me conclude this section by emphasizing that not only have Kripke's claims been derived from more general principles, but that the claims follow from a theory of fictional individuals and species. That fictional individuals and species have natures distinct from ordinary individuals and species is made reasonable by the fact that fictional individuals and fictional species are identified as entities that are incomplete along the dimension of their encoded properties. Of course, Kripke reaches his conclusions by arguing from both the assumption that names like 'Holmes' and 'being a hobbit' have unique denotations and the assumption that no member of the domain of possible entities could be uniquely singled out as their denotation. But our derivation of these Kripkean claims comes from a general theory and fills in the metaphysical blanks underlying Kripke's argument. And, as we shall see in the next section, the preceding ideas offer a precise way of spelling out Kripke's view that the names of fictional characters denote abstract objects.

#### 4. Validating Kripkean Claims About Language

In this section, we examine how the (semantic interpretation of the) formalism in which object theory is couched preserves and validates Kripke's analysis of natural language sentences when these sentences are represented in our formalism. We shall be examining Kripkean claims about the following features of natural language: rigid designation, modes of predication, and "levels of language" when storytelling. However, I should like to stress here none of our results undercuts the significance of Kripke's arguments for some his claims about these features of language. His arguments are intriguing and often compelling, and they focus on facts concerning natural language that we do not consider here. For example, Kripke's arguments for the rigid designation of names are in part grounded on certain assumptions about the way names are introduced into natural language. I shall have nothing to say here about those assumptions. The purpose of the following is rather to fill in Kripke's picture by providing a

precise background theory in which to formulate and represent certain semantical claims. I only attempt to show that Kripke's analyses of natural language sentences are preserved or even predicted by the formal representations of those sentences in object theory (sometimes in light of the semantic interpretation of that theory). This may provide independent confirmation of Kripke's views.

##### 4.1 Rigid Designation

It is an interesting fact about the construction of object theory that in its default state, all of its terms are rigid designators. Once the language of object theory is specified (and here we may revert to discussing the simpler, second-order modal version of the theory), a formal semantics can be defined. In previous work on the theory, we have specified that interpretations of the language include a single, fixed domain of individuals and a single, fixed domain of  $n$ -place relations. The use of 'fixed' domains draws attention to the fact that the system validates both the first- and second-order Barcan formulas. (For a philosophical defense of this kind of system, see Linsky & Zalta [1994].)

Now each individual term and relation term (simple or complex) of the language is assigned a denotation in the appropriate domain, relative to a given interpretation and an assignment to the variables. In other words (ignoring terms with free variables and suppressing the relativization to interpretations and variable assignments), the denotation of an individual constant or definite description is an element of the domain of individuals, and the denotation of a simple predicate or  $\lambda$ -expression is an element of the appropriate domain of relations. Thus, the denotation function is *not* relativized to the possible worlds taken as primitive in the semantics. Definite descriptions and predicates, as well as proper names (constants) are rigid designators. Although definite descriptions are rigid, the presence of encoding predications gives us a means for representing the definite descriptions of English which appear to function non-rigidly (see Zalta [1988], Chapter 5.5). Of course, it would be a routine exercise to revise the semantics so as to include non-rigid definite descriptions among the terms, but it is unclear whether there is a need to do so.

The fact that the denotation function is not relativized to possible worlds is a central metatheoretic feature of object theory. Although the denotation function maps predicates to properties or relations as their

denotations, the semantics of the system can still treat modal claims by supposing that the *extensions* of the properties vary from world to world. This makes the definition of truth extremely simple: An atomic exemplification formula such as ‘ $P^n a_1 \dots a_n$ ’, for example, is true at a world  $\mathbf{w}$  just in case the  $n$ -tuple of individuals denoted by ‘ $a_1$ ’,  $\dots$ , ‘ $a_n$ ’ is an element of the exemplification-extension, at world  $\mathbf{w}$ , of the relation denoted by ‘ $P^n$ ’. So the denotations of the terms of the language do not vary even in the context of modal operators. The truth definition stipulates that the atomic exemplification sentence ‘ $\Box Pa$ ’ is true just in case, at every world  $\mathbf{w}$ , the individual denoted by ‘ $a$ ’ is in the exemplification-extension, at  $\mathbf{w}$ , of the property denoted by ‘ $P$ ’. The question of what ‘ $a$ ’ and ‘ $P$ ’ denotes at other possible worlds doesn’t even arise.

These features of object theory have important consequences for the representation of terms of English. Clearly, when proper names of English are represented by rigid constants like ‘ $a$ ’ and ‘ $b$ ’ in object theory, the representations of identity claims involving proper names, such as ‘ $a=b$ ’, are necessarily true whenever true.<sup>27</sup> Moreover, this last fact generalizes to predicates and thereby becomes more significant. If we represent species terms of natural language as property-denoting expressions, then a term like ‘tiger’ will be analyzed as rigidly designating the species. Similarly, the species term ‘unicorn’ will rigidly designate a fictional species (i.e., an abstract property), just as ‘giraffe living in the Arctic Circle’ will rigidly designate an ordinary property. In general, identity statements of the form ‘ $P=Q$ ’ become necessarily true if true. Thus, when we represent identity claims of English in the system, they have the modal characteristics which Kripke argues that they have.

The representations of terms denoting species in the language of object

<sup>27</sup>In contrast to Kripke [1963], object theory doesn’t leave open the question as to how rigidly-designating constants are to be included in the system. The inclusion of constants in the language of object theory is unremarkable. The system employs a modal logic which preserves the theorems for the necessity of identity even when rigidly-designating constants are included in the system. By contrast, constants (rigidly-designating or otherwise) can’t be added to Kripke’s modal system in [1963] without undermining the steps he takes to invalidate the proofs of the Barcan Formula, the Converse Barcan formula, and other ‘offending’ theorems of modal logic. In his system, Kripke banished constants from the language and employed the generality interpretation for the variables. It is not immediately obvious how his system should be revised so as to capture the idea that names are rigid designators. Philosophers have debated about the best way of reintroducing constants back into Kripke’s system. See, for example, Deutsch [1990], [1994], and Menzel [1991].

theory suggest how we should analyze certain modal claims of natural language. First, consider a possibility claim with respect to an ordinary property. There is a clear sense in which the claim “There might have been giraffes in the Arctic circle” is true. If we represent the property denoted by ‘giraffe living in the Arctic Circle’ as  $[\lambda y Gy \ \& \ Lya]$ , then the true English sentence would be represented by the following sentence of object theory:

$$\diamond \exists x([\lambda y Gy \ \& \ Lya]x)$$

Here, the  $\lambda$ -predicate denotes an ordinary property, and the sentence is true just in case, in some possible world, something falls in the exemplification extension of that property at that world.

However, the claim “There might have been unicorns” turns out to be false if represented in an analogous way as:

$$\diamond \exists x Ux$$

It is not hard to see why this should turn out false, given our work in the previous section. Being a unicorn is a fictional property and, as a general hypothesis, we asserted that fictional properties couldn’t possibly be exemplified. Thus, ‘ $\diamond \exists x Ux$ ’ is false.

But, surely, there is at least one sense in which “There might have been unicorns” is true. Indeed there is. To represent this truth in object theory, consider first the simpler case of Sherlock Holmes. Although Holmes is not identical with any possible object, it is still true to say that there might have been a Sherlock Holmes. In object theory, to say this is to say that there might have been a (concrete) object which exemplifies all the properties which Holmes exemplifies in the story (i.e., that there might have been a concrete object which exemplifies all the properties which Holmes encodes). Similarly, to assert truly that there might have been unicorns is to assert that there might have been a concrete species  $F$  which both (i) exemplifies every property  $G$  that the fictional species *unicorn* encodes, and (ii) is itself exemplified by some concrete object. In formal terms, the following claim of object theory correctly represents the truth conditions of the claim that there might have been unicorns, where the first occurrence of ‘ $E!$ ’ has type  $\langle\langle i \rangle\rangle$  and the second occurrence has type  $\langle i \rangle$ , and the other terms are typed appropriately:

$$\diamond \exists F[E!F \ \& \ \forall G(UG \rightarrow GF) \ \& \ \exists x(E!x \ \& \ Fx)]$$

This claim may be consistently asserted in object theory; it semantically requires that, at some possible world  $\mathbf{w}$ , there exists a property  $F$  which (i) is concrete at  $\mathbf{w}$ , (ii) exemplifies at  $\mathbf{w}$  all of the properties of properties encoded by (the fictional property of) being a unicorn, and (iii) is exemplified at  $\mathbf{w}$  by some concrete object. Note that nothing in these truth conditions for “there might have been unicorns” requires us to consider what the terms of our language denote at other possible worlds. The truth conditions involve the fictional and ordinary properties that are in fact rigidly denoted by the predicates in the sentence—they consider whether there are ordinary properties which exemplify at other possible worlds the properties encoded by a certain fictional property. So when terms for fictional species, in addition to terms for ordinary species, are represented as rigid designators, we have a way of representing and preserving the truth value of intuitive but philosophically problematic claims of natural language.

## 4.2 Predication

The most striking way in which the present system validates Kripkean claims about natural language concerns its treatment of predication. In [1973], Kripke develops an informal analysis of the language of fiction by drawing a distinction between kinds of predication in natural language. In Lecture 3 (pp. 20–21), Kripke claims that there are two types of predication:<sup>28</sup>

But here there is a confusing double usage of predication which can get us into trouble. Well why? Let me give an example. There are two types of predication we can make about Hamlet. Taking ‘Hamlet’ to refer to a fictional character rather than to be an empty name, one can say ‘Hamlet has been discussed by many critics’; or ‘Hamlet was melancholy’, from which we can existentially infer that there was a fictional character who was melancholy, given that Hamlet is a fictional character. (p. 20)

One will get quite confused if one doesn’t get these two different kinds of predication straight. . . . the fictional people who live on Baker Street are not said to live on Baker Street in the same sense that real people are said to live on Baker Street. . . . (p. 21)

It should be mentioned here that on pp. 20–21, Kripke says things which suggests that instead of two kinds of *predication*, he had two kinds of *predicates* in mind, for he says:

These two predicates should be taken in different senses. The second predicate, ‘is melancholy’, has attached to the implicit qualifier ‘fictionally’, or ‘in the story’ [sic]. Whereas of course the first, ‘is discussed by many critics’, does not have this implicit qualifier. (p 20)

And later, just after the passage from p. 21 quoted above, he says:

In the one case one is applying the predicate straight; in the other case one is applying it according to a rule in which it would be true, if the people are so described in the story. And ambiguities can arise here because of these two uses of the predicate. (p. 21)

Now if we take *all* of these passages into consideration, there is some question as to whether Kripke is endorsing an ambiguity in two kinds of predication or in two kinds of predicate. But, given this ambiguity in Kripke’s discussion, he is subject to interpretation. Clearly, the two-modes-of-predication theory constitutes a legitimate interpretation of the view Kripke is outlining here, at least in so far as he finds that there is some kind of ambiguity in statements about fictions.

Here is how Kripke’s distinction is preserved in object theory. The claim ‘Hamlet was melancholy’ is contrasted with the claim ‘Hamlet has been discussed by (many) critics’ as follows:

$$hM$$

$$[\lambda y \exists x(Cx \ \& \ Dxy)]h$$

The first is an encoding predication of the form ‘ $xF$ ’, the second an exemplification predication of the form ‘ $Fx$ ’. Indeed, the language of object theory recognizes a structural ambiguity in natural language predications of the form ‘ $x$  is  $F$ ’. ‘Hamlet was melancholy’ can be represented either as the false exemplification claim ‘ $Mh$ ’ or as the true encoding claim ‘ $hM$ ’. The latter is provably equivalent to the claim of object theory which represents ‘In the play, Hamlet was melancholy’, namely,  $s \models Mh$  (see notes 15 and 25 for the definition of  $\models$ ). By contrast, ‘Hamlet was discussed by (many) critics’ is not true in the play (we are assuming). We may treat it

<sup>28</sup>See also Kripke [1973], Lecture 3 (p. 7), Lecture 4 (p. 18), and Lecture 5, pp. 1-2.

as similarly ambiguous; in this case, the exemplification reading displayed above is the true reading, while the encoding reading is false.

This distinction between two ways in which an abstract object may ‘have’ a property can be traced back to Ernst Mally ([1912], §33).<sup>29</sup> Mally’s distinction between the notion of an object ‘satisfying’ a property and that of a property ‘determining’ an object was formally captured in object theory in terms of the distinction between exemplifying and encoding a property. As far as I have been able to discover, J. Findlay ([1933], 110-112) was the first English speaking philosopher to explicitly recognize this distinction in Mally’s work.<sup>30</sup> No doubt, some philosophers here will point out that Mally was a student of Meinong and that Kripke explicitly disassociates his analysis of fiction from Meinongian object theory ([1973], 23). But, clearly, Mally’s theory is *not* a strict Meinongian theory, since Mally postulates no objects that exemplify the properties attributed to Pegasus, Zeus, the fountain of youth, etc. Moreover, Meinong has a *univocal* notion of predication, and since Mally’s ideas in the above passages point to an equivocal notion of predication, Kripke’s theory is a ‘Mallyan’ theory if our interpretation is right.

Finally, note that Kripke endorses a principle which is made precise and explicit in object theory. He says, in Lecture #3:

Using predicates according to their use in fiction, that is according to the rule which applies a predicate to a fictional character if that character is so described in the appropriate work of fiction, we should conclude that Hamlet was not a fictional character. In fact, paradoxical as it may sound, in this sense no fictional person is a fictional person. For no fictional person is said in his own work of fiction to be a fictional person. (p. 21)

Note first that the ‘rule’ Kripke produces is basically the principle of identification we use for characters  $\kappa_s$  which are native to story  $s$ :

$$\kappa_s = \iota x(A!x \ \& \ \forall F(xF \equiv s \models F\kappa))$$

<sup>29</sup>Translations of the crucial passages by Alfons Süßbauer and myself can be found in Zalta [1998].

<sup>30</sup>See Findlay [1933], 110-112, 183. See also Rapaport 1978, where a similar distinction is developed, and Boolos 1987, which suggests that this distinction might be present in Frege as well.

This principle identifies the native fictional character  $\kappa$  of story  $s$  with the abstract object that encodes just the properties attributed to  $\kappa$  in story  $s$ . So, given a body of data of the form “In story  $s$ ,  $F\kappa$ ”, this principle gives us a systematic way to identify  $\kappa$ .

Note second that the ‘paradoxical’ way of speaking Kripke mentions has a natural explanation in object theory. Fictional characters will typically not encode the property of being a fictional character (assuming there is such a property), but they will exemplify it. If natural language predication is ambiguous in the way we claim, then we would read the true technical claim “Holmes doesn’t encode being a fictional person” in terms of the following ambiguous English sentence: Holmes isn’t a fictional person. Thus, the ‘paradoxical’ use of language is predicted on the present view.

### 4.3 ‘Levels’ of Language

We conclude this section by noting how object theory incorporates one other feature of language that Kripke takes to be important for the proper analysis of fiction. In Lecture 6 of [1973], Kripke talks about “levels” of language:

One should bear in mind here that one should not confuse levels of language. Where I said originally that an empty name was just a pretence, as in the case of Hamlet, or a mistake, as in the case of Vulcan (where one thought a name had been properly introduced when it had not), that was one level of language. An extended level of language was set up by the invention of an ontology of fictional characters, of legendary objects; and this level uses just the same names for them as were originally empty. This happens especially in the case of pretence in fiction. But one shouldn’t confuse this level of language with the previous one. One should not say that, when one is just pretending to refer to a man though really one is not, that that pretence was in and of itself naming a fictional character. That was *creating* a fictional character. p. 19

Although others have suggested that the use of language in creating a fiction is a “special” use, it is important to the theme of the present paper to recall that the “levels” of language view was independently developed in object theory in Zalta [2003 (1985), 1987]. I argued there that when authors are in the process of authoring a story, they are using language

not to refer, but rather to “baptize” their characters. I invoked Kripke’s [1972] causal theory of reference to trace the subsequent uses of fictional names back to the storytellings in which the names were introduced. However, the storytelling itself was viewed as an extended baptism. Once the storytelling is complete, the names and descriptions involved in the storytelling become analyzable in terms of the background ontology of object theory. The comprehension principle for abstract objects and the definitions rehearsed in this and the previous section allow us to theoretically identify the denotations of the names and descriptions used in the storytelling. This idea, then, captures at least one relevant sense in which there are two “levels” of language.<sup>31</sup> Of course Kripke here also explicitly suggests that authors “create” fictional characters when authoring their stories. This suggestion (and related ones) form the subject matter of the final section.<sup>32</sup>

## 5. The Contingency of Fictions

In [1973], Kripke argues that ordinary language quantifies over a realm of fictional entities. (He says this explicitly in Lecture 3, on p. 16 and again on p. 18.) However, Kripke also holds further views about fictional characters which seem inconsistent with the theory of abstract objects. He asserts the following claims:

- (A) It is an empirical question whether there was such and such a fictional character. (Lecture 3, p. 18)
- (B) [Fictions] exist in virtue of more concrete activities of telling stories. (Lecture 3, p. 19)
- (C) There are fictional characters in the actual world, right here in the ordinary concrete world. (Lecture 3, p. 16)

It is not too difficult to see why these claims might be thought inconsistent with object theory. The present theory defines an “abstract” object to be an object that couldn’t exemplify concreteness. If being spatiotemporal necessarily implies being concrete, then objects which couldn’t exemplify

<sup>31</sup>See also Kroon [1992], for an important investigation which shows that the “two-levels-of-language” view is found in Meinong.

<sup>32</sup>See Deutsch [1991] for an in-depth analysis of the sense in which authors create their characters. The view proposed in the final section of the present paper, however, offers yet another sense in which authors create characters.

concreteness couldn’t exemplify spatiotemporality.<sup>33</sup> It would seem that the existence of such objects is not a contingent matter or an empirical question. But since the present theory identifies fictional characters as abstract objects, it also seems that object theory implies that it is not an empirical question whether there was such and such a fictional character, that fictions do not exist in virtue of more concrete activities such as telling stories, and that fictional characters are not ‘in’ the ordinary concrete world.

But things are not quite what they seem. There are several ways in which object theory can accommodate the idea that fictions are contingent entities. One way is to note that whenever an abstract object satisfies the definition of “ $x$  is a story”, it does so contingently. Consider a particular case, such as Günther Grass’s novel *The Tin Drum*. To say that *The Tin Drum* is a story is to say (given the definition in §3) that (i) every property *The Tin Drum* encodes is a propositional property, and (ii) some concrete object authored *The Tin Drum*. But the abstract object which satisfies this definition does so contingently, since it is a contingent fact that someone authored this novel. Moreover, if it is a contingent fact that certain abstract objects are stories, it is a contingent fact that certain other abstract objects are characters (which originate) in those stories. For example, our definitions identify a certain abstract object as the fictional character Oskar Mazerath of *The Tin Drum*. This abstract object, namely Mazerath, only contingently satisfies the definition of “ $x$  originates in *The Tin Drum*”. If no one had authored *The Tin Drum*, then the latter would not have been a story and Mazerath would not have been (identifiable as) a character which originates in the story. Note also that it is a contingent fact that Mazerath is not a character of any earlier story. This is another part of the definition of “originates in” which contingently obtains when it does obtain.

So this is one way in which the idea that fictions are contingent entities can be accommodated within the present theory, namely, it is a contingent fact about the abstract objects that are fictions *that* they are fictions. We can’t label the abstract objects described by object theory *as* “fictions”, “legends” or “mythical characters” unless authors behave in the ways that

<sup>33</sup>If being spatiotemporal ( $S$ ) necessarily implies being concrete, then  $\Box(Sx \rightarrow E!x)$ . Since it is a theorem of modal logic that  $\Box(\varphi \rightarrow \psi) \rightarrow (\Diamond\varphi \rightarrow \Diamond\psi)$ , it then follows that  $\Diamond Sx \rightarrow \Diamond E!x$ . So an abstract object  $x$  is, by definition, such that  $\neg\Diamond E!x$ . Therefore,  $\neg\Diamond Sx$ , i.e., abstract objects couldn’t exemplify spatiotemporality.

are necessary to author stories. But this suggestion may be refined further to reflect the observation that some fictional characters are baptized *together* and are identifiable in terms of the relations that they bear to one another in the story. Holmes is not just an abstract object which encodes being a detective, living in London, being extremely clever, etc. (Such an object would be independent of the contingent storytelling behavior of any authors.) Rather, Holmes also encodes such properties as being a friend of Watson, tracking down the evil genius Professor Moriarty, etc. The properties denoted by ‘being a friend of Watson’ and ‘tracking down Moriarty’ are *not* ordinary properties that can be assumed in the system *prior* to the storytelling acts of authors such as Conan Doyle. For we can assert the existence of such properties only when names such as ‘Watson’ and ‘Moriarty’ are (a) used in a special way in a storytelling to baptize characters and (b) become referential once the storytelling is complete. These are conditions that obtain contingently.

If this is correct, then in many cases of storytelling, the author introduces new names and expressive power into the language. Fictional characters whose extended baptisms involve the names of other fictional characters in the story may therefore become fictional only when authors (contingently) introduce new names and expressive power into the language during the course of a storytelling.<sup>34</sup>

<sup>34</sup>Indeed, this new expressive power makes it more difficult to analyze the truth that there might have been a Sherlock Holmes. As we saw above, this was to be represented as: possibly, some concrete object, say *b*, exemplifies all of the properties that Holmes encodes. But one of the properties that Holmes encodes is being a friend of Watson. Now if Watson is identified as an abstract object, then it would follow that *b* would exemplify being a friend of an abstract object. Given that we aren’t talking about the philosophical sense in which someone might be a friend of abstract objects, one might conclude that since no ordinary object could possibly exemplify the property of being a friend of some abstract object, there couldn’t be a Sherlock Holmes.

To address the concern, the object theorist may follow Currie’s [1990] (pp. 150–155) strategy of Ramseyfying. We first suppose there is a finite sequence of formulas  $\varphi_1, \dots, \varphi_n$ , which constitute the truths expressed in the Conan Doyle novels. Let  $\psi$  be the conjunction of all of these sentences. In this scenario, a proposition is true in the story if it is relevantly entailed by the proposition denoted by  $\psi$ . Now replace each name *n* in  $\psi$  other than ‘Sherlock Holmes’ with a distinct variable, yielding the formula  $\psi^*$ , and then prefix  $\psi^*$  with an existential quantifier for each new variable. The resulting Ramsey sentence relative to ‘Holmes’ is therefore true in the story. It follows that Holmes encodes the property that is denoted by the  $\lambda$ -expression that is formed by dropping out all of the occurrences of ‘Holmes’ from the Ramsey sentence and replacing them with a single variable bound by the  $\lambda$ . Call the resulting property “the individual concept of Holmes”. The sense in which “There might have been a

The above ideas go some way towards accomodating Kripke’s claims (A) and (B) above. To accommodate claim (C), however, a more radical suggestion is needed, namely, a *reinterpretation* of the formalism of object theory on which fictions and other abstract objects are conceived as somewhat more concrete parts of our world. Before we describe this reinterpretation, note that the theory of abstract objects has traditionally had *two* interpretations. On one interpretation, the theory offers a version of Meinongianism (though not a strict version of Meinongianism, as we noted at the end of §4.2). On this interpretation, the quantifier ‘ $\exists$ ’ is interpreted as existentially *unloaded*—it simply means ‘there is’ rather than ‘there exists’. This interpretation exploits a subtle distinction in natural language (which we can recognize in sentences like “There are fictional detectives that criminologists admire, but none of them exist” and “There are lots of fictional characters which have never been discussed by English professors.”) We can appeal to this distinction to interpret and distinguish simple quantified claims of the form  $\exists x\varphi$  from *existential* claims of the form  $\exists x(E!x \ \& \ \varphi)$ . On the Meinongian interpretation of the formalism, the predicate ‘*E!*’ serves as an existence predicate which is not co-extensive with the quantifier, and when object theory asserts that there *are* abstract objects (in terms of the quantifier in the comprehension principle), the abstract objects in question, by definition, necessarily fail to exist.

On the second traditional interpretation, the one we’ve used in the present paper, object theory offers a kind of Platonism. On this interpretation, the quantifier ‘ $\exists$ ’ is given a Quinean and existentially loaded reading. The predicate ‘*E!*’ denotes the property of being concrete, and when object theory asserts (in terms of the quantifier of the comprehension principle) that there exist abstract objects, it is asserting the existence of necessarily nonconcrete objects.

In recent work, however, another interpretation of the formalism has been proposed.<sup>35</sup> This interpretation is somewhat different in kind from the first two, since it identifies the abstract objects postulated by the

Holmes” is true, then, is that there might be some ordinary object which exemplifies the individual concept of Holmes.

<sup>35</sup>See Zalta [2000], for example. This new interpretation has also been the subject of a presentation (“A Solution to the Problem of Abstract Objects”) delivered at the 1998 Australasian Association of Philosophy conference at Macquarie University and at the 22nd International Wittgensten Symposium in Kirchberg, Austria, in August 1999.

theory with a somewhat more familiar kind of entity. The proposal is to reinterpret the formalism of object theory so that it becomes a theory of contingent property patterns which supervene on the ways in which properties are exemplified in the actual world. On this interpretation, certain abstract objects, namely the fictions, are simply identified with certain large-scale patterns of properties which are grounded in the systematic linguistic and behavioral patterns of authors.<sup>36</sup> So when some philosophers say that sentences involving the name ‘Holmes’ are just *manners of speaking*, the suggestion is to take this idea literally. The existence of such large-scale patterns of properties is a contingent matter. If authors had not behaved in certain ways, the property patterns would not have existed. To complete the view, we: (a) interpret the comprehension principle for abstract objects as a comprehension principle that circumscribes the existence of large scale patterns of properties, and (b) interpret ‘ $xF$ ’ as ‘ $F$  is an element of the pattern  $x$ ’. Consequently, on the proposed interpretation, we are no longer to think of the comprehension principle as an *a priori* principle of metaphysics, but rather are to conceive it as a principle which *systematizes our linguistic and behavioral practices*. After all, the comprehension principle can be instantiated to assert the existence of particular fictions *only* after authors tell stories and described characters that can be explicitly named and referred to in instances of comprehension.

If something like this interpretation is viable, then fictional characters and stories alike become identified as contingently existing patterns of properties. With such an identification, we go some way towards preserving Kripke’s suggestion that there are fictional characters ‘right here in the ordinary concrete world’. The theory of abstract objects may not be inconsistent with this part of his analysis of fictions. Of course, if it turns out that this reinterpretation of object theory cannot be reasonably sustained, then one might challenge Kripke to give us an account of fictions that is consistent with the views he expresses in [1973] and which explains how fictions can be ‘right here in the ordinary concrete world’.

<sup>36</sup>One might even try to take this idea further by adjusting the underlying property theory to make it consistent with the empirical conception of properties canvassed in Swoyer [1993].

## Appendix: Derivations of the Theorems

**Derivation of:**  $\neg\exists w\exists p(w \models (p \& \neg p))$ .

**Proof:** Suppose, for reductio, that  $\exists w\exists p(w \models (p \& \neg p))$  and that  $w_1$  and  $q$  are an arbitrary such world and proposition, respectively; i.e.,  $w_1 \models (q \& \neg q)$ . By the definition of world, we also know  $\diamond\forall p(w_1 \models p \equiv p)$ . Let  $\varphi$  represent the embedded modal claim  $\forall p(w_1 \models p \equiv p)$ . So we know  $\diamond\varphi$ . Now we prove a claim of the form  $\Box(\varphi \rightarrow \psi)$ , from which we can derive  $\diamond\psi$  (by a theorem of S5). Let  $\psi$  be  $\neg w_1 \models (q \& \neg q)$ . To derive the modal conditional, we first derive the embedded conditional, so assume  $\varphi = \forall p(w_1 \models p \equiv p)$  and instantiate the universal quantifier to the proposition  $(q \& \neg q)$ . It follows that  $(w_1 \models (q \& \neg q)) \equiv (q \& \neg q)$ . But it is a theorem of logic that  $\neg(q \& \neg q)$ . So it follows that  $\psi = \neg w_1 \models (q \& \neg q)$ . Thus, the conditional  $\varphi \rightarrow \psi$  is provable, and by the Rule of Necessitation, we have that  $\Box(\varphi \rightarrow \psi)$ . From this latter fact and  $\diamond\varphi$ , it follows, by a theorem of modal logic, that  $\diamond\psi = \diamond\neg w_1 \models (q \& \neg q)$ , i.e.,  $\neg\Box w_1 \models (q \& \neg q)$ .

Now, the Logic of Encoding ensures that  $\diamond xF \rightarrow \Box xF$  (encoding is rigid and not relative to any circumstance). The following is an instance of the Logic of Encoding, given the definition of  $\models$ :

$$[\diamond w_1 \models (q \& \neg q)] \rightarrow [\Box w_1 \models (q \& \neg q)]$$

Now given that

$$[w_1 \models (q \& \neg q)] \rightarrow [\diamond w_1 \models (q \& \neg q)]$$

is an instance of the T-schema dual, we may assemble the previous two instances to conclude:

$$[w_1 \models (q \& \neg q)] \rightarrow [\Box w_1 \models (q \& \neg q)]$$

It now follows by Modus Tollens that  $\neg w_1 \models (q \& \neg q)$ , contrary to our initial assumption for reductio.  $\times$

**Derivation of:**  $\forall p[(\neg p \& \diamond p) \rightarrow \exists w(\neg Actual(w) \& w \models p)]$ .

**Proof:** Pick an arbitrary proposition  $q$  such that both  $\neg q$  and  $\diamond q$ . Since  $\diamond q$ , and  $\diamond p \equiv \exists w(w \models p)$  is a theorem (noted in the text), we can infer that there is a world, say  $w_1$ , where  $q$  is true, i.e.,  $w_1 \models q$ . We then simply show that  $w_1$  is not actual. But the definition of actuality tells us that  $w_1$  is actual iff every proposition true at  $w_1$  is true (and vice versa). But  $q$  is a proposition true at  $w_1$  which, by hypothesis, is not true. So  $w_1$  is not actual.  $\times$



**Derivation of (7):**  $x = x$ .

**Proof:** By Disjunctive Syllogism from the fact that  $O!x \vee A!x$ . From each disjunct, we establish that  $x = x$ .

(i) Suppose  $O!x$ . Then, by definition (6) in our text, to show that  $x = x$ , we have to show:

$$(6') [O!x \& O!x \& \Box\forall F(Fx \equiv Fx)] \vee [A!x \& A!x \& \Box\forall F(xF \equiv xF)]$$

It suffices, then, to show the first disjunct. But it is a theorem of logic that  $\forall F(Fx \equiv Fx)$ . And by the Rule of Necessitation, this fact must be necessary. So  $\Box\forall F(Fx \equiv Fx)$ . Since we know  $O!x$  by assumption, we have established that:

$$O!x \& O!x \& \Box\forall F(Fx \equiv Fx),$$

which is what we had to show.

(ii) Suppose  $A!x$ . Then, again by definition (6) in our text, to show that  $x = x$ , we have to show (6'), as described above. This time, however, it suffices, then, to show the second disjunct. But it is a theorem of logic that  $\forall F(xF \equiv xF)$ . And by the Rule of Necessitation, this fact must be necessary. So  $\Box\forall F(xF \equiv xF)$ . Since we know  $A!x$  by assumption, we have established that:

$$A!x \& A!x \& \Box\forall F(xF \equiv xF),$$

which is what we had to show.

Now since  $O!x \vee A!x$ , it follows that  $x = x$ .  $\bowtie$

**Derivation of (4'):**  $x =_E y \rightarrow \Box(x =_E y)$ .

**Proof:** Assume  $x =_E y$  (to show:  $\Box(x =_E y)$ ). So, by the axiom governing  $=_E$  and the definition of identity in (6), (we've seen that)  $x = y$ . Note that the following claim is now derivable indirectly from (1) or assertible directly as an instance of (5):

$$x = y \rightarrow [\Box(x =_E x) \rightarrow \Box(x =_E y)]$$

(To derive this claim from (1), we instantiate both occurrences of the variable ' $F$ ' in (1) by the  $\lambda$ -expression  $[\lambda z z =_E x]$  and then use  $\lambda$ -conversion to simplify.) So, since we've established  $x = y$ , it follows that  $\Box(x =_E x) \rightarrow \Box(x =_E y)$ . So we need only prove that  $\Box(x =_E x)$ . We do this by building up the definition of ' $x =_E x$ '. From the axiom governing

' $=_E$ ', it follows from our initial assumption ( $x =_E y$ ) that  $O!x$ . Now in object theory, ordinary objects (' $O!x$ ') are, by definition, possibly concrete (' $\diamond E!x$ '). So, by S5, we know  $\Box\diamond E!x$ , i.e.,  $\Box O!x$ . Now, put this intermediate conclusion temporarily aside and consider next that it is a theorem of logic that  $\forall F(Fx \equiv Fx)$ . So by the Rule of Necessitation, we know  $\Box\forall F(Fx \equiv Fx)$ . And, again by S5, it follows that  $\Box\Box\forall F(Fx \equiv Fx)$ . So, putting things that we already know into a conjunction, we have established:

$$\Box O!x \& \Box O!x \& \Box\Box\forall F(Fx \equiv Fx)$$

Now a conjunction of necessities is a necessary conjunction:

$$\Box[O!x \& O!x \& \Box\forall F(Fx \equiv Fx)]$$

So, by definition of ' $=_E$ ', we have therefore established that  $\Box(x =_E x)$ , which is what we had to show to complete our derivation.  $\bowtie$

**Derivation of (10):**  $p = q \rightarrow \Box(p = q)$

**Proof:** Assume  $p = q$ . So, by definition of proposition identity in footnote 13,  $[\lambda y p] = [\lambda y q]$ . Again, by the definition of property identity (9), this is equivalent to:

$$\Box\forall x(x[\lambda y p] \equiv x[\lambda y q])$$

But, then, by S5,

$$\Box\Box\forall x(x[\lambda y p] \equiv x[\lambda y q])$$

So, by definition,  $\Box(p = q)$ .  $\bowtie$

**Derivation of (12):**  $Fictional(x) \rightarrow A!x$ .

**Proof:** Assume that  $x$  is fictional. Then, by definition of 'fictional', there is a story  $s$  such that  $x$  originates in  $s$ ; i.e.,

$$\exists s[Story(s) \& Originates(x, s)]$$

So pick an arbitrary such story, say  $s_1$ . Then since  $x$  originates in  $s_1$ , it follows by the definition of 'originates in' that  $x$  is an abstract object which is a character of  $s_1$  and which is not a character of any earlier story. So  $x$  is an abstract object.  $\bowtie$

**Derivation of (14):**  $Fictional(x) \rightarrow \neg\exists y(\diamond E!y \& y = x)$ .

**Proof:** Assume  $x$  is fictional. Then, by the previous theorem,  $x$  is an abstract object. So, by the definition of ‘abstract’,  $\neg\Diamond E!x$ ; i.e.,  $x$  is not the kind of thing that could be concrete or spatiotemporal. So  $x$  is not identical with any possibly concrete object, on pain of contradiction.  $\boxtimes$

**Derivation of (16):**  $Fictional(x^t) \rightarrow A!(^t)x^t$ .

**Proof:** (We derive the most general case, where  $x$  is an object of an arbitrary type  $t$ .) Assume that  $x^t$  is fictional. Then, by the typed version of definition of ‘fictional’, there is a story  $s^i$  such that  $x^t$  originates in  $s$ ; i.e.,

$$\exists s^i[Story(s) \ \& \ Originates(x^t, s)]$$

So pick an arbitrary such story, say  $s_1$ . Then since  $x^t$  originates in  $s_1$ , it follows from the typed version of definition ‘originates’ that  $x^t$  is an abstract object which is a character of  $s_1$  and which is not a character of any earlier story. So  $x^t$  is an abstract object of type  $t$ .  $\boxtimes$

**Derivation of (17):**  $Fictional(x^t) \rightarrow \neg\exists y^t(\Diamond E!(^t)y^t \ \& \ y^t = x^t)$ .

**Proof:** (We derive the most general case, where  $x$  is an object an arbitrary type  $t$ .) Assume  $x^t$  is fictional. Then, by the previous theorem,  $x^t$  is an abstract object. So, by the typed definition of ‘abstract’,  $\neg\Diamond E!(^t)x^t$ ; i.e.,  $x^t$  is not the kind of thing that could be concrete. So  $x^t$  is not identical with any possibly concrete object of type  $t$ , on pain of contradiction.  $\boxtimes$

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