Logic and Metaphysics*

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1. Introduction

In this article, we canvass a few of the interesting topics that philosophers can pursue as part of the simultaneous study of logic and metaphysics. To keep the discussion to a manageable length, we limit our survey to deductive, as opposed to inductive, logic. Though most of this article will focus on the ways in which logic can be deployed in the study of metaphysics, we begin with a few remarks about how metaphysics might be needed to understand what logic is.

When we ask the question, "What is logic and what is its subject matter?", there is no obvious answer. There have been so many different kinds of studies that have gone by the name 'logic' that it is difficult to give an answer that applies to them all. But there are some basic commonalities. Most philosophers would agree that logic presupposes (1) the existence of a language for expressing thoughts or meanings, (2) certain analytic connections between the thoughts that ground and legitimize the inferential relations among them, and (3) that the analytic connections and inferential relations can be studied systematically by investigating (often formally) the logical words and sentences used to express the thoughts so connected and related. For example, analytic connections give rise to various patterns of inferences expressed by certain logical words and phrases like 'not', 'if-then', 'and', 'every', 'most', 'the', 'equals', etc., or expressed by predicates like 'red' and 'colored', or expressed by modal words such as 'necessarily', 'possibly', and actually', or expressed in the subjunctive mood or with tenses, etc.

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If we proceed from this rough outline of what logic is to a more specific statement of the patterns and laws that a logician might recognize and formulate, respectively, we start to do metaphysics. For example, consider the pattern of inference, or logical law, that might be expressed as follows: the thought expressed by the sentence 'P *and* Q' logically implies the thought expressed by the sentence 'P'. We might express this law somewhat differently by saying that the proposition *that* P *and* Q logically implies the proposition *that* P. However we express it, the law in question doesn't merely relate uninterpreted pieces of syntax. It is a law in virtue of the thoughts or propositions expressed by the sentences involved, i.e., in virtue of the meanings of those sentences. Thoughts, propositions, and meanings are objects of study in metaphysics. Moreover, it is often said that by systematizing a body of inferences in terms of laws like the one just described, logic becomes the study of the logical consequence relation.[1] Not only does saying this much posit an abstract *relation*, but a moment's reflection suggests that logical consequence relates thoughts, propositions, or sentence meanings, etc., not uninterpreted sentences. One important part of metaphysics involves formulating theories of these abstracta, taking them as objects of study in their own right.

Of course, a logician might eschew talk of thoughts and propositions and formulate the example law by saying: the sentence 'P *and* Q' logically implies the sentence 'P'. But this doesn't help them avoid doing metaphysics, for this reformulated version of the law is not about the particular physical inscriptions of the expressions 'P', 'Q' and '*and*' now printed in the book before you (or being presented electronically in the display before you). Rather, it is a generalization about the *types* of expressions that the particular physical tokens before you are instances of: symbol types, types of sounds, letter types, word types, sentence types, etc. Expression types, in contrast to expression tokens, are abstract objects, and once we invoke types to help us describe the subject matter of logic, we

[1]See Etchemendy 1990 and Hanson 1997 for studies of the concept of logical consequence.

are again in the domain of metaphysics. (For an excellent study of the type-token distinction and its ubiquity, see Wetzel 2009.) Moreover, it seems reasonable to suppose (a) that each of the subsentential parts of the sentence 'P *and* Q' has a semantic significance, or meaning, which combine with the meanings of the other subsentential parts to produce the meaning of the whole sentence, and (b) that neither the expression types, nor the meanings, of the subsentential expressions referenced in the analysis of the meaning of the whole sentence are particular physical objects or obviously isolatable parts of the physical world.

Consequently, though logicians are frequently fond of supposing that the principles of logic themselves should not imply the existence of anything, the attempt to say what logic is may reveal that it presupposes the existence of one or more of the following: thoughts (or propositions), meanings (sentential and subsentential), the logical consequence relation (or other relations), truth-values, expression types, symbol types, and possibly other logical objects. This is a fruitful area of research for philosophers interested in the metaphysical underpinnings of logic.

It might be thought that the foregoing remarks are to be understood as a discussion of how metaphysics might be needed in *the philosophy of logic*, as opposed to *logic* itself. Let's then turn specifically to how metaphysics might be needed for logic itself. One crux of the interaction between the two is in the concept of predication. In the linguistic, formal mode, we might say that a predication is a sentence in which a predicate is asserted to hold of some subject. But in the more metaphysical material mode, we might say that a predication is a statement to the effect that an object *exemplifies* a property (or instantiates a universal, or has an attribute, or falls under a concept). More generally, a predication is a statement to the effect that n objects exemplify, or stand in, an n-place relation (properties are henceforth treated as 1-place relations). Predications form the basis of thought and without atomic thoughts (or, as we shall say, atomic propositions), we wouldn't have a basis either for the logical consequence relation or for the principles of propositional logic and first-order logic. In the remainder of this introductory section, we focus on the metaphysics of predication and logic.

Some logicians and mathematicians believe that no special metaphysics is required for the analysis of predication. At the beginning of the modern era of logic, Frege (1891) used mathematical functions to analyze both predication in natural language and the structure of our thoughts. Frege's

analysis of the simple predication 'John is happy' treated the predicate 'is happy' as signifying a function that maps all happy objects to the truth-value The True and everything else to the truth-value The False (Frege called functions whose values are truth-values 'concepts'). So when the object denoted by 'John' is mapped by the concept signified by 'is happy' to The True, Frege suggested that the whole sentence 'John is happy' becomes a name of The True. He introduced general rules of inference that ensured that the sentence 'John is happy' is derivable from the sentence 'Something is happy' (the latter being analyzed as the result of applying the second-level concept 'something', which is also a function, to the first-level concept 'happy', which is the argument of the function).

Logicians and mathematicians following Frege recognized that (a) functions could in turn be analyzed as sets of ordered pairs, (b) that functions which map their arguments to a truth-value could be replaced by the set of the objects which are mapped by the function to The True, and (c) any residual role that might require 'truth values' could be played by the numbers 1 and 0 (which themselves could be given a set-theoretic definition). Thus, by working within the mathematical framework of set theory, one could develop an analysis on which the natural language predication 'John is happy' is true just in case the object denoted by 'John' is an element of the set of things denoted by 'is happy'. This analysis generalized to relations, by treating n-place relations as sets of ordered n-tuples. For example, 'John loves Mary' becomes analyzed as: $\langle$John,Mary$\rangle$ is an element of the set of pairs denoted by 'loves'. By applying set theory in this way, and independently developing a mathematically-based proof theory for studying inferential relations, researchers could pose and solve interesting questions in logic without doing any metaphysics. Thus, logic became like the other sciences—only mathematics is needed for its study.

A more philosophically-informed tradition, however, has recognized that metaphysics is still quite relevant to this task. Statements about set membership, of the form $x \in y$, are themselves particular predications of the form Rxy and set theory itself is formulable within first-order predicate logic, the simplest system for studying predication. Indeed, set membership is less general than predication, since it is only one of many examples of relations that can be used in a predication. Though it does prove useful, for some purposes, to *model* relations and predication using sets and set membership, the latter can't be all there is to a theory of relations and predication. The concepts of *relation* and *predication* have features that would be lost if we were to suppose that they were completely reducible to sets and set membership: (1) whereas sets simply contain, or at best *classify*, their members, relations and properties *characterize* the objects of which they are truly predicated (i.e., relations and properties provide the *characteristic* the objects exhibit in virtue of which they may be classified together), and (2) whereas sets are regarded as identical when they have the same members, relations are not (Quine 1951). These features may become more vivid with an example involving a property: (1) the property of *being a creature with a heart* characterizes an object x in a way that is not captured by saying that the set of creatures with a heart contains or classifies x (indeed, one might wonder how the set in question can be specified without somehow appealing to the property that characterizes the members of the set), and (2) the property of *being a creature with a heart* is not identical to the property of *having a circulatory system*, though if we were to reduce these properties to sets, they would collapse, since they would contain the same members (let us suppose).

Frege's analysis of predication in terms of functional application also fails with respect to (1) and (2): (1) functions simply *map* their arguments to values, and don't characterize their arguments in any way, and (2) even though Frege wouldn't identify functions with sets, he would have identified extensionally-equivalent functions, i.e., ones that map the same arguments to the same values, thereby collapsing the functions *being a creature with a heart* and *being a creature with a circulatory system*. Moreover, Frege's background logic is not metaphysics free; it not only postulates functions (i.e., 'unsaturated' entities that Frege thought were distinct from sets), but also postulates the two truth-values to represent truth and falsity. These are entities that can't be arbitrarily identified with the numbers 0 and 1, for otherwise they wouldn't have the metaphysical significance that truth and falsity have. Though Whitehead and Russell (1910–1913) analyzed predication in terms of propositional functions, their notion of propositional function is more like our concept of a relation, since a proposition can be considered a 0-place relation, something which results when an n-place relation (propositional function) is predicated of n arguments. Consequently, the logic of their famous treatise (1910–1913) is based on certain metaphysical notions (Linsky 1999).

With this brief review of how metaphysics is needed to understand (the subject matter of) logic, we may turn to the ways in which logic

itself can be useful in the study of metaphysics. We examine three topics in greater depth in Section 2, 3, and 4, and conclude with a section that provides more breadth through an overview of a variety of other topics. In our first example, we see how logic can be used in the study of the metaphysics of predication, which as we've seen, is a concept that plays a role in our understanding of logic itself.

2. Logic for Properties and Relations

Once a philosopher has mastered the basics of propositional logic and first-order predicate logic (with identity, function terms, and definite descriptions), it is natural to extend the quantification theory couched in first-order logic so as to be able to quantify over relations. Leibniz's Law of the identity of indiscernibles makes use of such a quantification. It asserts: *if* for every property F, x exemplifies F if and only if y exemplifies F, *then* x and y are identical. In the language of second-order logic, this would be expressed: $\forall F(Fx \equiv Fy) \rightarrow x = y$. This introduction of a quantifier over properties can be traced to the legitimacy of an inference that has its roots in Plato's One-Over-Many Principle: from the facts that x exemplifies F and y exemplifies F, it follows that there is something that x and y both exemplify.[2] There is a body of *data* that is constituted by (1) statements that quantify over, or refer to, properties and relations, and (2) the facts about the logical consequences of such statements. A real question arises about the meaning, or semantic significance, of gerundive expressions like *being so and so* that are so ubiquitous in natural language and in the sciences. It is perfectly natural in mathematics, for example, to discuss and formulate principles or theorems about the relationship between properties of numbers, e.g., theorems about the relationship between the property of *being prime and greater than 2* and the property of *being odd*. The simplest logical analysis of such data involves the use of terms that denote, and quantifiers that range over, relations and properties.

As an example of quantifying over complex relations, consider all the objects x and y which are such that (a) x is odd, (b) y is even, and (c) $x > y$. It would be natural to speak generally about a relation that holds between all and only those objects x and y for which all three conditions

[2]The nominalistic tradition in philosophy denies this is a valid inference, but we shall be focusing only on the tradition that accepts the inference.

hold. In the language of second-order logic, we might assert that there exists such a relation as follows:

$$\exists R \forall x \forall y (Rxy \equiv Ox \,\&\, Ey \,\&\, x > y)$$

Such a claim asserts the existence of a complex relation R that relates, for example, the numbers 3 and 2, 5 and 2, 5 and 4, etc., but not the numbers 3 and 1, 4 and 3, etc. One can ask the question, is this complex relation reflexive, symmetrical, or transitive?[3] The existence claim displayed above might form part of the metaphysics of relations, and it is straightforward to state such a theory using second-order logic. (For a good general overview of second-order logic, see Enderton 2009.)

To give a metaphysical theory of relations, one must, at a minimum, state (i) the conditions under which such (complex) relations can be said to exist, and (ii) the conditions under which relations F^n and G^n are to be identified. While philosophers have come to some agreement about (i), few philosophers have a good theory concerning (ii). A well-known principle that achieves (i) is the cornerstone for the theory of relations, from a logical point of view, namely, the comprehension principle for relations:

$\mathbf{CP}_n$ $\exists R^n \forall x_1 \ldots \forall x_n (R x_1 \ldots x_n \equiv \phi)$, where $n \geq 1$ and ϕ has no free occurrences of the variable R^n.

Since the expression ϕ in $\mathbf{CP}_n$ is a metavariable ranging over formulas of the language of second-order logic, $\mathbf{CP}_n$ is an axiom schema: whenever ϕ is a formula with no free occurrences of the variable R^n, one can formulate an instance of this schema and each such instance is then considered an *axiom* of the theory of relations. We've seen an instance of $\mathbf{CP}_2$ in the second paragraph of this section, and here are some examples of $\mathbf{CP}_3$ and $\mathbf{CP}_2$ (where R is a 3-place relation variable in the first instance, and a 2-place relation variable in the last two instances):

[3] A relation R is reflexive iff $\forall x Rxx$. R is symmetrical iff $\forall x \forall y (Rxy \rightarrow Ryx)$. R is transitive iff $\forall x \forall y \forall z (Rxy \,\&\, Ryz \rightarrow Rxz)$. The relation asserted to exist in the displayed sentence fails to be reflexive (since no number x is odd, even, and greater than itself). It also fails to be symmetric (since if $Ox \,\&\, Ey \,\&\, x > y$, then it fails to be the case that $Oy \,\&\, Ex \,\&\, y > x$). It proves to be trivially transitive, however, since the conjuncts in the antecedent of the definition of transitivity ($Rxy \,\&\, Ryz$) never simultaneously hold for arbitrary numbers (if x is odd, y is even, and $x > y$ then there is no number z such that y is odd, z is even, and $y > z$). If the antecedent of the definition of transitivity is always false, the entire condition required for transitivity is trivially true.

$$\exists R \forall x \forall y \forall z (Rxyz \equiv \neg Qxyz)$$

$$\exists R \forall x \forall y (Rxy \equiv \exists z Txyz)$$

$$\exists R \forall x \forall y (Rxy \equiv Sxy \vee Wxy)$$

The first axiom asserts the existence of a relation that objects x, y and z stand in just in case they fail to stand in the 3-place relation Q. The second asserts the existence of a relation R that objects x and y exemplify just in case they stand in the 3-place T relation to something. The third asserts the existence of a relation that x and y exemplify just in case they either exemplify the relation S or the relation W.

Similarly, $\mathbf{CP}_1$ yields existence conditions for properties. Here is a variety of such instances, each of which is taken as an axiom:

$$\exists F \forall x (Fx \equiv \neg Gx)$$

$$\exists F \forall x (Fx \equiv Gx \,\&\, Hx)$$

$$\exists F \forall x (Fx \equiv Gx \vee Hx)$$

$$\exists F \forall x (Fx \equiv \forall y Rxy)$$

The first asserts the existence of a property that objects exemplify when they fail to exemplify G; the second the existence of a property that objects exemplify when they exemplify both G and H; the third the existence of a property that objects exemplify when they exemplify either G or H, and the fourth the existence of a property that an object x exemplifies whenever x bears R to everything.

Once we lay down the comprehension principle for relations as the cornerstone of a theory of relations and properties, it becomes important to find a principle that captures the conditions under which properties and relations are to be identified. Everyone agrees that the following principles are *not* correct:

$$R^n = S^n \equiv \forall x_1 \ldots \forall x_n (R^n x_1 \ldots x_n \equiv S^n x_1 \ldots x_n)$$

$$P = Q \equiv \forall x (Px \equiv Qx)$$

We can now put Quine's (1951) observation in logical terms in the case of properties: the identity claim immediately above will fail in the right-to-left direction when P is *being a creature with a heart* and Q is *being a* *creature with a circulatory system*, since P and Q in this case are exemplified by all and only the same objects but are not identical properties. Indeed, since the suggested identity principle identifies P and Q whenever they are extensionally equivalent, it turns our budding property theory into one on which little would be lost by modeling (a) P and Q as sets and (b) predication as set membership. The conclusion to draw is that, at present, our second-order language doesn't yet offer enough expressive power for stating correct identity conditions for properties and relations. One important avenue of metaphysical investigation is to study the question: what facts about the nature of properties can be systematized, possibly by adding expressive power to the language of second-order logic, so as to find an identity principle that is materially adequate to the data?

Putting aside the question of identity, metaphysicians with an interest in logic should also become familiar with the relational version of the λ-calculus, the functional version of which was the subject of Church 1941. The relational λ-calculus is a system that allows us to name complex relations by systematically using the gerundive expression 'being such that'. Its cornerstone principle, λ-conversion (discussed below) is slightly stronger than the comprehension principle for relations. To follow up on an example mentioned earlier: a mathematician might ask whether there is anything special about the property *being prime and greater than 2*, which we might represent using a λ-expression as: $[\lambda x\ Px\ \&\ x > 2]$. Logicians have applied these gerundive expressions more generally, and allow the following expressions, in which G and H can be any property whatsoever, and R any relation:

failing to be G

being both G and H

being either G or H

bearing R to everything

The following formal expressions, in which the symbol λx stands for the phrase 'being an x such that', are used to represent the above English phrases:

$$[\lambda x\, \neg Gx],\ \ [\lambda x\, Gx\ \&\ Hx],\ \ [\lambda x\, Gx \vee Hx],\ \ [\lambda x\, \forall y Rxy]$$

Here are examples that correspond to the relations we introduced previously:

$$[\lambda xyz\, \neg Qxyz],\ \ [\lambda xy\, \exists z Txyz],\ \ [\lambda xy\, Sxy \vee Wxy]$$

The first may be read: being objects x, y, and z that fail to exemplify (in that order) the 3-place relation Q; the second: being objects x and y such that x and y bear the relation T to something; the third: being objects x and y such that either x bears S to y or x bears W to y.

These formal λ-expressions are governed by the three main principles of the λ-calculus, known as λ-conversion, η-reduction, and α-conversion:

λ-conversion: $\forall y_1 \ldots \forall y_n([\lambda x_1 \ldots x_n\, \phi]y_1 \ldots y_n \equiv \phi^{y_1,\ldots,y_n}_{x_1,\ldots,x_n})$

η-reduction: $[\lambda x_1 \ldots x_n\, F^n x_1 \ldots x_n] = F^n$

α-conversion: $[\lambda x_1 \ldots x_n\, \phi] = [\lambda x'_1 \ldots x'_n\, \phi']$,
where ϕ, ϕ' are alphabetic variants in x, x'

The instances of these principles are taken as axioms of the λ-calculus of relations. Here are two examples of λ-conversion:

$$\forall t \forall u \forall v([\lambda xyz\, \neg Qxyz]tuv \equiv \neg Qtuv) \qquad (\theta)$$

$$\forall z([\lambda x\, Px\ \&\ \exists y(Myx\ \&\ Sxy)]z \equiv Pz\ \&\ \exists y(Myz\ \&\ Szy)) \qquad (\vartheta)$$

We may read (θ) as follows: any objects t, u, and v, are such that they exemplify (the relation) *being an x, y, and z that fail to exemplify the relation Q* if and only if t, u, and v fail to exemplify Q. If we suppose that 'P' denotes the property of being a philosopher, 'M' denotes the *motherhood* relation, 'S' denotes the *supports* relation, and z denotes the person John, then (ϑ) would assert: for any object z, z is a philosopher who supports his mother if and only if z is a philosopher and something which is a mother of z is supported by z. This tells us something about the logic of predication, namely, that an object exemplifies a complex property just in case the complex logical condition implied by the structure of the complex property holds true.

As to examples of η-reduction and α-conversion, consider the instance of the principle of η-reduction that asserts: $[\lambda xy\, Rxy] = R$. Semantically, this tells us that the elementary λ-expression '$[\lambda xy Rxy]$' denotes the same relation as the relation symbol 'R'. Finally, consider this instance of α-conversion: $[\lambda xy\, \neg Qxy] = [\lambda yz\, \neg Qyz]$. Semantically, this tells us that the relation denoted by the λ-expression is independent of the particular variables bound by the λ; this makes sense since the property denoted by the λ-expressions isn't the kind of thing to have variables as part of its nature and so it shouldn't matter which variables we use to denote it. What is important is how the λ-bound variables and the structure of the formula constituting the body of the λ-expression jointly signify the logical structure of the property denoted.

In light of this last remark, a few words about the semantical interpretation of the logical system just described are in order. First, it is important to recognize that even though the $\mathbf{CP}_n$ appears to be a very strong existence principle, in fact it can be true in very small domains. If there is only one element in the domain of individuals, say object b, then a domain with only two properties (one property which is exemplified by b, and one which is not) makes $\mathbf{CP}_1$ true. Such a model would identify all extensionally equivalent properties (i.e., properties that are exemplified by the same objects), and so might be called an extensional model.

Since it is important that we avoid identifying properties (and n-place relations generally) with sets of objects (or with sets of n-tuples generally), we should investigate intensional models of the axioms described, which don't identify extensionally equivalent properties and relations. Quine (1960) inadvertently provided a piece of the puzzle, by applying the combinatory logic of Schönfinkel (1924) to eliminate variables from the language of first-order logic (see also Curry & Feys 1958). However, work by McMichael & Zalta (1979), Bealer (1982), Zalta (1983), and Menzel (1986) showed that the predicate functors Quine used could be reconstructed so as to apply to properties rather than predicates. In these works, a λ-expression such as $[\lambda x\, \neg Px]$, for example, is interpreted as denoting a new property that can be semantically described as $\mathbf{neg}(\mathbf{d}(P))$, i.e., a complex property the structure of which is the result of applying the negation operator **neg** to the property denoted by the predicate 'P'. When defining interpretations of our second-order language, we must stipulate that the domain of relations is closed under all the analogous logical operations needed to interpret the other logical constants (e.g., &, $\vee$, $\rightarrow$, etc.) that might appear in instances of $\mathbf{CP}_n$ and λ-conversion. Instead of eliminating variables from the syntax of the language (where they have standardly played an important role, e.g., in mathematics), such a stipulation eliminates the variables from the semantic representations of complex properties and relations (where variables have no natural role to play). A philosophical logician would, at the same time, introduce an extension function **ext** that maps each relation to a set constituting its extension. For example, **ext** maps $\mathbf{neg}(\mathbf{d}(P))$ to the set of objects that

fail to be in the extension of the property denoted by 'P', or formally, $\mathbf{ext}(\mathbf{neg}(\mathbf{d}(P))) = \{\mathbf{x} \mid \mathbf{x} \notin \mathbf{ext}(\mathbf{d}(P))\}$.[4] Notice how such semantic stipulations distinguish properties, whether simple or complex, from their extensions.[5] This allows one to consistently assert, for example, both $P \neq Q$ and $\forall x(Px \equiv Qx)$. Of course, a proper, materially adequate analysis of $P = Q$ remains a promissory note. The reader may find the following useful investigations on the topic: Zalta 1983; Cocchiarella 1986; Chierchia & Turner 1988; and Swoyer 1996, 1998, 2009.

Finally, it is important to mention that once this logic of properties and relations is in place, metaphysicians can carry out investigations into the extent to which it has to be modified to capture Plato's theory of predication, participation, and Forms (Pelletier and Zalta 2000), Leibniz's calculus of concepts (Zalta 2000a), Frege's theory of concepts, and other systematic theories of universals.

3. Logic for Propositions

Propositions are often introduced as the meanings of sentences, the bearers of truth and falsity, the objects of belief, and the denotations of relative clauses. For the present purpose of studying the interaction of logic and metaphysics, we might also say that propositions are the entities that are related by the relation of logical consequence. Philosophers early in the 20th century were concerned about the nature of propositions and their relationship to *facts*: whether facts are simply true propositions, whether this implied there were negative facts (given that there are true negative propositions) in addition to atomic facts (Russell 1918), whether propositions (or thoughts, as Frege called them) had ordinary objects or only concepts as constituents (Russell 1904, Frege 1904), whether we could do without propositions, say, in favor of states of affairs (e.g., Wittgenstein 1922), etc. In what follows, however, we shall examine how logic can be used to formulate a simple theory of propositions. This simple theory will

[4] Here the boldface variable $\mathbf{x}$ is a semantic variable ranging over the objects in the domain.

[5] The semantics just outlined though leaves open certain questions, such as whether $\mathbf{d}(P)$ and $\mathbf{neg}(\mathbf{neg}(\mathbf{d}(P)))$ are distinct. Though these two semantic descriptions of the property are different, axioms could be laid down so as to identify them. The metaphysician must do more work to determine whether such properties should be identified or kept distinct. Note also that though set theory is used to help structure the semantic representations of relations and their extensions, the sets themselves are not denoted, or referred to, by the predicates of the language.

regard facts as true propositions, and assumes that facts are not physical objects (given they have logical structure), so that one won't be inclined to ask whether negative facts are physical objects.

Though propositional logic is usually introduced to students before predicate logic, it is almost always interpreted in such a way that a truth value serves as the semantic significance of the propositional letters 'p', 'q', etc. That is, logic texts almost never interpret these propositional letters as denoting propositions, since their authors often prefer to remain neutral on the metaphysical question about the existence of propositions. But more metaphysically-minded logicians will find it interesting to consider such an interpretation of propositional logic. The key to the success of such an interpretation is to have a background theory of propositions upon which to draw. Such a theory forms a natural extension of the logic of properties and relations.

Let us define a proposition as a 0-place relation. Intuitively, if we predicate the 2-place relation *loves* of two objects a and b, then we have asserted the proposition *that a loves b*. Now if we let n go to 0 in the principle $\mathbf{CP}_n$, and abbreviate the 0-place relation symbols $P^0, Q^0, \ldots$ by the more familiar propositional letters $p, q, \ldots$, then we may take the instances of the following schema $\mathbf{CP}_0$ (i.e., the 0-place version of $\mathbf{CP}_n$) as axioms for the theory of propositions:

$\mathbf{CP}_0$ $\exists p(p \equiv \phi)$, where ϕ has no free p variables.

Here are some examples:

$$\exists p(p \equiv \neg Pa)$$

$$\exists p(p \equiv Pa \,\&\, Qb)$$

$$\exists p(p \equiv \forall y Ray)$$

The first asserts that there exists a proposition which is the negation of the proposition that Pa (i.e., that is true if and only if a fails to exemplify P); the second that there exists a (conjunctive) proposition that is true just in case both Pa and Qb are true; and the third that there is a proposition that is true if and only a bears R to everything. These examples show how the existence of propositions is implied by the theory of relations.

Though the simple terms $p, q, \ldots$ have been introduced to denote propositions in our language, we may also introduce complex terms for denoting them, namely, λ-expressions in which no variables are bound by

the λ. Thus, we might use the following λ-expressions corresponding to the three instances of $\mathbf{CP}_0$ immediately above:

$$[\lambda\, \neg Pa]$$

$$[\lambda\, Pa \,\&\, Qb]$$

$$[\lambda\, \forall y Ray]$$

We read these, respectively, as: that a fails to exemplify P, that a exemplifies P and b exemplifies Q, and that a bears R to everything. In other words, where λx formalizes 'being an x such that', λ itself without any variables formalizes the relative-clause operator 'that' and produces a name of a proposition when prefixed to a formula of our language.

These new λ-expressions are well-behaved: λ-conversion still applies to them. For example, the following is an instance of λ-conversion:

$$[\lambda\, \neg Pa] \equiv \neg Pa$$

This asserts: (the proposition) that a fails to exemplify P is true if and only if a fails to exemplify P. Note here that we explicitly use the phrase 'is true' in this reading. This treats the concept of truth as the 0-place case of the n-place concept of exemplification. The λ-expression $[\lambda\, \neg Pa]$ is therefore to be regarded as a formula as well as a complex term! As a formula, it can stand in isolation on the left hand side of the biconditional sign $\equiv$, which is defined so that whenever ϕ and ψ are formulas, so is $\phi \equiv \psi$.

Moreover, now that our theory of relations has been extended to include propositions, our logic allows us to form new predicates denoting properties that objects exemplify in virtue of a proposition's being true. For example, the following is now a perfectly well-defined instance of $\mathbf{CP}_1$:

$$\exists F \forall x(Fx \equiv Pa)$$

This asserts that there is a property that objects exemplify if and only if the object a exemplifies the property P. Indeed, we might use the λ-expression $[\lambda x\, Pa]$ (where the x bound by the λ is *not* free in the ensuing formula) to denote the property: being an x such that a exemplifies P. Such a λ-expression is well-behaved logically, since it obeys the following instance of λ-conversion:

$$\forall y([\lambda x\, Pa]y \equiv Pa)$$

This asserts that every object y is such that y exemplifies the property of being an x such that a exemplifies P iff a exemplfies P (thus, if Pa is true, everything exemplifies $[\lambda x\, Pa]$, and if it is false, nothing does). These new λ-expressions of the form $[\lambda x\, \phi]$ are important because even though we don't yet have an adequate principle governing the identity conditions for properties, we may use them to define the identity conditions for propositions in terms of the identity of properties, as follows (Myhill 1963):

$$p\!=\!q \equiv [\lambda x\, p]\!=\![\lambda x\, q]$$

In other words, the propositions p and q are identical whenever the properties *being such that* p and *being such that* q are identical. Such a definition allows the metaphysician to consistently assert that propositions p and q may be distinct even if they are materially equivalent, i.e., to consistently assert both $p \neq q$ and $p \equiv q$. This immediately implies that propositions can't be identified with truth values.

Of course, the theory of propositions just outlined would benefit from a semantical model of how equivalent propositions can be distinct. Such a model can be given by extending the interpretations discussed in the previous section for the theory of relations. The very same logical operation, **neg**, that maps a property to its negation, can also map a proposition **p** to its negation $\mathbf{neg}(\mathbf{p})$. Similarly, the operation **cond** would map propositions **p** and **q** to the conditional proposition $\mathbf{cond}(\mathbf{p},\mathbf{q})$. In what follows, let us suppose that **p** is the denotation of the propositional letter 'p' and **q** is the denotation of the propositional letter 'q'. Then the denotation of the λ-expression $[\lambda\, \neg p]$ is $\mathbf{neg}(\mathbf{p})$, and the denotation of the λ-expression $[\lambda\, p \rightarrow q]$ is $\mathbf{cond}(\mathbf{p},\mathbf{q})$. Moreover, the extension function **ext** can be expanded so that in addition to mapping relations to their extensions, it maps propositions to truth values. Thus, for example, the **ext** function would be defined so that the extension of the negative proposition $\mathbf{neg}(\mathbf{p})$ is the truth value The True if and only if the extension of the negated proposition **p** is The False. Here is a formal statement of the constraint on **ext** in the case of conditional propositions, where **T** and **F** are the two truth values: $\mathbf{ext}(\mathbf{cond}(\mathbf{p},\mathbf{q})) = \mathbf{T}$ if and only if either $\mathbf{ext}(\mathbf{p}) = \mathbf{F}$ or $\mathbf{ext}(\mathbf{q}) = \mathbf{T}$. Clearly, on this model, propositions are distinct from their extensions.

See King (2008) for a description of other theories of structured propositions, and Fitch (1988) for more on the theory of singular propositions

(i.e., propositions that have objects themselves, as opposed to concepts of objects, as constituents). For a different approach to the topic of propositions, which employs states of affairs and situations, see Barwise & Perry 1983, Perry 1986, Stalnaker 1986, and Zalta 1993.

4. Logic for Possibilia, Contingent Beings, and Worlds

Logic and metaphysics interact in a significant way in the attempt to systematize our modal beliefs expressed with the sentential operators 'necessarily', 'possibly', and 'actually'. In his *Monadology* and *Theodicy*, Leibniz introduced the idea that necessary truths are propositions true in all possible worlds. This insight was one of the key elements of Kripke's (1959) semantics for modal logic.[6] He postulated a domain of possible worlds that included a distinguished actual world $\mathbf{w}_0$. He interpreted the primitive operator 'necessarily' in the language of modal logic as a quantifier of the form, 'in every possible world'. The picture that resulted, based on the idea that predication becomes world-relative (i.e., that objects exemplify properties with respect to worlds), led philosophers to question whether the semantics of modal logic rests upon a sound metaphysical basis. They became interested in the following issues: (1) what are the fundamental principles governing possible worlds? (e.g., what are their existence and identity conditions, and what other axioms and definitions are needed to give a systematic theory of them?); (2) is a statement like 'Obama might not have been President' true in virtue of the fact that Obama himself exists at some other world without being President there or in virtue of some other fact?; and (3) is a statement like 'Obama might have a son' (i.e., 'Possibly, Obama has a son') true because there exists some nonactual, but possible, object which is Obama's son at some other possible world or because some other truth conditions, not involving the existence of nonactual but possible objects, obtain? Issue (3) has especially received a lot of attention, for Quine expressed extreme skepticism

[6] Another key insight was the use of an accessibility relation among the various possible worlds: where $\mathbf{w}$ and $\mathbf{w}'$ are two possible worlds, $\mathbf{w}'$ is *accessible* from $\mathbf{w}$ just in case whenever a proposition, say **necessarily p**, is true at $\mathbf{w}$, the proposition **p** is true at $\mathbf{w}'$. The validity of statements in modal logic such as the T axiom ('if necessarily p, then p'), the 4 axiom ('if necessarily p, then necessarily, necessarily p'), the 5 axiom ('if possibly p, then necessarily possibly p'), etc., is grounded in the the properties of the accessibility relation. For example, the T axiom is valid if the accessibility relation is reflexive, the 4 axiom is valid if accessibility is transitive, the 5 axiom is valid if if accessibility is Euclidean, etc.

about such possible but nonactual objects (Quine 1948).

Probably the most widely endorsed metaphysical view about possible worlds and possible objects is called *actualism*, namely, the thesis that everything there is (i.e., everything that exists) is actual. To develop this view consistently: (1) possible worlds must be regarded as some sort of existing abstract object (rather than as a possible object) and, moreover, one that embodies, in some sense, a maximal (i.e., complete) group of propositions that might have been true together, and (2) an analysis of modal claims has to be developed that doesn't postulate possible but nonactual objects, like Obama's possible but nonactual sons. (For an excellent overview of actualism, see Menzel 2010.) Actualism is often contrasted with Lewis's (1968, 1986) view that endorses both possible but nonactual worlds and possible but nonactual objects like Obama's possible sons, possible million carat diamonds, possible talking donkeys, etc. (On Lewis's view, though each individual x that exists at our world exists only at our world, there are possible but nonactual objects that serve as x's counterparts at other possible worlds, whose properties at those worlds ground the modal properties that x has at our world.)

Kripke's well-known semantics (1963) for quantified modal logic has been taken to be an actualist stance on these problems. Though Kripke didn't axiomatize the notion of *possible world* or state existence and identity conditions for them, he did take worlds as primitive in his semantic metalanguage. What he says about them is consistent with the assumption that they are actual, abstract objects of some kind. Moreover, his semantics clearly supposes (a) that one and the same object exists in multiple possible worlds, (b) that contingent objects that exist at our world fail to exist at other worlds, and (c) that contingent objects that exist at other worlds fail to exist at our world (though since Kripke restricted the quantifiers of our language so that they don't range over those objects, we can't validly conclude that there are any nonactual possible objects). Features (a) – (c) bring us to a discussion of the methods Kripke used to invalidate certain metaphysically significant theorems about possible objects that would otherwise be provable when one combines the principles of classical quantification theory with the principles of classical S5 modal logic to form the simplest quantified modal logic.[7]

[7]The principles of S5 include the K axiom ('if necessarily if p then q, then if necessarily p then necessarily q'), and the axioms T, 4, and 5 mentioned in a previous footnote.

Kripke's methods focused on the interpretation of the quantifier $\forall x$ (i.e., 'every x') in the language of quantified modal logic. He denied that this quantifier ranges over everything whatsoever in the domain of discourse. Instead of assuming a single, fixed domain of discourse, Kripke assumed that each possible world should be associated with its own domain of objects (intuitively, the objects that exist at that world). He then he restricted the interpretation of the quantifier $\forall x$ in two ways: (i) if $\forall x$ stands outside the scope of the modal operator 'necessarily' (represented as $\Box$) in a formula, as in $\forall x\Box\phi$, then the variable x ranges only over the objects in the domain at the distinguished actual world $\mathbf{w}_0$ (the sentence $\forall x\Box\phi$ is thus true iff every object in the domain of $\mathbf{w}_0$ satisfies ϕ at every possible world), and (ii) if $\forall x$ stands inside the scope of the modal operator, as in $\Box\forall x\phi$, then as you evaluate the truth of $\forall x\phi$ at each possible world $\mathbf{w}$ (as part of the process of determining whether $\forall x\phi$ is true at all possible worlds), you consider only whether all the objects in the domain at $\mathbf{w}$ satisfy ϕ (the sentence $\Box\forall x\phi$ is thus true iff at every possible world $\mathbf{w}$, all of the objects in the domain at $\mathbf{w}$ satisfy ϕ at $\mathbf{w}$).

Using these techniques, Kripke was able to *invalidate* the following three important theorems of the simplest quantified modal logic:

BF $\forall x\Box\phi \to \Box\forall x\phi$

CBF $\Box\forall x\phi \to \forall x\Box\phi$

NE $\forall x\Box\exists y(y = x)$

The first of these, the Barcan Formula **BF**, asserts that if everything is necessarily such that ϕ, then necessarily, everything is such that ϕ. Clearly this becomes invalid in Kripke's semantics, since it doesn't follow from the fact, that every object in the domain at the actual world $\mathbf{w}_0$ satisfies ϕ in every possible world, that at every possible world, every object in the domain there satisfies ϕ. For consider the scenario where every object in the domain of $\mathbf{w}_0$ satisfies ϕ at every possible world, but some object in the domain of some other world $\mathbf{w}_1$ (but not in the domain of $\mathbf{w}_0$) fails to satisfy ϕ at $\mathbf{w}_1$. In such a scenario, the antecedent of **BF** is true but the consequent false. Similarly, **CBF** fails to be valid because it doesn't follow from the fact, that at every possible world every object in the domain there satisfies ϕ there, that every object in the domain of $\mathbf{w}_0$ satisfies ϕ in every possible world. For consider the scenario where at every world $\mathbf{w}$, every object in the domain at $\mathbf{w}$ satisfies ϕ, but some object in the domain of $\mathbf{w}_0$ but not in the domain of world $\mathbf{w}_1$, say, fails to satisfy ϕ at $\mathbf{w}_1$. In such a scenario, the antecedent of **CBF** is true, but the consequent is false. Finally, **NE** becomes invalid because there are scenarios where an object in the domain at $\mathbf{w}_0$, say b, fails to be in the domain at some other world, say $\mathbf{w}_1$. In such a case, the formula $\forall x \exists y (y = x)$ fails to be true at $\mathbf{w}_1$ (when b is assigned as value to the variable x), and so fails to be necessary.

Once Kripke had a semantics that invalidated **BF**, **CBF**, and **NE**, he had to weaken the axioms of quantified modal logic in various ways (this included the elimination of proper names from the language) so that instances of these sentences were no longer derivable as theorems. Such a weakening of quantified modal logic has left many philosophers and logicians dissatisfied. Moreover, many actualists were dissatisfied with Kripke's semantics on metaphysical grounds: (1) unless one provides a specific theory of possible worlds that shows them to be abstract objects, the nonactual possible worlds of Kripke's semantics would seem to violate the principles of actualism; (2) Kripke's semantics allows objects in the domain of other possible worlds that don't appear in the domain of $\mathbf{w}_0$, suggesting that the semantics was committed to nonactual possible objects (like Obama's possible sons), again contrary to the spirit of actualism; and (3) Kripke's semantics allows an object to have a property at a world $\mathbf{w}$ even when the object isn't in the domain of $\mathbf{w}$ (in other worlds, Kripke semantics assumes that it makes sense to predicate properties of objects even at worlds where those objects don't exist), which violates the principle of *serious actualism* that many actualists hold, namely, the thesis that objects have properties only at worlds where they exist (serious actualists believe this should be properly reflected in a modal logic).

An extremely interesting literature has developed around these dissatisfying features of Kripke's logic (the literature is summarized nicely in Menzel 2010). Some philosophers have investigated the principles that govern possible worlds, thereby taking them as theoretical objects in their own right rather than as primitive objects (Plantinga 1974, Chisholm 1976, Fine 1977, Lycan & Shapiro 1986, and Zalta 1993); some have developed systems that avoid weakening quantified modal logic (Fine 1978, Menzel 1991, Deutsch 1994); some have attempted to avoid the semantic commitment to nonactual possibles (Plantinga 1976, Prior 1977, Adams 1981, McMichael 1983, and Menzel 1990); and others have investigated the alternative offered by Lewis's theory of possible worlds and the metaphysics of counterpart theory (Hazen 1979, Bricker 1996, 2001). One

recent trend has been to reconceptualize, along actualistic grounds, the simplest quantified modal logic, which validates **BF**, **CBF**, and **NE**. If this logic (S5 modal logic with classical quantification theory) is interpreted by a single, fixed domain of objects, and the quantifier $\forall x$ is interpreted as ranging over everything in the domain, then one might argue that this treats every object in the domain on a par, as an actual, existing object (Cresswell 1991, Linsky & Zalta 1994, 1996, and Williamson 1998, 2000). It is argued that Kripke semantics (with domains that vary from world to world) are unnecessary and that no metaphysical problems are created by the validity of **BF**, **CBF**, and **NE**.

5. Other Topics of Interest

5.1 The Logic of Fiction

One of the most fascinating areas requiring the interaction of logic and metaphysics concerns the meaning of language used in stories and fiction more generally. There is a rich body of data to be explained, in connection with the inferences we draw involving names and descriptions that appear to denote fictional characters. It follows from "Dionysus worshipped Zeus" that "Dionysus worshipped something", and it follows from "Teams of scientists searched for the Loch Ness monster" that "Teams of scientists searched for something". Even more complicated inferences can be described; the following argument, for example, is valid:

> Dionysus worshipped Zeus.
> Zeus is a mythical character.
> Mythical characters don't exist.
> Therefore, Dionysus worshipped something that doesn't exist.

It seems hard to understand how these inferences can be valid if the proper names and definite descriptions fail to denote anything at all.

It used to be commonplace in philosophy to say that Russell's famous theory of descriptions (1905) solved the problem of analyzing sentences about fictions. But many philosophers now agree that Russell's theory doesn't solve this problem, but rather creates a problem. If we eliminate the description "the fountain of youth" ('$\imath xFx$') in the sentence "Ponce de Leon searched for the fountain of youth" ('$Sp\imath xFx$') by applying Russell's theory, then we get the analysis: there exists a unique object that exemplifies being a fountain that confers everlasting life and Ponce de Leon searched for it, i.e.,

$$\exists x(Fx \,\&\, \forall y(Fy \rightarrow y = x) \,\&\, Spx)$$

But this latter is clearly false, while the original English sentence is true. No object exemplifies being a fountain that confers everlasting life. For this same reason, Russell's theory of descriptions fails to provide a good analysis of proper names occurring in sentences about fictions, for on his view, such proper names just abbreviate definite descriptions. Such an analysis would turn true sentences involving names of fictions into falsehoods.

Some logicians have suggested that 'free logic' (Hailperin 1953, Morscher & Simons 2001, Lambert 2003) is needed for the analysis of fiction. Free logic is the variant of the classical first-order predicate calculus which allows for constants that fail to denote and allows for an empty domain. In such a logic, one may not instantiate a name for an individual, say 'a', into a universal claim like '$\forall xPx$' to infer 'Pa', since such an inference could move you from a universal generalization which is true to a falsehood ('Pa' would be false if 'a' is a constant that fails to denote). But free logic suffers from problems similar to those of Russell's theory of descriptions. It can't successfully represent "Dionysus worshipped Zeus" as true, much less successfully represent the inference from "Dionysus worshipped Zeus" to "Dionysus worshipped something" or the argument displayed in the previous paragraph. Nor can it explain why "Zeus is the most powerful god according to Greek myth" is true while "Zeus is the god of war according to Greek myth" is false. It offers no theory of the semantic significance of "Zeus" that might explain why the former sentence is true while the latter is false.

One of the first systematic treatments of fiction that took seriously the fact that names like 'Zeus' and descriptions like 'the fountain of youth' denote fictional objects can be found in Parsons 1980, which is an elegant combination of metaphysics and logic. Using Meinong's (1904) theory of objects as a guide, Parsons developed an axiomatic theory of nonexistent objects, couched in a second-order language that was modified so as to admit two kinds of properties, nuclear and extranuclear properties. A comprehension principle guarantees that for every condition ϕ on nuclear properties, there is an object that exemplifies exactly the nuclear properties satisfying the condition. Some of these objects exemplify

the extranuclear property of existence, while others fail to exemplify this property. Parsons shows how his system addresses Russell's famous objections to Meinong's naive theory, and then shows how sentences and inferences involving fictional names and descriptions can be represented in the logic of his system. Other logics for fiction have been developed as well (Zalta 2000b, and Woods & Alward 2004). See Sainsbury 2009 for a recent summary of the different theories of fiction that have been advanced recently.

5.2 Logic, Metaphysics, and Mathematics

Logic has an important role to play in the analysis of the metaphysical underpinnings of mathematics. Mathematical statements such as "$3 > 2$" and "$\emptyset \in \{\emptyset\}$" seem to be true and seem to be about numbers and sets, as well as about abstract relations such as *greater than* and *set membership*. The work we did in Section 2 puts the reader in a position to understand Frege's insightful definition of the (immediate) predecessor relation among natural numbers. It involves the operator # which operates on a property-denoting term to form a singular term that denotes the number of objects exemplifying the property denoted:

$$\mathit{Precedes}(x, y) \;=_{df}\; \exists F \exists u(Fu \,\&\, y = \#F \,\&\, x = \#[\lambda z\, Fz \,\&\, z \neq u])$$

In other words, x precedes y just in case there is a property F and an object u such that u exemplifies F, y is the number of Fs, and x is the number of the property *being an F-thing other than u*. One can see that this correctly predicts that 1 precedes 2 if we let F be the property *being an author of Principia Mathematica* and let u be A.N. Whitehead. Since Whitehead is an author of *Principia Mathematica*, 2 is the number of the property F, and 1 is the number of the property *being an author of Principia other than Whitehead*, the definition for *Precedes*(1,2) holds. This is an instructive example of how a logician employed quantification over properties to define a significant mathematical concept. (See Zalta 2010 for a thorough discussion of the results Frege was able to achieve in terms of this definition.)

Though some philosophers have supposed that the singular terms in mathematical statements (e.g., '3', '$\emptyset$', etc.) fail to denote anything (e.g., Field 1980, Hellman 1989), these views mostly fail to preserve the logic of such statements; analyses in which these terms systematically contribute a denotation to the truth conditions of, and inferences among, the statements in which they appear, do a better job capturing this logic. One obvious place where metaphysics and logic combine for the analysis of mathematics is in the view known as *logicism*, the idea that mathematics is reducible to logic alone, i.e., that the concepts of mathematics are definable in terms of logical concepts, and that the axioms of mathematics can be derived as theorems of logic. If this view were true, the metaphysics needed for mathematics would simply be that needed for logic. Though the pursuit of a logicist analysis of mathematics was a goal of philosophers and logicians in the early 20th century, most philosophers nowadays see logicism as unachievable. For it clearly fails for mathematical theories that have strong existence axioms (e.g., axioms that require an infinite domain if the theory is to be true). Few logicians would accept that the axioms of logic are strong enough to imply the existence of an infinite number of objects, and given that logic doesn't require infinite domains for the truth of its axioms, it would seem impossible to reduce the axioms of strong set theories and number theories, which are true only in infinite domains, to theorems of logic.

Recently, however, philosophers have considered whether some weakened form of logicism might be true. If logicism is the view that mathematics is reducible to logic alone, then the view could be weakened, holding mathematics fixed, by either (1) expanding the scope of what counts as logic, (2) supplementing logic with analytic truths that express technical analyses of mathematical concepts, or (3) revising the standard of reduction. See Hodes 1984 and 1991, and Tennant 2004, for developments along the lines of (1), Wright 1983, Hale 1987 and 2000, and Boolos 1986 for developments along the lines of (2), and Linsky & Zalta 2000 for developments along the lines of (3). Fine 2002 and Burgess 2005 offer interesting technical discussions of the topic.

5.3 Other Noteworthy Connections

We conclude this survey by listing, without much discussion, other fruitful areas of interaction between logic and metaphysics. The study of tense logic helps to clarify the metaphysics of time. See Prior 1967, Burgess 1979, and van Benthem 1983. Logical techniques have been brought to bear in the study of the structure of *events* in such works as Parsons 1990 and Link 1998. The study of intentionality (the property of the

mind and mental events that makes them *about* or *directed upon* things), has led to theories of the objects of thought, and in particular, the nature of those objects of thought which turn out to be impossible in some sense. See Zalta 1988, Paśniczek 1997, and Priest 2007. Fine 1985 offers an interesting investigation of *arbitrary objects*, which play an important role in logical and mathematical reasoning. Finally, a deeper investigation into the nature of properties has led Yi (2005, 2006) to postulate *plural attributes* in his study of the logic of plurals, and Leitgeb (2007) to examine the extent to which properties might be abstracted or reconstructed from similarity relations. Finally, the reader may find that the study of metaphysics computationally using automated reasoning engines has many interesting benefits, such as the independent derivation of interesting theorems so as to confirm and validate reasoning, the discovery of countermodels to hypotheses and errors in reasoning, and the discovery of facts about the strength of axioms and premises needed to derive metaphysical conclusions. See Fitelson & Zalta 2007 and Oppenheimer & Zalta 2011a, and also Oppenheimer & Zalta 2011b for an idea of how work in computational metaphysics has led to new perspectives on the foundations of logic, including the discovery of a limitation on functional type theory (and automated reasoning engines based on it).

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