

# Fregean Senses, Modes of Presentation, and Concepts\*

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Ever since Frege postulated senses in [1892] and conceived of them as (containing) modes of presentation, serious studies in the philosophy of language have often appealed to these entities. In the recent philosophy of language and mind, however, the notions of ‘mode of presentation’ and ‘concept’ have become more important. It seems fair to say that much of the theorizing involving senses, modes of presentation, and concepts has taken place at a very general level. Frege never told us what senses are; they were simply stipulated to be entities that play certain roles in his philosophy of language. Philosophers today still work with both senses and modes of presentation without having a systematic and viable theory of them. And although the notion of a concept has been employed in various ways, there are not all that many precise theories of concepts.

In this paper, I try to clear away some of the mystery surrounding these three kinds of entities by offering a precise theory of them. The theory of Fregean senses developed in my previous work will be extended to yield a more general theory of modes of presentation and concepts. Modes of presentation and concepts will be identified, therefore, in terms

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of my axiomatic theory of abstract objects.<sup>1</sup> The theory asserts the existence not only of ordinary properties, relations, and propositions, but also of abstract individuals and abstract properties and relations. The concepts by which ordinary individuals, properties, and relations are conceived will be analyzed as abstract individuals, properties, and relations, respectively.<sup>2</sup> In what follows, we take the denotation of a predicate to be a property or relation (intensionally conceived), and we analyze the sense of that predicate, relative to some person, as a *concept* by which that person conceives of the property or relation denoted by the predicate. We shall, therefore, be distinguishing properties and concepts. Moreover, our theory of concepts will predict that the concepts of individuals can combine with concepts of properties and relations so as to form complex concepts. These complex concepts are entities by which ordinary ‘Russellian’ propositions (or states of affairs) can be conceived.

In Section 1 of the paper, I’ll focus on senses and modes of presentation, and explain why the more general notion of a mode of presentation may be more useful for the analysis of propositional attitude reports than the notion of the sense of a term. In Section 2, I explain how to extend my earlier work on senses to produce a theory of modes of presentation and concepts. Since the resulting theory offers precise existence conditions for modes of presentation and for concepts, we’ll see that it addresses Schiffer’s [1990] ‘candidate problem’ for modes of presentation.

In Section 3 of the paper, our theory will be used to ‘plug the theoretical gap’ which exists in the work of those philosophers who employ senses and/or modes without having a precise theory of them. (I’ll reserve, for another occasion, a similar study of some of the work on concepts which has been conducted in the absence of a precise theory.) The work of the following four philosophers will be discussed in more detail in that section. The first is D. Kaplan, who acknowledged the elegance of Frege’s theoretical explanation of intensional contexts, but pointed out that we must continue looking for the ‘peculiar’ intermediate entities involved.<sup>3</sup>

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<sup>1</sup>The axioms for the theory have been presented in various works, including Zalta [1983], [1988a], [1993b] and [1999].

<sup>2</sup>It is important to note that the notion of ‘concept’ we consider in the present paper is to be distinguished from the notion that was the subject of Zalta [2000a]. In the present paper, I am interested in the notion that comes to us from the contemporary tradition in the philosophy of mind and language. In [2000a], I was interested in systematizing the Leibnizian notion of a concept.

<sup>3</sup>In Kaplan [1969], we find (p. 119):

The second is G. Forbes, who acknowledges at the end of a recent essay that his neo-Fregean theory of attitude reports awaits a more precise theory of modes of presentation.<sup>4</sup> Despite this absence of theory, Forbes introduces ([1987], 8; and [1990], 548) the variables  $\alpha, \beta, \dots$  to range over senses and uses the uninterpreted notation  $\alpha \hat{=} \ulcorner F \urcorner$  to indicate that  $\alpha$  is a mode of presentation which has somehow combined with the sense of the predicate ' $F$ ' to form a complex sense or Fregean thought. Not only does this notation need interpretation, but also modes of presentation need a precise identification if we are to accept that they can be combined in this way. The third example is N. Salmon, who in [1986] eschews senses in his analysis of belief reports as 2-place relations between persons and Russellian propositions, but appears to employ modes ('ways of taking a proposition') as the third relatum of his theoretical *BEL* relation.<sup>5</sup> But he then admits that he has said very little about what this third relatum is.<sup>6</sup> The final example is M. Crimmins, who in [1998] develops a pretense-theoretic analysis of modes of presentation and belief ascriptions. Crimmins agrees that an account of modes of presentation is required for

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My own view is that Frege's explanation, by way of ambiguity, of what appears to be the logically deviant behavior of terms in intermediate contexts is so theoretically satisfying that if we have not yet discovered or satisfactorily grasped the peculiar intermediate objects in question, then we should simply continue looking.

It should be mentioned, however, that in recent years, Kaplan has changed his view on these matters.

<sup>4</sup>In [1987], Forbes says (p. 31):

My overall conclusion is that a Fregean theory of the semantics of attitude contexts is from the structural point of view the best that is available. Its ultimate viability depends of course on how successful the efforts to develop a detailed theory of the nature of modes of presentation will be.

Such a theory is not developed in Forbes' paper.

<sup>5</sup>In Salmon [1986], a belief report ' $x$  believes that  $S$ ' is analyzed as  $B(x, p)$  (where  $p$  is the Russellian proposition denoted by the sentence  $S$ ). But this in turn is equivalent to  $\exists y(BEL(x, p, y))$ , where  $y$  is something like a mode of presentation of the proposition  $p$ .

<sup>6</sup>Near the end of [1986], we find (p. 126):

The major problem remaining for the sort of theory I have advocated here is to provide a more complete account of the things corresponding to propositional recognition failure, the things that serve as the third relatum for the *BEL* relation.

When canvassing the possibilities (p. 120), Salmon suggests that this third relatum could be a 'way of taking' the proposition, a 'mode of presentation', or a 'mental file'.

his analysis of belief reports to succeed,<sup>7</sup> but suggests that his view is not committed to standard conceptions of modes of presentation. Nevertheless, he quantifies over, individuates, and introduces notation for modes in his paper.<sup>8</sup>

In Section 4 of the paper, I'll discuss, but not resolve, a technical issue about how to best enhance our formalism so that we can refer to modes of presentation in the analysis of belief reports. We will work within a theory of belief reports on which they are analyzed as asserting 2-place relations.<sup>9</sup> In Section 5 of paper, we conclude with a few remarks about concepts.

Let me emphasize before we begin that the theory of modes of presentation and concepts developed in what follows is *neutral* with respect to the different analyses of belief reports that various authors have proposed. The theory that I offer here provides a theoretical underpinning for any theory of belief or belief reports which appeals to modes of presentation or concepts. Even if the analysis of belief reports discussed here turns out to be flawed, the underlying theory of modes of presentation could still ground the other analyses of attitude reports that invoke these entities.

Finally, I should mention that my earlier work on the theory of abstract objects will be presupposed in what follows. I shall assume that the reader is familiar with the typed theory of abstract individuals and abstract relations, with our previous discussions of how these abstract individuals and abstract relations can play the roles that Frege assigned

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<sup>7</sup>In [1998] (25), Crimmins writes:

The pretense account employs the notion of a mode of presentation, and nothing could be more important in assessing the account than determining the prospects for satisfying explanations of what modes of presentation are, and of the access to them that speakers and hearers rely on in producing and understanding attitude reports.

Crimmins then refers us to his earlier work on this question. See the following footnote.

<sup>8</sup>In [1998] (25, note 17), Crimmins refers us his [1992] for more discussion about modes of presentation. The discussion in [1992] is about 'representations' (i.e., notions and ideas), which Crimmins takes to be concrete mental particulars. He says explicitly there ([1992], 78) that these are not ways of thinking or modes of presentation. But in [1998], he seems to be talking about Fregean modes of presentation, not concrete particulars. See, for example, the first paragraph of Section 2 ([1998], 8).

<sup>9</sup>This will contrast with analyses which treat such reports as asserting 3-place relations. See, for example, Crimmins and Perry [1989], or Perry [1979] and Fitch [1987]. Our analysis will also contrast with the 2-place Russellian analyses found in McKay [1981], Salmon [1986], and Soames [1989], since we shall treat belief reports as ambiguous.

to senses in his philosophy of language, with the special notation we have introduced into intensional logic for denoting the sense of a term with respect to an individual, and with our previous approach to the analysis of belief reports, in which such reports are treated as ambiguous and in which any of the terms within the scope of propositional attitude verbs can be given either a *de re* or a *de dicto* reading depending on the facts about substitution. I'll refer to this overall theory as 'ILAO' ('the intensional logic of abstract objects') in what follows.<sup>10</sup>

### §1: From Fregean Senses to Modes of Presentation

In this section, we examine the reasons why the appeal to the more general modes of presentation, as opposed to Fregean senses, might give one more flexibility when analyzing propositional attitude reports. It is important to understand just how far the notion 'the sense of a term' can take you, and where the more general notion 'mode of presentation' becomes useful. Before we look at examples where the more general notion becomes important, let us look at an example which, it has been claimed, cannot be analyzed in terms of Fregean senses. Schiffer [1992, 507-508] describes a case that he claims is problematic for the Fregean. He begins his discussion of the problematic case by first pointing out how the Fregean would represent the following, unproblematic sentence:<sup>11</sup>

- (1) Ralph believes that Fido is a woodchuck.

On behalf of the Fregean, Schiffer offers the following representation:

- (S)  $B(\text{Ralph}, \langle m_f, m_w \rangle)$

Schiffer uses ' $\langle m_f, m_w \rangle$ ' to denote the "mode-of-presentation-containing proposition" referred to by the occurrence of 'that Fido is a woodchuck' in (1), where  $m_f$  and  $m_w$  are the ways Ralph has of thinking of Fido and the property of being a woodchuck, respectively. With this understanding

<sup>10</sup>The intensional logic for abstract objects was developed in its most sophisticated form in my [1988a] and [1988b]. However, readers familiar with Zalta [1983], [1989], or [1993a] should be in a good position to understand the material in this paper.

<sup>11</sup>I have revised Schiffer's example from 'Ralph believes Fido is a dog' to 'Ralph believes that Fido is a woodchuck'. To discuss further aspects of the case, Schiffer introduces the fictional species name 'schmog', as a second name for the species *dog*. In case we have to remark on the further aspects of the case, I prefer to use the real example of 'woodchuck' and 'groundhog', which are names of the same species.

of the Fregean analysis, Schiffer then says the following sentence poses a problem:

- (2) Everyone who has ever known her has believed that Madonna was musical.

Schiffer points out [1992, 507]:

According to the Fregean proposal, there is a particular mode of presentation  $m$  of Madonna and a particular mode of presentation  $m'$  of the property of being musical such that the foregoing utterance of (2) is true only if everyone who has ever known Madonna has believed the proposition  $\langle m, m' \rangle$ . Yet this is surely too strong a requirement on the truth of (2). It requires that everyone who has ever known Madonna shared a single way of thinking of her and a single way of thinking of the property of being musical. . . .

Since there is no single mode of presentation by which those who know Madonna conceive of her, Schiffer concludes that the classic Fregean view cannot successfully represent (2).

Clearly, Schiffer is correct when he says that there is no single mode of presentation by which people who have known Madonna conceive of her. To make the example even harder, we can replace 'known' in (2) by 'seen'. Even if someone were to argue that people who have known Madonna do have a single way of thinking of her, it is hardly likely that something similar can be said about everyone who has ever seen Madonna. Moreover, in what follows, we accept the claim that there is no single *sense* of the term 'Madonna' in terms of which everyone conceives of the person denoted by the name. We assume, in general, that the sense of a name varies from person to person.<sup>12</sup>

However, we've developed a more flexible notation and theory of senses than the one Schiffer offers on behalf of the Fregean. Consider first how

<sup>12</sup>Some philosophers might claim that this assumption is too strong. They argue that there is a very weak kind of sense on which each proper name has a single sense that is shared by all the competent speakers of a language. But if this is true, the present theory could still offer an account of such senses. These weak senses could be identified as abstract objects that encode the minimally informative properties involved. For example, the universal sense of 'Madonna' could be an abstract object which encodes such properties as being a name, being a name of Madonna, or being a name of someone called 'Madonna', etc. No circularity worries accompany such a view, since we would not use the sense of a term to determine the denotation of a term. But I shall not pursue this alternative theory of sense in what follows.

it differs from Schiffer's representation of the *unproblematic* case of (1). (1) is ambiguous from the point of view of ILAO, and among the possible readings of (1), there is a pure *de re* reading and a pure *de dicto* reading. These are as follows, respectively:

$$(1') B(r, [\lambda Wf]) \quad (\text{pure } de\ re)$$

$$(1'') B(r, [\lambda \underline{W}_r \underline{f}_r]) \quad (\text{pure } de\ dicto)$$

(In (1') and (1''), we read the  $\lambda$ -notation of the form  $[\lambda \phi]$  as 'that- $\phi$ ', and since (1') and (1'') are supposed to disambiguate the English, they read exactly the same when read back in *ordinary* English.) Clearly, (1'') is the counterpart of Schiffer's (S), but notice the difference in the notation for representing the thought to which Ralph is related. (1'') relates Ralph to a Fregean thought containing the sense of 'woodchuck' for Ralph ( $\underline{W}_r$ ) and the sense of 'Fido' for Ralph ( $\underline{f}_r$ ) as constituents. This notation acknowledges the fact that the sense of a term may vary from person to person.

This is the key to a representation of (2) which doesn't require that everyone who has known Madonna share a single way of thinking about her. In ILAO ([1983], 144; [1988a], 178), we allowed our special sense-denoting terms like ' $\underline{W}_r$ ' and ' $\underline{f}_r$ ' to be indexed by variables as well as names. Contrast (i) Schiffer's notation ' $m_M$ ', in which ' $m$ ' is a variable ranging over modes and in which the whole expression ' $m_M$ ' denotes the sense of the name 'Madonna', with (ii) our notation ' $\underline{m}_x$ ', in which ' $m$ ' names Madonna, ' $\underline{m}$ ' is a term that ranges over possible senses of the name 'Madonna' and the subscripted index ' $x$ ' serves to identify the particular sense the term 'Madonna' has for  $x$ . So relative to an assignment to the variable  $x$ , ' $\underline{m}_x$ ' denotes the sense of the name 'Madonna' for person  $x$ .<sup>13</sup>

Similarly, we symbolize the mode of presentation that  $x$  associates with the predicate 'is musical' as:  $\underline{M}_x$ . So a Fregean with a formalism that can represent how the sense of a term may vary, could use the following representation for Schiffer's sentence (2):

$$(2a) \forall x (Kxm \rightarrow B(x, [\lambda \underline{M}_x \underline{m}_x]))$$

<sup>13</sup>See Zalta [1983], Chapter VI, and [1988a], Chapters 9 – 12. To handle more complex examples, such as Kripke's [1979] 'Paderewski' case, ILAO is designed so that the sense of a term can vary from time to time for any given person. But I will not examine such complicated cases here. See Zalta [1988a], 195-6.

On this analysis, (2) is true just in case each person  $x$  that has known Madonna stands in a relation to a certain Fregean thought, namely,  $\underline{M}_x \underline{m}_x$ . This thought is a proposition-like logical complex in which  $\underline{M}_x$  and  $\underline{m}_x$  are constituents. Note that from a formal point of view, such an analysis is immune to Schiffer's objection, for it doesn't imply that there is a single mode of presentation under which everyone thinks of Madonna, or a single mode under which everyone thinks of musicality, or a single complex Fregean thought by which everyone thinks that Madonna is musical.

Of course, it seems reasonable to suggest that (2) has a reading on which it includes people who in some sense know Madonna and think she is musical but who don't know her name or know her as 'Madonna'. Such a reading becomes even clearer if we replace 'known' by 'seen' in (2); many people have seen Madonna and believe that she is musical without having learned the name 'Madonna' as the name of the woman they have seen. There is, at present, one way to represent this reading in ILAO. On that reading, the terms in the context of attitude verbs denote their ordinary denotation. Let us say that the occurrence of a term  $\tau$  in an ordinary language attitude report is a *de re* occurrence whenever substitutions of codenotational terms for  $\tau$  are truth preserving. When such substitutions for  $\tau$  are not truth preserving, we say that the occurrence of  $\tau$  is a *de dicto* occurrence. Therefore, in addition to the above reading of (2), ILAO also permits the following *de re* reading:

$$(2b) \forall x [Kxm \rightarrow B(x, [\lambda Mm])]$$

On this reading, (2) is true iff every person  $x$  that has known Madonna stands in the belief relation to a certain singular proposition, namely the one having the property of being musical and Madonna as constituents. This gives us one reading of (2) on which it quantifies even over those acquainted with Madonna without knowing her name. On this reading of (2), if we substitute for 'Madonna' any term that has the same denotation, truth is preserved. This analysis survives even when we replace 'known' by 'seen' in (2), and replace ' $K$ ' by ' $S$ ' in (2a).

However, this second reading (2b) raises a question about the first reading (2a). One might wonder whether (2a) can cover even those people who don't know Madonna by name. Right now, (2a) asserts that for any person  $x$  that knows Madonna,  $x$  is belief-related to the Fregean thought which has the following constituents:  $x$ 's sense of the name 'Madonna' and

$x$ 's sense of the predicate term 'is musical'. But what if we replace 'known' by 'seen' in (2) and consider those people who have seen Madonna (say on a video) and think she is musical, but who don't know her by name and, indeed, have never heard the name 'Madonna' used to refer to Madonna? By hypothesis, such people won't have a sense for the name 'Madonna', or if they do, it won't be a sense which represents Madonna. But such people will have at least one mode of presentation for Madonna, since there will be such modes in connection with their past episodes of seeing Madonna. Shouldn't there be reading of this variant of (2), in which every such person has a belief about Madonna's musicality which is effected by way of their mode of presentation for Madonna? Can we produce a variant of (2a) and regard it as asserting: for any person  $x$  that has known or seen Madonna,  $x$  is belief-related to a Fregean thought which has as constituents some mode of presentation that  $x$  has for Madonna and some mode of presentation that  $x$  has for the property of being musical?

This is, in fact, one of the reasons for adding flexibility to ILAO so that we may analyze belief reports in terms of the more general notion of a mode of presentation. Although ILAO addresses the problem Schiffer raised for the Fregean, it is still not quite flexible enough. In Section 4, we'll consider the technical issues that arise in when we consider adding new notation to our formalism, to obtain the needed flexibility. Our goal will be to extend the formalism, so that it can be used in analyses (of attitude reports) that are tied less closely to the notion of a sense and more closely to the notion of a mode of presentation.

Here is another kind of example which makes the point. We often report the beliefs of a person  $x$  in terms of proper names and predicates which  $x$  would not or could not have used. For example, we might reasonably report either of the following:

- (3) Hammurabi believed that Hesperus is not made of water.
- (4) Hammurabi didn't believe that Hesperus is not made of  $H_2O$ .

Given that the the property of being made of water is identical to the property of being made of  $H_2O$ , and that the Russellian proposition denoted by "that Hesperus is not made of water" is identical to the one denoted by "that Hesperus is not made of  $H_2O$ ", we'll have to appeal to the distinct modes of presentation by which Hammurabi conceived of the property of being made of water, if we are to represent these belief reports as consistent. From the terms ' $W$ ' ('is made of water'), ' $H$ ' ('is

made of  $H_2O$ ') and ' $b$ ' ('Hammurabi'), ILAO allows us to create special terms ' $\underline{W}_b$ ' and ' $\underline{H}_b$ '. We have interpreted these latter terms respectively, as the sense of the predicate 'is made of water' for Hammurabi and the sense of the predicate 'is made of  $H_2O$ ' for Hammurabi. But clearly, we don't want to analyze the belief reports in terms of the senses of these expressions for Hammurabi, since Hammurabi would not have used the expressions 'is made of water' and 'is made of  $H_2O$ '. Instead, the analyses of the reports should be in terms of the modes of presentation by which Hammurabi conceptualized the property of being made of water.

For these reasons, then, we plan to investigate the more general notion of a 'mode of presentation' within the context of ILAO. Our background theory of abstract objects doesn't need to be changed, for abstract individuals and abstract relations can just as easily play the more general role of modes of presentations. In the next section, we see one of the reasons why this is so.

## §2: Modes of Presentation and Concepts

In Frege's theory, the sense of a term is, or contains, a mode of presentation. Since the category *mode of presentation* is wider than the category *sense* (given that not all modes of presentation function as the sense of some term of natural language), it is possible to tease them apart. Modes of presentation are not introduced to explain problems in the philosophy of language such as the informativeness of identity statements or substitution failures in attitude reports, but rather to explain problems in the philosophy of mind such as our frequent recognition failures of familiar objects and properties and the apparent contradictory attitudes we sometimes have with respect to one and the same object. Modes of presentation may therefore be introduced and connected with the different ways by which we can conceive of an object or property (relation). One and the same object or property (relation) can be conceived in different ways and the various conceptions need not be recognized as conceptions of the same entity. Moreover, our conceptions of objects combine somehow with our conceptions of properties (relations) to form various complex conceptions. One and the same Russellian proposition or state of affairs can be conceived via different complex conceptions but these complex conceptions need not be recognized as conceptions of the same Russellian proposition or state of affairs.

It should come as no surprise to readers familiar with my work that my candidates for the role of modes of presentation are the abstract individuals and abstract relations of our typed intensional logic ILAO.<sup>14</sup> Recall that this logic is based on the following typing scheme: where ‘ $i$ ’ is the type for individuals, and where  $t_1, \dots, t_n$  are any types, then ‘ $\langle t_1, \dots, t_n \rangle$ ’ is the type for relations having arguments of type  $t_1, \dots, t_n$ , respectively. We sometimes let ‘ $p$ ’ stand for the type  $\langle \rangle$  (i.e., where  $n = 0$ ); this is the type for propositions, or 0-place logical complexes. In terms of this scheme, the typed comprehension principle for abstract objects asserts the existence not only of abstract individuals, but also of abstract properties and abstract relations (at every relational type).<sup>15</sup> Abstract objects of type  $t$  may encode the same properties that ordinary objects of type  $t$  exemplify. So abstract properties encode properties of properties, and abstract relations encode properties of relations, etc. In particular, abstract properties can encode properties that ordinary properties exemplify, and abstract relations encode properties that ordinary relations exemplify. In what follows, I’ll sometimes refer to ‘A-individuals’ and ‘A-properties’ (‘A-relations’) to talk about these abstract entities, and refer to them generally (both A-individuals and A-relations) as ‘A-objects’.

In our previous work, we used this typed theory of abstract objects as follows. If a term of natural language is a term of type  $t$  (i.e., denoted an object of type  $t$ ), then we represent the sense of that term (with respect to some individual) as an A-object of type  $t$ . For example, the sense of a term denoting an individual is an abstract individual; the sense of a term denoting a relation is an abstract relation. So the sense of a term of natural language was analyzed as an abstract object of the very same logical type as denotation of the term.<sup>16</sup> We showed that this identification

<sup>14</sup>These are not on the list of suspects that Schiffer ([1990], 254-265) rejects as answers to the question ‘What are modes of presentation?’.

<sup>15</sup>On this typing scheme,  $F^{(t_1, \dots, t_n)} x^{t_1} \dots x^{t_n}$  is a well-formed exemplification formula and  $x^t F^{(t)}$  is a well-formed encoding formula. For any logical type  $t$  (whether individual, relation, or proposition), the theory postulates abstract things of type  $t$  as follows:

$$\exists x^t (A!^{(t)} x^t \ \& \ \forall F^{(t)} (x F \equiv \phi)), \text{ where } \phi \text{ has no free } x\text{s.}$$

The domain of each type  $t$  therefore divides up into ordinary things of that type (which do not encode properties) and the abstract things of that type. The abstract things of a type will encode and exemplify the same kinds of properties that the ordinary things of that type just exemplify.

<sup>16</sup>Consider, then, how this stands in contrast to the typing schemes in Church [1951] and Montague [1974]. On those schemes, the sense or intension of a term is always an

would explain how Fregean senses could be modes of presentation.

But, in present essay, we are interested in showing more generally that modes of presentation can be identified as A-objects. There are two basic reasons for thinking that modes of presentation can be identified as A-individuals and A-relations. The first is the fact that an A-object of type  $t$  can *represent* an ordinary object of type  $t$ , and the second is that ILAO predicts that A-individuals and A-relations can combine to form logical complexes which can represent structured (Russellian) propositions or states of affairs. In the remainder of this section, we examine the first reason, and we take up the second reason in the next section.

The first reason for thinking that modes of presentation can be identified as A-objects is that there is a clear sense in which A-objects can represent ordinary individuals and properties, namely, by encoding properties that ordinary individuals and properties exemplify. By saying that A-objects ‘represent’ ordinary objects, I do not mean to imply that A-objects are ‘in the head’. A-objects are *not* in the head, though they can be used to classify what is in the head. As we shall see, A-objects can objectify the cognitive content of the mental tokens that are in the head. Of course, philosophers often suppose that the (intrinsically meaningless) mental tokens in our brains are the entities which ‘represent’ ordinary individuals and properties, and certainly there is a sense of ‘represent’ on which this may be true. But I am not using this sense of ‘represent’. I am using ‘represent’ to mean a *way* of conceiving an object or property, and though an intrinsically meaningless mental token can be a vehicle by which an object or property is conceived, it is not a *way* of conceiving them if it is intrinsically contentless. I am using a sense of ‘represent’ on which the object which is doing the representing does so in virtue of some *content* which it has. A *way* of conceiving an ordinary individual or property involves some cognitive content by which that individual or property is conceived.

Let’s look at an example of how an A-object can represent. Consider a person  $x$  having a veridical perception of some ordinary individual, say  $d$ , in  $x$ ’s visual field. In  $x$ ’s brain, some (intrinsically meaningless) mental token, call it ‘ $d$ ’, becomes a mental symbol of  $d$ . In the perceptual encounter with  $d$ , certain properties strike  $x$ ’s cognitive apparatus as features of  $d$ . Just which properties strike  $x$  depend on the situation in which  $x$  encounters  $d$ , on how attentive  $x$  is, and on  $x$ ’s capacity for observation of higher or different type than that denoted by the term.

tion and discrimination. So the mental token  $d$  gets cognitively *linked* to (mental tokens of) those properties that  $x$  registers in the perceptual encounter.

Now I claim that the A-individual that encodes those properties, call it  $\mu_1$ , just is the cognitive content of  $x$ 's mental token  $d$ , whereas  $d$  itself is the objective content of  $d$ .<sup>17</sup> The objective content of a mental token is the object that is causally responsible for the token's existence and which stands at the beginning of the chain leading back to the first perceptual encounter. But whereas the token  $d$  is in the head,  $\mu_1$  is not. The A-individual  $\mu_1$  represents  $d$  for  $x$  because it objectifies the (cognitive) content that the mental token  $d$  has or plays in  $x$ 's mental life. The token  $d$  then represents  $d$  to  $x$  both in the sense that it denotes  $d$  ( $d$  is its objective content), and more importantly for the present investigation, in the sense that  $\mu_1$  is one way by which  $x$  conceives of  $d$  (namely, the way which is defined by the particular group of properties it encodes).

Note that the A-individual  $\mu_1$  can represent the ordinary individual  $d$  for person  $x$  even though  $\mu_1$  encodes properties that  $d$  does not exemplify! Indeed,  $\mu_1$  may encode properties which individuate nothing, or which individuate something other than  $d$ . In cases of misinformation, such as non-veridical perceptions, properties which  $d$  doesn't really exemplify are presented in the context in which a mental token for  $d$  is created. Person  $x$  may take these properties to be characteristic of  $d$  and so these properties can be encoded by the A-individual  $\mu_1$  that is a mode of presentation of  $d$  for  $x$ . It is an *extrinsic* fact about  $\mu_1$  that it represents  $d$  for  $x$ ; this fact is not a result of  $\mu_1$ 's intrinsic (encoded) properties. But note that since A-objects encode properties, they have an intrinsic content. For example, the properties that  $\mu_1$  encodes constitute its intrinsic content. That is what allows it to *be* a mode of presentation. It re-presents  $d$  to  $x$  by encoding properties that  $x$  takes to be characteristic of  $d$ . The more vivid the properties that are encoded, the more vivid a mode of presentation it is.

Before we extend these ideas to modes of presentation for properties and relations, it is worth pausing for a moment to note how easy it is to introduce the language of 'concepts' at this point by just identifying

*concepts* with A-objects. Concepts of individuals may be identified as A-individuals that encode properties of individuals. (We'll see, in just a few paragraphs, that concepts of properties may be identified as A-properties that encode properties of properties.) Returning to the example we have been developing, we could say that  $\mu_1$  is one of the concepts that  $x$  has of  $d$ , namely, the concept of  $d$  that associated with the mental token  $d$ . If  $x$  encounters  $d$  (without knowing it is  $d$ ) in some completely different situation, some new mental token  $d'$  could become created in  $x$ 's cognitive architecture. The cognitive content of  $d'$ , call it the concept  $\mu_2$ , would encode different properties from those encoded by  $\mu_1$ .  $x$  might fail to recognize  $d$  because  $\mu_2$  would bear little resemblance to  $\mu_1$  (i.e., they would encode few properties in common). This identification of concepts has the added virtue that the precise existence and identity conditions for A-individuals now become existence and identity conditions for possible concepts.

We now extend these ideas to modes of presentation, and concepts, of properties and relations. Recall that we will be distinguishing ordinary properties and relations, conceived intensionally, from the hyperintensional modes of presentation, and concepts, by which we represent these entities.<sup>18</sup> Consider a being  $x$  that has the mental architecture to detect or perceive instances of some particular property, say  $P$ . Suppose  $x$  has perceptually encountered an exemplification instance of  $P$  for the first time and has registered that  $P$  is involved in the perception or is cognizing that the perception involves  $P$ . It seems that  $x$ 's cognitive architecture must take steps to represent the property  $P$  by first creating a mental token, say 'P'. Since various properties of properties will strike  $x$ 's cognitive architecture as being characteristic of  $P$ , we can take the cognitive content of the mental token P to be the A-property which encodes these properties of properties. This A-property serves to represent  $P$  to  $x$  by encoding some of the properties that  $P$  itself may exemplify. For example, consider the property of being a circle, and let 'C' denote the mental token for this property in  $x$ 's brain, and let  $\xi_1$  be an A-property that encodes one or more of the following properties of properties (all of which

<sup>17</sup>Note that this distinction between the objective content and cognitive content of the mental token  $d$  is the same distinction which we developed in previous work ([1988a], 158) with respect to the terms of natural language. A term of natural language has both an objective content (its denotation) and a cognitive content (its sense) for the person using or hearing the term.

<sup>18</sup>Ordinary properties and relations have precise existence and identity conditions. Their existence conditions are governed by a comprehension principle ([1983], 31; [1988a], 46; [1993b], 405; [1999], 623), while their identity conditions have been defined in terms of the notion of encoding. The identity conditions for properties are quite easy to state: ordinary properties  $F$  and  $G$  (of any complex type  $\langle t \rangle$ ) are identical just in case necessarily,  $F$  and  $G$  are encoded by the same objects (of type  $t$ ).

might have been cognized by  $x$  when  $x$  was becoming acquainted with the property of being a circle): being a shape that involves no straight lines, being a shape that is always uniformly curving, being a shape which can be inscribed with the help of a compass, being the geometric property exemplified by this particular figure on this particular printed page, etc.  $\xi_1$  is clearly a mode of presentation for  $x$  of the property of being a circle. It is one of the concepts that  $x$  has for the property of being a circle.

Now of course,  $x$  may subsequently cognize the property of being a circle in a different way, say by a description, without realizing it. For example,  $x$  might learn some geometry, and encounter the shape which, in analytic geometry, is defined by the equation  $a^2 + b^2 = c^2$ , or the shape which, in Euclidean geometry, is defined as being a closed, plane figure every point of which lies equidistant from some given point. Now the properties in question (being a circle, being defined by the equation  $a^2 + b^2 = c^2$  in analytic geometry, being a closed, plane figure . . . ) are all identical. But  $x$  may have different conceptions (i.e., different concepts) of this property. Each conception is a different mode of presentation, a different way of representing the property of being a circle in  $x$ 's cognitive architecture. Consider, for example, the A-property, call it ' $\xi_2$ ', which encodes the following properties of properties: being a property  $F$  which can be defined by a certain equation in analytic geometry, being a property  $F$  which can be defined by a certain condition in Euclidean geometry, etc. Here we might have a case of 'recognition failure';  $x$  may not recognize that the concept  $\xi_1$  and the concept  $\xi_2$  are concepts of the same property. Notice our switch here to the language of concepts. It seems natural to suppose that A-properties serve as our concepts of ordinary properties.

The foregoing remarks should be sufficient to establish that A-individuals and A-relations can represent ordinary objects and relations. This, you may recall, was the first reason for analyzing modes of presentation and concepts as A-objects. Before we turn to the second reason for doing so, we may summarize our analysis explicitly by way of the following equivalences:

$z^t$  is a *mode of presentation* [concept] of  $y^t$  for person  $x^i$  iff  $z^t$  is an abstract object of type  $t$ ,  $y^t$  is an object of type  $t$ , and  $z^t$  is the cognitive content of some mental token in  $x^i$ 's cognitive architecture which has  $y^t$  as its objective content

$z^t$  is a *mode of presentation* [concept] iff  $\exists y^t \exists x^i (z^t$  is a mode of

presentation [concept] of  $y^t$  for  $x^i$ )

Note that by supposing  $x$ 's cognitive architecture to contain mental tokens that correspond to the linguistic expressions in  $x$ 's vocabulary, then the cognitive content of a linguistic expression for  $x$  can be identified with the cognitive content of the corresponding mental token. It would follow that the sense of a linguistic expression for an individual is a mode of presentation.

It is important to point out that our analysis satisfies what Schiffer calls the 'Intrinsic Description Constraint' ([1990], 253; and [1992], 511). The entities in terms of which modes of presentation have been identified can be characterized independently of their role. As abstract objects, modes and concepts are intrinsically characterized by the axioms of our theory, and these are stated in terms of the notion of encoding. Encoding is the central notion of object theory, just as set membership is the central notion of set theory. If one accepts (as Schiffer does) that the notion of set membership and the axioms of set theory suffice to intrinsically characterize sets, one should accept that the notion of encoding and the axioms of object theory intrinsically characterize A-objects.<sup>19</sup>

### §3: Plugging the Theoretical Gap

The second reason for thinking that modes of presentation, and concepts, of individuals and relations can be identified with A-individuals and A-relations, respectively, is that under such an identification, ILAO predicts that modes of presentation (concepts) of relations can be combined with modes of presentation (concepts) of individuals to yield modes of presentation (concepts) of propositions or states of affairs. The formal semantics developed for ILAO in previous work makes use of a logical operation, the **PLUG** operation,<sup>20</sup> which can take objects  $o_1, \dots, o_n$ ,

<sup>19</sup>In addition, our theory of modes of presentation does not suffer from Schiffer's 'regress problem' ([1990], 68). The reason is that they serve to characterize the cognitive contents of mental tokens. Our mental tokens exist in a finite network in which tokens are linked to one another: tokens of individuals are linked to tokens of properties, tokens of properties are linked to tokens of properties of properties, etc. But there is no regress because the structure of tokens is a finite one. So there is a finite set of logical types of A-objects that serve to characterize the contents of the various tokens. The structure branches only into those higher-typed entities of which we have actual conceptions.

<sup>20</sup>The function **PLUG** is an algebraic operation that semantically corresponds to exemplification predication. It has been defined in numerous technical publications.



having types  $t_1, \dots, t_n$ , respectively, and ‘plugs’ them into a relation of type  $\langle t_1, \dots, t_n \rangle$ , producing a 0-place logical structure (of type  $p$ ). The relation here need not be a primitive relation (it maybe complex) and the objects here need not be individuals (but they have to be objects of a type which is appropriate for a given argument place of the relation). In the simplest case, the **PLUG** function can combine ordinary individuals and an ordinary relation among individuals to produce an ordinary ‘Russellian’ proposition (or, if you prefer, a ‘state of affairs’).<sup>21</sup> It can also combine A-individuals and an A-relation to produce a logical complex which is structurally identical to a Russellian proposition yet which contains only abstract constituents. But note that an A-object of type  $t$  can occupy any place of a 0-place logical complex that an ordinary object of type  $t$  can occupy. So **PLUG** can even combine any individuals, ordinary or abstract, with either an ordinary or an abstract relation to produce a logical complex which is structurally identical to a Russellian proposition but which has a mixture of abstract and ordinary constituents. These ‘mixed’ complexes will prove to be useful.

The point here is that by identifying modes of presentation (and concepts) with A-objects, ILAO predicts the existence of complex modes of presentation (and complex concepts). These latter can be used to analyze ‘ways of conceiving’ Russellian propositions or states of affairs; they are structurally identical to Russellian propositions but they may have an abstract constituent of type  $t$  where the Russellian proposition has an ordinary constituent of type  $t$ . In terms of the language of concepts, we could say that a way of conceiving an (atomic) proposition is defined whenever **PLUG** combines the concepts of individuals with a concept of a relation.

In previous work, this analysis gave us a clear theory of Fregean thoughts. The Fregean sense of a predicate was identified as an A-relation, and the Fregean senses of individual terms were identified as A-individuals. So the existence of Fregean thoughts was predicted by

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Compare the *pred* and **PLUG** functions in the following publications: Bealer [1979], Parsons [1980], McMichael and Zalta [1980], Bealer [1982], Zalta [1983], and Menzel [1986].

<sup>21</sup>On the present usage, ‘Russellian proposition’ and ‘state of affairs’ are used interchangeably. Some philosophers may wish to reserve the word ‘proposition’ to refer to complexes consisting of *concepts* of individuals and relations. Those philosophers will have to remember that in what follows, our usage of ‘Fregean thought’ corresponds to their usage of ‘proposition’.

ILAO.<sup>22</sup> Since Frege never told us what senses and thoughts were, our identification fills the theoretical gap in his work.

Notice also, that if we focus on ILAO’s analysis of Fregean senses as A-objects, we can easily interpret, and fill the theoretical gap, in the works of Kaplan and Forbes cited at the outset. These two philosophers discuss Fregean senses (as modes of presentation) in their work without providing a theory of them. Consider first Kaplan’s work, in which we find the following analyses of a ‘relational’ (as opposed to ‘notional’) belief ([1969], item (44)):

(K)  $\exists \alpha [\mathbf{R}(\alpha, \text{Ortcutt}, \text{Ralph}) \ \& \ \text{Ralph} \mathbf{B}^\top \alpha \text{ is a spy}^\top]$

In this analysis,  $\mathbf{R}(\alpha, \text{Ortcutt}, \text{Ralph})$  asserts that  $\alpha$  represents Orcutt to Ralph. Although the variable  $\alpha$  at this point in Kaplan’s paper stands for an expression, Kaplan later admits that this is inadequate to the task ([1969], Section XII):

When earlier I argued for Frege’s method—seek the intermediate entity—it was on the grounds that a clarified view of the problem was worth at least momentary ontological risk. But now it appears that to give adequate expression to the epistemological situation requires explicit quantification certification of the status of such entities. I am undismayed and even would urge that the conservative course so far followed of taking expressions as the intermediate entities is clearly inadequate to the task.

Kaplan then discusses the cases in which the mode of presentation involves not words but sensory images, impressions, sounds, etc. These are part of what makes names *vivid*.

We can now identify the ‘missing intermediate entities’ which Kaplan discusses if we take the variable  $\alpha$  to range over our A-individuals. We’ve seen how these objects can *represent* ordinary objects to individuals. That gives us an interpretation of the clause ‘ $\mathbf{R}(\alpha, \text{Ortcutt}, \text{Ralph})$ ’ in (K) above. Moreover, we can interpret the clause ‘ $\text{Ralph} \mathbf{B}^\top \alpha \text{ is a spy}^\top$ ’ in (K) as follows. Let  $\mathbf{S}$  be the A-property which serves as the sense of ‘is a

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<sup>22</sup>Though Frege would say that the sense of the predicate ‘ $F$ ’ maps the sense of the individual term ‘ $a$ ’ to a thought, in ILAO, it is the **PLUG** function that maps the sense of ‘ $F$ ’ and the sense of ‘ $a$ ’ to the thought expressed by the sentence ‘ $a$  is  $F$ ’. Moreover, each of the senses involved is relativized to a person and **PLUG** maps those senses to the thought the sentence expresses for that person.

spy'. Then, given our previous identification of  $\alpha$  in (K) as A-individual, the second clause simply relates Ralph to the proposition  $\mathbf{PLUG}(\mathbf{S}, \alpha)$ . This is the propositional complex which has  $\alpha$  plugged into Ralph's sense of the predicate 'is a spy'. Finally, Kaplan's talk of *vivid* names now makes more sense—if the mode of presentation Ralph associates with a name of Ortcutt is an A-individual that encodes vivid (i.e., visual or other perceptual) properties, then that name will become a 'vivid name' of Ortcutt for Ralph.

The theory also provides an interpretation for the uninterpreted notation used by Forbes. Recall that Forbes uses the variables  $\alpha, \beta$  to range over modes of presentation and represents the fact that a mode of presentation  $\alpha$  has combined with the sense of the predicate ' $F$ ' to form a complex sense by using the notation:  $\alpha \hat{\ } \ulcorner F \urcorner$ . There is a natural interpretation of this on the present view. The variables  $\alpha, \beta$  may be interpreted to range over A-individuals. We may take  $\ulcorner F \urcorner$  to be the A-property which serves as the sense of the predicate ' $F$ '.<sup>23</sup> Then the notation  $\alpha \hat{\ } \ulcorner F \urcorner$  can be interpreted as representing the Fregean thought  $\mathbf{PLUG}(\ulcorner F \urcorner, \alpha)$ . With these suggestions, then, we eliminate a theoretical gap in this recent work in defense of Frege's theory of senses.

We next consider how ILAO now provides a framework for those analyses of attitudes and attitude reports which appeal to modes of presentation without appealing to Frege's notion of sense. Salmon, for example, eschews senses and works directly with modes of presentation in analyzing attitudes. Although Salmon uses a 2-place ' $B$ ' relation to analyze belief reports, he uses a 3-place  $BEL$  relation to analyze beliefs. Our identification of modes of presentation fills the theoretical gap in Salmon's analysis since it offers a precise understanding of the 'ways of taking Russellian propositions' which serve as one of the arguments to the  $BEL$  relation. Here is how.

Consider, for example, the following claims:

- (3) Hammurabi believed that Hesperus is not made of water.
- (5) Hammurabi didn't believe that Phosphorous is not made of  $H_2O$ .

The pure *de re* readings of (3) and (5), in ILAO, are:

$$(3a) B(b, [\lambda \neg Wh])$$

<sup>23</sup>Note that Forbes is assuming that the sense of a predicate does not vary from person to person.

$$(5a) \neg B(b, [\lambda \neg Hp])$$

Given the identity statements ' $W = H$ ' and ' $h = p$ ', these are inconsistent reports. But Salmon analyzes the underlying beliefs as consistent. Consider how we can develop further analyses of (3a) and (5a) which are very much in the spirit of Salmon's analysis ([1986], 111) even if they don't follow it to the letter. In ILAO, Salmon's  $BEL$  relation would be of type  $\langle i, p, p \rangle$  and so holds between an individual and two 0-place logical complexes of type  $p$ .  $BEL(x, q, r)$  holds just in case the person  $x$  believes  $q$  by means of the complex mode of presentation  $r$ . Now, following Salmon's basic idea, we could claim that (3a) and (5a) respectively entail such claims as (6) and (7), respectively, where  $ModeOf(z^t, y^t, x^i)$  is a predicate representing the notion " $z^t$  is a mode of presentation (concept) of  $y^t$  for  $x^i$ " (which we analyzed at the end of Section 2):

$$(6) \exists z^i \exists F^{(i)} \exists q (ModeOf(z, h, b) \ \& \ ModeOf(F, W, b) \ \& \ q = [\lambda \neg Fz] \ \& \ BEL(b, [\lambda \neg Wh], q))$$

$$(7) \exists z^i \exists F^{(i)} \exists q (ModeOf(z, p, b) \ \& \ ModeOf(F, H, b) \ \& \ q = [\lambda \neg Fz] \ \& \ \neg BEL(b, [\lambda \neg Hp], q))$$

These assert, respectively:

- (8) There exists a concept  $z$  of type  $i$ , a concept  $F$  of type  $\langle i \rangle$ , and a logical complex  $q$  of type  $p$  such that (i)  $z$  is a mode of presentation of Hesperus for Hammurabi, (ii)  $F$  is a mode of presentation of being made of water for Hammurabi, (iii)  $q$  is the complex *that  $z$  doesn't exemplify  $F$* , and (iii) Hammurabi stands in the  $BEL$  relation to the proposition *that Hesperus exemplifies being visible in the evening* by means of the complex  $q$
- (9) There exists a concept  $z$  of type  $i$ , a concept  $F$  of type  $\langle i \rangle$ , and a logical complex  $q$  of type  $p$  such that (i)  $z$  is a mode of presentation of Phosphorus for Hammurabi, (ii)  $F$  is a mode of presentation of the property of being made of  $H_2O$  for Hammurabi, (ii)  $q$  is the complex *that  $z$  doesn't exemplify  $F$* , and (iii) Hammurabi fails to stand in the  $BEL$  relation to the proposition *that Phosphorus doesn't exemplify being made of  $H_2O$*  by means of the complex  $q$ .

(6) and (7) are clearly formulable in ILAO, and they show how an analysis similar to the one proposed by Salmon, reflecting the consistency of Hammurabi's beliefs, can be spelled out in precise detail.

So it remains for us to examine the recent work by Crimmins. Here is where our hybrid, or ‘mixed’ logical complexes, in which A-individuals are plugged into the argument places of ordinary relations, come into play. In [1998], Crimmins considers Hammurabi’s belief that Hesperus is visible in the evening. He says ([1998], 11):

Where  $m_H$  is the mode of presentation in question, the fictional truth turns on the claim diagrammed in (10):

(10) Hammurabi believed:  $[m_H]$  is visible in the evening.

... In (10), we are using the bracket notation in a formula that, in describing a belief, partly describes what is allegedly believed to be so, and also describes the agent’s alleged way of thinking of the subject matter of the belief. The formula (10) portrays a state of affairs that obtains just in case Hammurabi had a belief ascribing evening visibility, and this belief involved the mode of presentation  $m_H$  in the “subject position.” (It does not entail that Hammurabi’s belief is *about* the mode of presentation.)

Crimmins then notes that (10) is not the most perspicuous notation, and that a more perspicuous (and more complex) notation would make it clear that the mode of presentation is not part of the propositional object of belief. But I am intrigued by the fact that Crimmins uses the simpler notation in his paper to regiment belief reports (cf. Zalta [1983], 130; [1988a], 161-173; and [1989], 461). Let us consider whether we might take this simpler regimentation at face value and address Crimmins’ reasons for not doing so.

Using the simpler notation, Crimmins analyzes the truth conditions of (11) as (12) ([1998], 12):

(11) Hesperus, but not Phosphorus, was thought by Hammurabi to be visible in the evening.

(12) Hammurabi believed:  $[m_H]$  is visible in the evening, but Hammurabi did not believe  $[m_P]$  is visible in the evening.

Now, as we’ve said, Crimmins explicitly disavows the idea that modes of presentation are constituents of thoughts. Nevertheless, we can interpret (12) directly in ILAO! Since the notation Crimmins uses doesn’t index the mode of presentation to Hammurabi, let us use  $\underline{h}$  as the semantic

name of the mode of presentation Crimmins refers to as ‘ $[m_H]$ ’, and use  $\underline{p}$  as the semantic name of the mode of presentation Crimmins refers to as ‘ $[m_P]$ ’. Then if we use ‘ $\mathbf{V}$ ’ as a semantic name of the property of being visible, ILAO asserts the existence of the propositions  $\mathbf{PLUG}(\mathbf{V},\underline{h})$  and  $\mathbf{PLUG}(\mathbf{V},\underline{p})$ . The most straightforward interpretation of (12), then, is to regard it as asserting that Hammurabi is related to the first, and not the second, of these two propositions.

Of course, it is essential here not to confuse these truth conditions of the belief report with the truth conditions of the beliefs being reported. The truth conditions of the beliefs being reported have to be stated in terms of the ordinary Russellian proposition having the property of being visible and Venus itself as constituents. We can say Hammurabi’s belief is ‘about’ these entities, and this respects Crimmins’ injunction that Hammurabi’s belief is not *about* the mode of presentation. So whereas the truth of the belief report in (12) relates Hammurabi to  $\mathbf{PLUG}(\mathbf{V},\underline{h})$ , the belief of Hammurabi’s which is being reported is true iff the proposition  $\mathbf{PLUG}(\mathbf{V},\underline{h})$  is true, i.e., iff Venus is visible in the evening. This distinction, between the truth of the belief report and the truth of the belief reported, will come up again in the next section<sup>24</sup>

It is important to point out here that Crimmins developed the analysis (12) from within the context of a ‘pretense-theoretic’ account of belief reports, one which extends the ideas in Walton [1990]. Crimmins certainly does not regard modes of presentation as abstract objects ([1998], 8). Instead, he supposes that a pretense-theoretic account of modes can be given. But even though it might seem that our understanding of (12) in ILAO is inconsistent with Crimmins’ own understanding of (12), this may not be the case. In [2000b], we develop a rapprochement between object theory and pretense theory. Abstract objects can be understood as entities which the pretense theorist accepts and over which she quantifies, namely, patterns of pretense behavior and manners of speaking. This rapprochement may be consistent with Crimmins’ own pretense-theoretic conception of modes of presentation.<sup>25</sup> I won’t, however, pursue this question any further in the present paper.

<sup>24</sup>The distinction between the truth of the belief report and the truth of the belief reported was defined precisely in [1983] (130), [1988a] (175), and [1989] (462).

<sup>25</sup>Consider the remarks that Crimmins makes about the connection between modes of presentation and pretense in [1998] (10, 26). He talks about manners of speaking and our access to certain modes of presentation.

#### §4: An Unresolved Technical Issue

There is an interesting technical issue that arises concerning the best way of developing a new analysis of attitude reports *within the logic* ILAO. In previous work, we have represented such reports as having multiple readings. Reconsider (3) and (5), for example.

(3) Hammurabi believed that Hesperus is not made of water.

(5) Hammurabi didn't believe that Phosphorous is not made of H<sub>2</sub>O.

According to the plan developed in our previous work, (3) and (5) could each have up to four readings. However, given that Hesperus is Phosphorus ( $h = p$ ) and that being made of water just is being made of H<sub>2</sub>O ( $W = H$ ), the readings on which (3) and (5) are consistent are the pure *de dicto* readings:

(3b)  $B(b, [\lambda \neg \underline{W}_b \underline{h}_b])$

(5b)  $\neg B(b, [\lambda \neg \underline{H}_b \underline{p}_b])$

The relation denoted by ' $B$ ' is of type  $\langle i, \langle \rangle \rangle$  and therefore relates an individual to a 0-place logical complex. The ' $\underline{W}_b$ ' and ' $\underline{H}_b$ ' denote distinct A-properties, while an ' $\underline{h}_b$ ' and ' $\underline{p}_b$ ' denote distinct A-individuals.

But the expressions ' $\underline{W}_b$ ', ' $\underline{H}_b$ ', ' $\underline{h}_b$ ' and ' $\underline{p}_b$ ' represent the senses of the expressions 'is made of water', 'is made of H<sub>2</sub>O', 'Hesperus', and 'Phosphorus' for Hammurabi, respectively. As we noted at the end of Section 1, we should like to find a way to avoid representing the truth conditions of reports like (3) and (5) in terms of Hammurabi's senses of English expressions. So, the question is whether we can introduce notation that picks out appropriate modes of presentation for the analysis of (3) and (5) *without* further identifying them as the Fregean senses of the terms in question. This is an interesting question because a problem of uniqueness arises. For example, if we cannot consider the sense of the expression 'Hesperus' for Hammurabi, how are we to understand 'Hesperus' in (3) so that it both picks out one of the many modes of presentation by which Hammurabi conceived of Venus and contributes that mode to the truth conditions of the belief report? The problem becomes even more acute in the case of predicates. How are we to pick out the relevant concepts by which Hammurabi conceived of the property of being made of water without incorrectly treating those concepts as the senses, for Hammurabi,

of the English expressions 'is made of water' and 'is made of H<sub>2</sub>O'? Can we introduce notation which somehow picks out the relevant modes of presentation, either by using a choice function to interpret the notation or by supposing that the context in which the report is uttered will suffice to isolate the relevant modes of presentation by which Hammurabi conceived of Venus and the property of being made of water?

To summarize, then, we are now interested in the question of whether we can develop in ILAO a *new reading* of belief reports which invokes modes of presentation without invoking senses. To present the question in a maximally explicit way, consider the notation ' $\underline{h}_x$ ', where this is supposed to pick out a mode of presentation by which  $x$  conceives of Hesperus. Suppose that we interpret this notation as follows: if given assignment function  $f$  to the variable  $x$ , a choice function  $g$  maps ' $\underline{h}_x$ ' to one of the modes of presentation by which  $f(x)$  conceives of Hesperus (i.e., Venus). If we enhance our formalism with  $\epsilon$ -expressions, we might even consider such notation as abbreviations of formal expressions of ILAO as follows. That is, we might introduce:

$$\underline{h}_x =_{abbr} \epsilon z (ModeOf(z, h, x))$$

We would read the  $\epsilon$ -term as "an individual  $z$  such that  $z$  is a mode of presentation of  $h$  (Hesperus) for  $x$ ". (We may suppose that if there is no object  $z$  that is a mode of presentation of Hesperus for  $x$ , then the  $\epsilon$ -term denotes the null abstract object of type  $i$ , i.e., the A-individual which encodes no properties.) Now suppose, further, that ' $\underline{p}_x$ ' is introduced in the same way. Similarly, if ' $W$ ' denotes being made of water and ' $H$ ' denotes being made of H<sub>2</sub>O (i.e., the same property), we might introduce:

$$\underline{W}_b =_{abbr} \epsilon F^{(i)} (ModeOf(F, V, b)),$$

and do something similar for ' $\underline{H}_b$ '.

In terms of this new notation, we might offer the following as a new analysis of (3) and (5):

(3c)  $B(b, [\lambda \neg \underline{W}_b \underline{h}_b])$

(5c)  $\neg B(b, [\lambda \neg \underline{H}_b \underline{p}_b])$

On this analysis, the (3) is true just in case Hammurabi stands in the 2-place belief relation to the negation of a logical complex in which some mode of presentation of Hesperus for Hammurabi is 'plugged' into some

mode of presentation of the property of being made of water for Hammurabi. Similarly, (5) is true iff Hammurabi fails to stand in the 2-place belief relation to the negation of a logical complex in which some mode of presentation of Phosphorus for Hammurabi is ‘plugged’ into some mode of presentation of the property of being made of H<sub>2</sub>O for Hammurabi.<sup>26</sup> And, if we generalize our new notation in the obvious way, we could return to Schiffer’s problem case (2) and offer the following new reading:

$$(2c) \forall x(Kxm \rightarrow B(x, [\lambda \underline{M}_x \underline{m}_x]))$$

On this analysis, the truth conditions of (2) would be that every (person)  $x$  who has known (seen) Madonna stands in the belief relation to a logical complex which has as constituents some mode of presentation of the property of being musical and some mode of presentation of Madonna.

Even though these new analyses look promising, there is a problem. Under their present interpretations, there is no guarantee that ‘ $\underline{h}_b$ ’ and ‘ $\underline{p}_b$ ’ will be assigned *distinct* modes of presentation. Without such a guarantee, we cannot rest assured that (3c) and (5c) are consistent. An analogous problem holds for ‘ $\underline{W}_b$ ’ and ‘ $\underline{H}_b$ ’. Is there some other way of interpreting ‘ $\underline{h}_b$ ’ and ‘ $\underline{p}_b$ ’ (and ‘ $\underline{W}_b$ ’ and ‘ $\underline{H}_b$ ’) which ensures that these pairs of expressions pick out different modes of presentation? Or can we relativize the interpretation of  $\epsilon$ -terms to a context, and legitimately suppose that such a relativization can ensure that distinct modes of presentation will be assigned whenever appropriate?

It is unclear to me, at present, whether this technical issue can be nicely resolved, and if so, what is the best resolution. So I am uncertain about what is the best way to extend ILAO so as to develop analyses of attitude reports which appeal to modes of presentation that aren’t conceived as senses. (At least, we have found a new reason to appreciate the tight theoretical connection between senses and modes in Frege’s theory.) Although further study is in order, it should be reiterated that even if no

<sup>26</sup>The logical complexes can be described semantically in terms of **PLUG** and the **NEG** operations as:

$$\begin{aligned} &\mathbf{NEG}(\mathbf{PLUG}(\mathbf{P}, \mathbf{a})), \text{ and} \\ &\mathbf{NEG}(\mathbf{PLUG}(\mathbf{Q}, \mathbf{b})), \end{aligned}$$

where ‘**P**’ is a semantic name of the mode of presentation denoted by ‘ $\underline{W}_b$ ’, ‘**Q**’ is a semantic name of the mode of presentation denoted by ‘ $\underline{H}_b$ ’, and ‘**a**’ is a semantic name of the mode of presentation denoted by ‘ $\underline{h}_b$ ’ and ‘**b**’ is a semantic name of the mode of presentation denoted by ‘ $\underline{p}_b$ ’. See Zalta [1983] or [1988a] for the semantic definition of **NEG**.

nice resolution can be found, our identification of modes of presentation and concepts as A-objects nevertheless provides a framework for other possible analyses of attitude reports which appeal to these entities, as we have seen.<sup>27</sup>

## §5: Concluding Remarks on Concepts

The present theory of concepts treats concepts of individuals as A-individuals and treats concepts of properties and relations as A-properties and A-relations. This gives us a *new* understanding of what it is to ‘possess a concept’. Consider the example of the property of being red. A sighted person with an ordinary visual system, assuming they haven’t been enclosed in a colorless room since birth, etc., will become acquainted with the property of being red and be able to learn properly how to use the predicate ‘is red’. When philosophers say that such a person ‘possesses the concept red’, they usually mean that the person in question has had visual experiences of red things, is (thereby) acquainted with the property of redness, and/or can properly use the term ‘red’. In the present essay, however, we are distinguishing properties and concepts. The present theory allows us to give a new explanation of how a blind person or individual deprived of red stimuli can understand and use the term ‘red’ and even be said to ‘possess the concept red’. This new explanation involves our theoretical notion of a concept and goes as follows.

Although the blind person, or person deprived of red stimuli, has no acquaintance with property *being red*, he or she can nevertheless have one or more concepts of the property of being red. Each concept is something that encodes properties that the property of being red may exemplify, such as being the color of fire engines, being the color of traffic lights which indicate ‘stop’, Bill’s favorite color, being the color of ultraviolet light with a certain frequency, etc. The properties of properties encoded can characterize the property of being red to a blind or sensory-deprived person who has had no visual experiences of red things. Concepts that encode such properties may serve as the cognitive content of some mental token for the property of being red. Now suppose that a blind or sensory-

<sup>27</sup>Recently, S. Boër (1994, 1995) has begun to modify and extend ILAO in new ways, in an attempt to clarify the idea of a language of thought and to interpret such a language. It is not yet clear to me how the work in the present paper aligns with Boër’s work. He has attempted to develop the syntax and semantics of Mentalese within an enhanced version of ILAO.

deprived person, say  $x$ , has such a mental token, say  $R$ , and that  $R$  has a certain cognitive content, say  $\xi_3$ . If the A-property  $\xi_3$  comes to serve as the cognitive content of a term of natural language, say the term 'red', then  $x$  can understand and use the term on the basis of the conception that  $x$  has of the denotation of the term. Similarly, a blind mathematician, with no visual experiences of circular objects, might possess several concepts of the property of being circle, one by way of the Euclidean definition and one by way of the definition in analytic geometry. Each definition yields distinct properties of the property of being a circle, and these distinct properties of properties are encoded by distinct A-properties.

This new explanation suggests that 'possessing a concept' can be a matter of degree. Our judgement that  $x$  does 'possess the concept red', for example, should be based on (i) the degree to which the properties of properties encoded by  $x$ 's concept of red are properties that are in fact exemplified by the property of being red, and (ii) the degree to which the properties of properties encoded reflect intrinsic properties of being red. Similarly, in the case where  $F$  is a more abstract property, such as being a brother, being a number, etc., then the question of whether  $x$  'possesses the concept' will depend on the degree to which the A-property which serves as  $x$ 's ' $F$ '-concept encodes defining or other properties of properties which are exemplified by the property  $F$ . This can be tested, since these A-properties will ground  $x$ 's propositional attitudes about the property  $F$ . By this I mean that the properties encoded by  $x$ 's concepts of  $y$  are the source of the beliefs  $x$  has about  $y$ .

Philosophers can define several notions here. One might be said to 'possess the concept  $F$ ' simply by having a concept of  $F$  (in the present sense) which encodes a single, defining property of properties. Or one might 'possess the concept  $F$ ' by having a concept of  $F$  (in the present sense) which encodes a sufficient number of properties of properties all of which the property  $F$  in fact exemplifies. Thus, the *failure* to 'possess the concept  $F$ ' can come about in various ways. This will often be a matter of degree, since our concepts of a property  $F$  may be inaccurate if they also encode properties of properties that  $F$  *fails* to exemplify. As the number of such properties becomes greater, the more inaccurate the concept becomes. In such cases, there may come a point where we have to say that  $x$  just fails to have the concept of  $F$ .

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