In Defense of the Law of Noncontradiction

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An important philosophical puzzle arises whenever we find a group of philosophically interesting sentences which individually appear to be true but jointly imply a contradiction. It is traditional to suppose that since the sentences in the group are jointly inconsistent, we cannot accept them all. This refusal to accept all the sentences in the group is not just grounded in (a) the problem of accepting the derivable contradiction, but also in (b) the problem that classical logic gives us the means to derive every sentence whatsoever once we have derived a contradiction. But with certain really hard puzzles of this kind, it is difficult to identify even one sentence in the puzzling group to reject. In such cases, there seems to be no good reason or argument for rejecting one of the sentences rather than another. We often find ourselves in the uncomfortable position of having to reject statements that have a strong claim to truth.

Paraconsistent logic and dialetheism constitute a fascinating body of doctrines for critically analyzing this kind of philosophical puzzle. Paraconsistent logic removes problem (b), noted above, concerning the presence of contradictions in classical logic. In contrast to classical logic, paraconsistent logic tolerates the derivation of a contradiction without thereby yielding a proof of every sentence. Dialetheism goes one step further, however, and addresses problem (a). It is the doctrine that, in some of these really hard cases, there are indeed true contradictions. Dialetheists argue that some sentences are both true *and* false, and that sometimes appearances are not deceiving—there just are special groups of true yet jointly incompatible sentences for which the contradiction they imply is both true and false. We shall suppose, for the purposes of this paper, that the law of noncontradiction is the claim that there are no true contradictions. Thus, dialetheism is the view that the law of noncontradiction is false. While there are plenty of philosophers who accept, and work within, paraconsistent logic, only a few count themselves as dialetheists.

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I take paraconsistent logic and dialetheism seriously, and think that they offer a philosophically worthy approach to these puzzling groups of sentences. The logical investigation of paraconsistent logic is certainly interesting and justified. We should endeavor to know what are the metatheoretical features of this logic. Dialetheism also deserves careful study. Truth value gluts may be no worse than truth value gaps,[1] and it is always good to investigate whether, or why, philosophers just take it on faith that no contradictions are true.

But I am not yet convinced that the *arguments* of the dialetheists for rejecting the law of noncontradiction are conclusive. The arguments that dialetheists have developed against the traditional law of noncontradiction uniformly fail to consider the logic of encoding. This extension of classical logic, developed in Zalta (1983), (1988) and elsewhere, offers us an analytic tool which, among other things, can resolve apparent contradictions.[2] In this paper, I'll illustrate this claim by considering many of the apparent contradictions discussed in Priest (1995) and (1987). In (1995), Priest examines certain interesting cases in the history of philosophy from the point of view of someone without a prejudice in favor of classical logic. He suggests that each case constitutes an example where there is no other good analysis except that offered by dialetheism. However, in each of these cases, the logic of encoding offers an alternative explanation of the phenomena being discussed while preserving the law of noncontradiction. But Priest fails to consider this explanation when he describes what options there are in classical logic for analyzing the problem at hand. In what follows, I'll reanalyze these examples from the history of philosophy and then move to the examples which form the

[1] See Parsons (1990).

[2] In addition to the two books just cited, readers will find applications of the logic of encoding in the following papers: Zalta (2000a), (2000b), (1999), and (1993).

heart of the case that Priest develops against the law of noncontradiction, namely, those embodied by his 'inclosure schema'.

I don't plan to undertake a systematic examination of all the arguments produced against the law of noncontradiction. Nor do I plan to consider Priest's excellent (1998) piece in which he undermines arguments attempting to establish why contradictions can't be true. Instead, it should suffice if I simply point out certain clearcut cases where the arguments by the dialetheists against the law of noncontradiction proceed too quickly. It should be of interest to see just how far the logic of encoding can be used to defend the law of noncontradiction. It may turn out, in the end, that there are some true (and false) contradictions and that paraconsistent logic is the correct logic. If so, the object theory discussed here could easily be recast in terms of such a logic and remain of interest. But whether or not this latter task is undertaken, our present concern is to discover exactly the point at which the law of noncontradiction allegedly fails. I think the following shows that more work has to be done to identify that point, should it exist.

Before I begin, let me note that I shall presuppose familiarity with one or another of the canonical presentations of the logic of encoding and the theory of abstract objects that is cast within this logic. My readers should know that the logic of encoding is a classical logic in which two kinds of atomic formulas ('$F^n x_1 \ldots x_n$' and 'xF^1') form the basis of a second-order, quantified modal language and logic (identity is not primitive but is instead defined for both individuals and relations). The language is extended to include (rigid) definite descriptions and λ-expressions, and the logic is extended with the usual axioms that govern these expressions. A single primitive notion 'x is concrete' ('$E!x$') is used to formulate the definition of 'x is abstract' ('$A!x$') and the axioms for abstract objects are stated within the resulting language and logic. The main axiom of the theory is a comprehension principle which asserts that for any condition ϕ without free xs, there is an abstract object that encodes just the properties satisfying ϕ ($\exists x(A!x \& \forall F(xF \equiv \phi))$). Readers unfamiliar with this system should consult one of the previously cited works in which the theory has been formally developed.

Discussion of Cusanus

In discussing the limits of thought in pre-Kantian philosophy, Priest examines the work of the fifteenth century German philosopher Nicolas of Cusa. In his (1995), Priest summarizes Cusanus' argument that God is beyond the limit of that which is expressible. Cusanus, according to Priest, argues that God cannot be (truly) described because God, being infinite, can fall under no finite category. After quoting Cusanus's explanation of this last claim (*Of Learned Ignorance*, I, 3), Priest observes in (1995) (p. 24):

> We see that Cusanus is operating with a 'mirror' conception of categorisation. An adequate category must share the relevant properties with the object categorised. . . . But clearly, from a modern perspective, it has no plausibility. Categories hardly ever share crucial properties with the objects categorised. The category of redness is not red; the notion of foreignness is not foreign; the notion of length is not long. And for good measure, the notion of a circle is not circular either.

Of course, Priest is quite right to point out that the property F doesn't share F with the objects categorized as F. However, the logic of encoding offers an analysis which shows that Cusanus wasn't completely off the mark.

In object theory and its logic of encoding, there is an object which is very closely related to the property of F and which does 'share' F with the objects that exemplify F. This analysis was developed in connection with the Self-Predication Principle (i.e., The Form of F is F) in Plato's Theory of Forms. In (1983), and in Pelletier & Zalta (2000), an analysis of Plato's theory was put forward on which the Form of F is identified not with the property F but with either the abstract object that encodes just F or the abstract object that encodes the properties necessarily implied by F.[3] This analysis turned Plato's One Over the Many Principle into a proper

[3] I proposed the former analysis (i.e., identifying the Form of F with the abstract object that encodes just F), in (1983), and Pelletier and I proposed the latter analysis (i.e., identifying the Form of F with the abstract object that encodes all of the properties necessarily implied by F), in Pelletier & Zalta (2000). But this subtlety will not play a role in what follows.

Note also that in what follows, I shall assume that 'necessary implication' is defined in the usual way, namely, that F necessarily implies G iff necessarily, everything exemplifying F exemplifies G.

thesis of metaphysics instead of a logical truth. Given the ambiguity of the copula 'is', the Self-Predication Principle then received two readings, one of which is *true*. The claim 'The Form of a Circle is circular' comes out true if we analyze 'The Form of a Circle' as denoting the abstract object that encodes circularity and we analyze the copula 'is' as 'encodes'.

So, from the point of view of the logic of encoding, the charitable way to interpret Cusanus is to identify the category of F with the abstract object that encodes F (or, what might be preferable, with the abstract object that encodes every property necessarily implied by F). When the category is understood in that way, it does, in some sense, *share* the property F with the objects that exemplify F. We can predicate redness, foreignness and length of 'the category of redness', 'the notion of foreignness', and 'the notion of length', respectively, if we understand the predication correctly and analyze the category as an abstract object. So although Cusanus's argument as to why God can't be truly described still fails, his 'mirror' principle (to which Priest alludes) does have one true reading. His intuitions weren't completely off base. However, the exemplification reading of the principle ('the category of F exemplifies F'), which is needed for the argument, is false.

Discussion of Anselm

Priest (1995) later goes on to analyze Anselm's *Proslogion*. In his analysis, Priest discusses what Routley (1980) called the Characterization Principle ('CP'), which states that 'the thing with property ϕ is a ϕ-thing'. The suggestion is that Anselm appeals to this principle when he argues that, that than which nothing greater can be conceived is such that nothing greater can be conceived. In criticising Anselm, Priest concludes (p. 64):

> The CP, then, is not a logical truth. I think that it appears so plausible because the claim that a (the) thing that is P is P is easily confused with the claim that everything that is P is P, which is a logical truth.

Actually, Priest doesn't defend CP by appealing to paraconsistent logic, but rather concludes that 'CP cannot be assumed in general' (p. 65).

But with the logic of encoding and the theory of abstract objects, one can defend CP by recognizing that it is subject to an ambiguity connected with the copula 'is'. (This is not to defend Anselm's ontological argument,

but only to give an analysis which might explain why Anselm might have been misled into grounding the crucial premise of his argument in something like the Characterization Principle.) Indeed, the Characterization Principle was shown to have one true reading in Zalta (1983) (47-48). I offered a reading of the principle 'The P-thing is P' which is true *in general*. The idea is to interpret 'the P-thing' as denoting the abstract object which encodes just the property P and read the copula as 'encodes'. The result is a truth, and indeed, one that is provable. We may take the analysis there one step further by instead reading 'the P-thing' as 'the abstract object that encodes all and only the properties necessarily implied by P'. Then, given that 'is' is ambiguous between exemplification and encoding predication, we have a reading of the Characterization Principle which turns out true, namely, the (abstract object) x which encodes all the properties necessarily implied by P encodes P. This is true in general because the comprehension principle for abstract objects guarantees the existence of a unique object that encodes all the properties necessarily implied by P . Moreover, the principle is easily generalized from the single property form 'The P-thing is P' to the general form 'The so-and-so is so-and-so', where 'the so-and-so' is any definite description of ordinary language. Then the general form of the Characterization Principle can be given the true reading: the abstract object that encodes all and only the properties necessarily implied by being so-and-so encodes being so-and-so. We may represent this formally by representing 'so-and-so' in the usual way as a complex exemplification condition ϕ and by using '$\Rightarrow$' to stand for necessary implication (where $F \Rightarrow G =_{df} \Box\forall x(Fx \rightarrow Gx)$). Then the general Characterization Principle can be formally represented as follows:

$$\imath x(A!x \,\&\, \forall F(xF \equiv [\lambda y\, \phi] \Rightarrow F))[\lambda y\, \phi]$$

This is an 'atomic' encoding predication with both a complex object term (namely, a description of the form '$\imath x\psi$') and a complex predicate (namely, '$[\lambda y\, \phi]$'). Moreover, it is a *theorem* of the theory of abstract objects.[4]

Now how does this give us a more charitable interpretation of Anselm? Consider a somewhat different example. Suppose someone asks the question, why did Ponce de Leon search for the fountain of youth? It it not too

[4]Note that the reason that it is a truth of metaphysics and not a truth of logic is that its truth depends on the fact that the description has a denotation. This fact is consequence of a proper axiom of metaphysics, namely, the comprehension principle for abstract objects. See Zalta (1983), 48, for further formal details.

helpful to appeal to CP by answering 'Because the fountain of youth is a fountain of youth'. Instead, we expand upon the truth given by CP and answer by saying 'Because the fountain of youth is a fountain the waters of which confer everlasting life on those who drink from it'. Now here we have what looks like a true statement, namely, the fountain of youth is a fountain the waters of which ... . The encoding logician assumes 'the fountain of youth' refers to a certain intentional object. Depending on the circumstances of utterance, there are two ways to identify this intentional object. In the simplest case, we identify it as the abstract object that encodes just the property of being a fountain of youth (i.e., it encodes just the property of being a fountain the waters of which confer everlasting life upon those who drink from it). In more complex cases, we would identify this object as the abstract object that encodes all and only the properties necessarily implied by being a fountain of youth.[5] On either identification, the object indeed does 'have' the property of being a fountain of youth, for it encodes the property of being a fountain of youth. Since encoding is a mode of predication, it is a way of having a property. An appeal to such an object therefore allows us to explain Ponce de Leon's behavior, for why would anyone search for the fountain of youth if there is no sense of 'is' on which it *is* a fountain that confers everlasting life? Although nothing exemplifies the property of being a fountain of youth, there is an intentional object which *is* (in the encoding sense) a fountain of youth.

Similarly, I think it is a more sympathetic analysis of Anselm to suppose that (1) there is an intentional object grounding this thought when he assumes that 'that than which nothing greater can be conceived is such that nothing greater can be conceived', and (2) this intentional object 'is' such that nothing greater can be conceived. You can't identify this intentional object using the standard exemplification reading of the ordinary definite description 'that than which nothing greater can be conceived'. But in the logic of encoding, we can read this description as denoting the abstract object that encodes all and only the properties necessarily implied by the property of being such that nothing greater can be conceived. This abstract object is governed by the general Characterization Principle. Where 'nothing greater (than y) can be conceived' is represented as '$\neg\exists z(Cz \,\&\, Gzy)$', then the following instance of the general Characterization Principle formulated above is derivable in object theory:

[5] The proper identification depends on the context of utterance and on the way the person uttering the sentence conceives of the fountain of youth.

$$\imath x(A!x \,\&\, \forall F(xF \equiv [\lambda y\, \neg\exists z(Cz \,\&\, Gzy)] \Rightarrow F))[\lambda y\, \neg\exists z(Cz \,\&\, Gzy)]$$

So the property of being such that nothing greater can be conceived does *characterize* the intentional object, and Anselm was correct to this extent.

Anselm's mistake was to fail to notice the subtle ambiguity in predication when forming descriptions of conceivable objects, namely, that the 'is' of predication for intentional objects is not quite the usual one. He assumed that the property involved in the definite description 'that than which nothing greater can be conceived' would characterize the object of his thought. But there are two modes of predication underlying natural language characterizations and only one of them (encoding) behaves the way the Anselm expected. Unfortunately, it is the other mode of predication (exemplification) that is needed for the ontological argument to succeed. While one can prove in the logic of encoding that there exists an intentional object that encodes the property of being such that nothing greater can be conceived, one cannot prove the existence of an object that *exemplifies* this property.[6]

Berkeley's Master Argument

Priest develops an extremely elegant reconstruction of Berkeley's Master Argument, as presented in *Three Dialogues Between Hylas and Philonous*. The puzzle consists of two premises and 3 principles, all of which appear to be true but which jointly appear to yield a contradiction. The first premise is the claim 'there exists something which is not conceivable' and since a contradiction can be derived from this premise (together with the second premise and the 3 principles), one could take the result to be a *reductio* which yields the negation of the first premise, namely, that nothing exists which is not conceivable. Priest nicely explains the subtle differences between this conclusion and the conclusion that nothing exists unconceived, but for the present discussion, however, we won't be distracted

[6] Some readers may be familiar with the formulation of the ontological argument developed in Oppenheimer & Zalta (1991), in which the distinction between 'being' and 'existence' is regimented by the distinction between '$\exists x\phi$' and '$\exists x(E!x \,\&\, \phi)$'. Oppenheimer and I thought this would give a more accurate representation of the argument. But however one reads the quantifier, the above discussion should prepare the reader to anticipate my reasons for rejecting Premise 1 of our formulation of the argument in that paper.

by this subtlety. The modality in 'conceivable' will not be represented in the argument and so the puzzle will involve no modal inferences.

With this proviso, the two premises of the argument are formulable with the predicate 'Cx' ('x is conceivable') and a propositional operator '$\mathbf{C}\phi$' ('it is conceivable that ϕ'). They are:

Premise 1: There exists something which is not conceivable.
$$\exists x \neg Cx$$

Premise 2: It is conceivable that there exists something which is not conceivable.
$$\mathbf{C}\exists x \neg Cx$$

Now using ϕ and ψ as metavariables for sentences, $\phi(x)$ as a metavariable for a sentence in which x may or may not be free, and Hilbert epsilon terms of the form '$\epsilon x \phi(x)$', we can state the 3 principles as follows:

Conception Scheme: If it is conceivable that ϕ holds of x, then x is conceivable.
$$\mathbf{C}\phi(x) \rightarrow Cx$$

Rule of Conception: If it is provable that ϕ implies ψ, then it is provable that the conceivability of ϕ implies the conceivability of ψ.
$$\text{If } \vdash \phi \rightarrow \psi, \text{ then } \vdash \mathbf{C}\phi \rightarrow \mathbf{C}\psi$$

Hilbert Scheme: If there exists something such that $\phi(x)$, then $\phi(x)$ holds of an-x-such-that-$\phi(x)$.
$$\exists x \phi \rightarrow \phi(\epsilon x \phi(x))$$

The reader should consult Priest's justification for these principles in (1995) (68-70). The argument then proceeds as follows:

1.	$\exists x \neg Cx$	Premise 1
2.	$\exists x \neg Cx \rightarrow \neg C(\epsilon x \neg Cx)$	Instance, Hilbert Scheme
3.	$\neg C(\epsilon x \neg Cx)$	Modus Ponens, 1,2
4.	$\mathbf{C}\exists x \neg Cx$	Premise 2
5.	$\mathbf{C}\exists x \neg Cx \rightarrow \mathbf{C}\neg C(\epsilon x \neg Cx)$	Rule of Conception, 2
6.	$\mathbf{C}\neg C(\epsilon x \neg Cx)$	Modus Ponens, 4,5
7.	$\mathbf{C}\neg C(\epsilon x \neg Cx) \rightarrow C(\epsilon x \neg Cx)$	Conception Scheme Instance
8.	$C(\epsilon x \neg Cx)$	Modus Ponens, 6,7
9.	$C(\epsilon x \neg Cx)\ \&\ \neg C(\epsilon x \neg Cx)$	&I, 3,8

Of course, before we conclude that Premise 1 is false (Berkeley) or that we have a true contradiction (Priest), we have to be justified in accepting the various premises and principles used in the argument. But

the justification for the various premises and principles strikes me as controversial. The Conception Scheme and the Rule of Conception can each be challenged on separate grounds. I won't spend the time here doing so, since I plan to accept them (below) for the sake of argument. Moreover, it seems reasonable to claim that if one accepts the Hilbert Scheme, one shouldn't accept the Conception Scheme. If the Hilbert Scheme legitimizes the inference from an existential claim to a claim involving a defined (but not necessarily well-defined) singular term for an arbitrary object satisfying the existential claim, then why think it *follows* from the *de dicto* conceivability of an existential claim that the thing denoted by the singular term is conceivable *de re*?[7]

But suppose we grant, for the sake of argument, that Premise 2 and the 3 principles (Conception Scheme, Rule of Conception, and Hilbert Scheme) are all true. Then it becomes important to point out that one can both accept the ordinary intuition that 'there exists something which is not conceivable' and develop an analysis on which it turns out true, without accepting Premise 1. Priest takes Premise 1 to be the only analysis of the intuition 'there exists something which is not conceivable'. But in the logic of encoding, the ordinary claim has additional readings, both of which are true. So one can reject the reading offered by Premise 1 without rejecting that there exists something which is not conceivable.

The ordinary claim 'there exists something which is not conceivable' has the following two additional readings in the logic of encoding:

$\exists x(x[\lambda y\ \neg Cy])$
(There exists something which encodes being inconceivable.)

$\exists x(\neg xC)$
(There exists something which fails to encode being conceivable.)

Both of these are true. The first one is true because the comprehension principle for abstract objects asserts the existence of abstract objects which encode the property of being inconceivable. The second is true because the comprehension principle asserts the existence of abstract objects which provably fail to encode the property of being conceivable.

[7]This question was inspired by Fred Kroon's presentation at the 'author-meets-critics' session on Priest's book, which took place at the July 1998 meetings of the Australasian Association of Philosophy (at Macquarie University) and in which we both participated along with Rod Girle. Kroon's presentation has now been published as Kroon (2001). I'd like to thank the organizers of that conference, and in particular, Peter Menzies, for agreeing to field that session.

Using an expression of natural language which is multiply ambiguous to describe these objects, we would say that these abstract objects 'are not conceivable'.[8]

The moral here is that we are not forced to conclude that the ordinary claims underlying the formal Premises 1 and 2, together with the 3 principles, jointly yield a true contradiction. When classical logic is extended by the logic of encoding and theory of abstract objects, we have options for analyzing apparent contradictions which the dialetheists have not considered. I am not claiming that the only correct approach to this puzzle is to accept Premise 2, accept the 3 principles, reject Premise 1, and analyze 'there exists something which is not conceivable' in terms of encoding predications. One might wish to reject one of the principles used in the puzzle. Rather, I am claiming only that *if* one accepts Premise 2 and the 3 principles involved in the contradiction, then one is not forced either to accept true contradictions or to reject the intuition that 'there exists something which is not conceivable'. An alternative is available.

The Inclosure Schema

Now one of the central parts of Priest's case for dialetheism concerns the so-called inclosure paradoxes. He classifies all of the set theoretic and semantic paradoxes as instances of a general recurring pattern called the 'Inclosure Schema'. This schema, when formulated at the most general level of abstraction, requires notions of set theory. Priest describes the Inclosure Schema as follows:

> We now require two properties, ϕ and ψ, and a function δ satisfying the following conditions:
>
> (1) $\Omega = \{y : \phi(y)\}$ exists and $\psi(\Omega)$ [Existence]
> (2) if $x \subseteq \Omega$ such that $\psi(x)$: (a) $\delta(x) \notin x$ [Transcendence]
> (b) $\delta(x) \in \Omega$ [Closure]
>
> Given that these conditions are satisfied we still have a contradiction. For since $\psi(\Omega)$, we have $\delta(\Omega) \notin \Omega$!. I will call any Ω that satisfies these conditions (for an appropriate δ) an *inclosure*.[footnote]

[8]The ambiguity in the ordinary predicate 'is not conceivable' is twofold. One ambiguity is between reading the negation as either internal to the predicate or external to the sentence. The other is between the exemplification and encoding readings of the copula. In the two new readings cited above, we have focused on the encoding readings, the first of which involves internal negation and the second of which external.

> The conditions themselves, I will call the *Inclosure Schema*, and any paradox of which this is the underlying structure, an *inclosure contradiction*.
>
> (1995), p. 147

The exclamation point at the end of the third sentence in this quotation indicates that both the sentence $\delta(\Omega) \notin \Omega$ and its negation are true (p. 142). Elsewhere, Priest explains that the δ function can be thought of as the 'diagonalizer' function. He then shows how a wide variety of logical paradoxes, vicious circles, and semantic paradoxes fit into the pattern of this schema, and summarizes his results nicely in (1995), Tables 7 (p. 144), 8 (p. 148) and 9 (p. 160). In these cases, Priest explains how contradictions stand at the limits of iteration, cognition, conception (definition), or expression.

Priest's book then becomes an extended argument to show that the traditional solutions to the paradoxes are not adequate. He concludes that we must accept dialetheism (and revise classical logic) to accommodate the contradictions at the limits of thought:

> In the last two chapters we have, *inter alia*, completed a review of all main contemporary solutions to inclosure contradictions. As we have seen, the solutions are not adequate, even in the limited domains for which they were generally designed. Moreover, not only do they tend to be incompatible, but the piecemeal approach initiated by Ramsey flies in the face of the PUS [Principle of Uniform Solution] and the fact that all such paradoxes instantiate a single underlying structure: the Inclosure Schema. The only satisfactory uniform approach to all these paradoxes is the dialetheic one, which takes the paradoxical contradictions to be exactly what they appear to be. The limits of thought which are the inclosures are truly contradictory objects. (1995), p. 186

An encoding logician, however, need not accept this. The logic of encoding offers a consistent, non-piecemeal and uniform solution to inclosure contradictions. The solution is to first reject (as false) the formal conditions which assert the existence of, or describe, objects which *exemplify* contradictory properties, and then appeal to objects that *encode* the relevant properties to formally analyze the informal existence claims and descriptions of what appear to be contradictory objects at the limits of thought. On Priest's analysis, either (a) informal existence claims for the existence of some object Ω become formalized in inclosure contradictions in the Existence condition for Ω, or (b) informal descriptions of contradictory objects become formalized in the well-definedness condition underlying the diagonalizer function δ and presupposed in the Transcendence and Closure conditions. In either case, the only option Priest considers for analyzing the informal existence claims and descriptions is to use the exemplification form of predication in classical logic. The alternative, however, is to (1) analyze Ω as encoding, rather than exemplifying, the contradicting properties embedded in the informal existence condition, or (2) analyze $\delta(x)$ as an object that encodes, rather than exemplifies, the properties defined in the function δ.

Consider, as an example, Priest's analysis of Russell's paradox. Priest analyzes an informal existence claim of naive set theory ('there exists a unique set of all sets') in terms of an (exemplification-based) Existence condition (this condition is one of the elements of the inclosure contradiction). He alleges that the contradictory object Ω at the limit of thought is the unique object V which exemplifies being a set of all sets.[9] In addition, Priest identifies the informal function 'the set of all elements of the set x which are not members of themselves' as a kind of diagonalizer function δ. (This function plays a role in the Transcendence and Closure conditions which form part of the inclosure contradiction associated with Russell's paradox.) Note that the formally defined function ρ_x ($= \{y \in x | \neg(y \in y)\}$) involves exemplification predications of the form '$y \in x$' and '$y \in y$'. The inclosure contradiction then becomes $\rho_V \in V$ & $\rho_V \notin V$.[10]

The analysis of Russell's paradox from the present theoretical perspective looks very different, however. Assume for the moment that membership ('$\in$') is an ordinary relation which has been added as a primitive to our background metaphysics. Then it follows immediately that nothing exemplifies the property of having as members all and only things which are non-self-membered. That is, the following is a theorem:

$$\neg\exists y([\lambda z\, \forall w(w \in z \equiv w \notin w)]y)$$

However, the comprehension principle for abstract objects gives us an object of thought for our naive thoughts about 'the object which has as

[9] See Priest (1995), Table 7, p. 144.

[10] Clearly, given that V is the set of all sets, $\rho_V \in V$. Now to show that $\rho_V \notin V$, note that if $\rho_V \in \rho_V$, then $\rho_V \notin \rho_V$, by definition of ρ_V. So $\rho_V \notin \rho_V$. But, for reductio, if $\rho_V \in V$, then since $\rho_V \notin \rho_V$, it follows, by definition of ρ_V, that $\rho_V \in \rho_V$, which is a contradiction. So $\rho_V \notin V$.

members all and only non-self-membered objects', namely, the abstract object that encodes all and only the properties necessarily implied by $[\lambda z\, \forall w(w \in z \equiv w \notin w)]$. The following instance of comprehension asserts the existence of this abstract object (where '$\Rightarrow$' represents necessary implication, as this was defined earlier):

$$\exists x(A!x \,\&\, \forall F(xF \equiv [\lambda z\, \forall w(w \in z \equiv w \notin w)] \Rightarrow F))$$

Readers familiar with the logic of encoding will know that the identity conditions for abstract objects guarantee that there is, in fact, a unique such abstract object. This object *has* (in the encoding sense) the property being-a-member-of-itself-iff-it-is-not, for this latter property is necessarily implied by the property of having as members all and only the non-self-membered objects.[11] But no contradiction is true.

Similarly, consider the naive existence principle for sets, which can be expressed informally as: for any condition expressible in terms of 'membership' and 'set', there is a set whose members are precisely the objects satisfying the condition. Now assume for the moment that the property of being a set ('S') is an ordinary property which has been added as a primitive to our background metaphysics. Then the ordinary formalization of the naive existence principle for sets is provably false. That is, the following is provably *not* an axiom or theorem schema of object theory:

$$\exists y(Sy \,\&\, \forall z(z \in y \equiv \phi)), \text{ where } y \text{ is not free in } \phi$$

But comprehension for abstract objects and the logic of encoding offers a general way of preserving the naive principle. It guarantees, for example,

[11] Here is the proof. Call the abstract object in question 'a'. So we know:

$$\forall F(aF \equiv [\lambda z\, \forall w(w \in z \equiv w \notin w)] \Rightarrow F)$$

So we want to show that a encodes $[\lambda z\, z \in z \equiv z \notin z)]$. To do this we have to show that:

$$[\lambda z\, \forall w(w \in z \equiv w \notin w)] \Rightarrow [\lambda z\, z \in z \equiv z \notin z)]$$

That is, we have to show:

$$\Box\forall x([\lambda z\, \forall w(w \in z \equiv w \notin w)]x \rightarrow [\lambda z\, z \in z \equiv z \notin z)]x)$$

So pick an arbitrary object, say c, and assume that $[\lambda z\, \forall w(w \in z \equiv w \notin w)]c$. Then, by λ-abstraction, $\forall w(w \in c \equiv w \notin w)$. Instantiating this univeral claim to c, it follows that $c \in c \equiv c \notin c$. So, by λ-abstraction, it follows that $[\lambda z\, z \in z \equiv z \notin z)]c$. Since c was arbitrarily chosen, we have proved that:

$$\forall x([\lambda z\, \forall w(w \in z \equiv w \notin w)]x \rightarrow [\lambda z\, z \in z \equiv z \notin z)]x)$$

and the Rule of Necessitation gets us the modal implication. Thus, a encodes being a member of itself iff it is not.

that for any condition expressible in terms of 'membership' and 'set' that is formally representable in the usual way as an exemplification formula ϕ (in terms of '$\in$' and 'S'), there is an abstract object which encodes all and only the properties necessarily implied by the property of being a set whose members are precisely the objects satisfying ϕ. The following schema, derivable from the comprehension schema, asserts this:

$$\exists x(A!x \;\&\; \forall F(xF \;\equiv\; [\lambda y\, Sy \;\&\; \forall z(z \in y \equiv \phi)] \Rightarrow F)),$$
where y is not free in ϕ

This constitutes a (true) reading of the naive existence principle because it yields, for any set-theoretic condition ϕ, an object which has (in the encoding sense) the property of having as members just those objects satisfying ϕ (and any property implied by this). So the informal, naive existence principle for sets gets both a false and a true reading in the logic of encoding. The true reading doesn't imply a contradiction. In the logic of encoding, the usual moral applies—we can't always assume that 'the set of all x such that $\ldots x \ldots$' is well-defined. But we can prove, for any ordinary sentence '$\ldots x \ldots$' of set theory representable as a (complex) exemplification formulas $\phi(x)$, that 'the (abstract) object which has (i.e., encodes) all and only the properties necessarily implied by $[\lambda x \phi]$' is always well-defined. The abstract object in question serves as both the object of thought and the denotation of the ordinary description 'the set of all x such that $\ldots x \ldots$' in those true sentences containing this phrase.

These facts concerning the analysis of Russell's paradox in terms of the logic of encoding stand in contrast to Priest's analysis of the paradox in terms of an inclosure schema leading to a true contradiction. An encoding logician may claim either that the formal assertion for the existence of V is false or that the formal conditions which imply the well-definedness of the function ρ_x are false (or both). He would reject either the idea that something exists which *exemplifies* all the properties described by the Existence condition or the idea that there exists a unique value for ρ_x (interpreted as an exemplification-based function), for each argument x, or both.

There is, of course, no space in the present essay to take up all of the inclosure paradoxes that Priest discusses in his book. But I think that the foregoing discussion gives us a general way of analyzing these inclosure paradoxes without accepting that there are true contradictions. For each inclosure paradox Priest considers, the encoding logician would suggest

that one should reject either the formal analysis of the Existence condition for Ω or the principles which guarantee the well-definedness of the formally-defined function $\delta(x)$ (i.e., the principle which guarantees that there exists a unique value for $\delta(x)$ for all arguments). Naive existence assertions of ordinary language can nevertheless be given true readings. If our intuitions, expressed in ordinary natural language, suggest that we should endorse some intuitive but contradictory existence condition, the logic of encoding offers us a reading that asserts the existence of an object which can consistently stand at the limits of thought. An encoding logician holds that the intuitive existence claim is false when analyzed as asserting the existence of an object which *exemplifies* the properties involved in the condition in question, but true when interpreted as asserting the existence of an object which encodes the properties involved. Similarly, an encoding logician holds that the principle which ensures that $\delta(x)$ is always well-defined (for each argument x) is false when interpreted as guaranteeing the existence of a value which exemplifies the (properties involved in the) defining condition of δ, but true when interpreted as guaranteeing the existence of a value which encodes the (properties involved in the) condition.

This solution, unlike the other attempts to deny the Existence condition of the Inclosure Schema or the well-definedness of the diagonalizer function δ, always provides us with an appropriate 'object of thought' for the alleged contradictory limit objects. By doing so, the logic of encoding overcomes the problem with classical logic that Priest finds so objectionable (bottom, p. 183). The objects (at the limits) of thought do 'have' the properties attributed to them, but not in quite the way the conditions imply (the conditions are contradictory, after all). The puzzling limit objects such as the set of all sets, the set of all ordinals, the set of all propositions, the set of all truths, etc., can all be analyzed as objects that encode the intuitive but contradictory properties attributed to them by the relevant conditions. Moreover, our analysis is consistent with classical logic, since xF doesn't imply Fx.

Furthermore, on the present view, the very properties and relations themselves involved in the description of these limit objects are by no means guaranteed either to exist or to exemplify the properties they are frequently assumed to have in dialetheism and elsewhere. Take the notion of 'membership', for example. Some notion of membership is essential to the formulation of the Inclosure Schema itself. Earlier, we simply assumed that we could add membership as a primitive, ordinary relation, for the purposes of the subsequent argument. But, strictly speaking, from the present point of view, there is no distinguished 'correct' membership relation, but rather as many abstract membership relations as there are conceptions of membership and theories of sets. Readers familiar with the applications of the typed theory of abstracta described in Zalta (1983) (Chapter 6), Linsky & Zalta (1995), and Zalta (2000b), will recognize that for each theory T in which '$\in$' is primitive or defined, there is an abstract relation (with type $\langle i, i \rangle$) which encodes just the properties of relations attributed to $\in$ in theory T.

To give an example, take Zermelo-Fraenkel (ZF) set theory. In the work cited at the end of the previous paragraph, we identified the membership relation of ZF as that abstract relation that encodes all the properties of relations attributed to $\in$ in ZF. We were able to do this by using the following procedure. We take the axioms of ZF to be 'true in' ZF and represent these facts, for each axiom p, as encoding predications of the form ZF$[\lambda y\, p]$ ('ZF encodes being such that p'). Here, we are taking ZF itself to be an abstract object that encodes only properties of the form $[\lambda y\, p]$, where p ranges over propositions (i.e., entities of type $\langle\,\rangle$). Then we assert that 'truth in', as just defined, is closed under logical consequence. That is, if $p_1, \ldots, p_n \vdash q$, and the p_i $(1 \leq i \leq n)$ are all true in ZF, then q is true in ZF. This principle allows us to infer, from each theorem of ZF, that it is true in ZF that $\in$ exemplifies a particular property of relations. (For example, from the fact that $\emptyset \in \{\emptyset\}$ is true in ZF, it follows that $[\lambda R\ \emptyset R\{\emptyset\}]{\in}$ is true in ZF.) Now, where 'ZF $\models p$' asserts that p is true in ZF (where this is defined as 'ZF$[\lambda y\, p]$'), we can give a theoretical identification of the membership relation of ZF ('$\in_{\mathrm{ZF}}$') as follows:[12]

$$\in_{\mathrm{ZF}} = \imath x^{\langle i,i \rangle}(A!x \,\&\, \forall F(xF \equiv \mathrm{ZF} \models F{\in}))$$

In other words, the membership relation of ZF is that abstract relation (among individuals) which encodes exactly the properties of relations F which $\in$ exemplifies in ZF. This is a rather interesting relation, assuming ZF is consistent.

The above procedure allows us to identify different membership relations for each of the theories Z, ZF, ZFC, the axioms of Aczel's (1988)

[12]In the following identification, the predicate '$A!$' and the variable 'F' both have type $\langle\langle i, i \rangle\rangle$. The former denotes a property of relations and the latter is a variable ranging over properties of relations.

nonwellfounded set theory, etc. Each theory is based on a different conception of the membership relation. For the purposes of the present paper, however, it is interesting to note that our procedure also allows us to theoretically identify the membership relation of naive set theory. Assume that naive set theory (NST) is constituted simply by the naive comprehension principle (as formalized in its inconsistent guise above) and the extensionality principle. By the procedure outlined in the previous paragraph, we may identify the membership relation of NST as follows:

$$\in_{\mathrm{NST}} = \imath x^{\langle i,i \rangle}(A!x \,\&\, \forall F(xF \equiv \mathrm{NST} \models F{\in}))$$

Clearly, this membership relation will encode all of the properties of relations expressible in the language of NST, since every sentence expressible in the language of NST is derivable as a truth of NST. As such, it is a rather uninteresting relation. But note that we have identified an object of thought, namely, the membership relation of NSF.

The important point here is that the world itself offers no distinguished, ordinary membership relation. It is only by assuming that there is one distinguished membership relation (governed by distinctively true principles) that a dialetheic logician can formulate the Inclosure Schema as an objective pattern and thereby argue that the law of noncontradiction must be revised. But an encoding logician need not accept this. From the present perspective, predicates such as 'membership', 'set', etc., denote different abstract relations depending on the (formal or informal) principles by which these relations are conceived. This may be the only conception of abstracta which can address the epistemological problems of Platonism, as Linksy and I argued in (1995).

Observations and Remarks

I take it that the solution to the inclosure paradoxes offered in the present paper satisfies Priest's Principle of Uniform Solution. Each object at a limit of thought is analyzed as an abstract object that encodes, rather than exemplifies, the contradictory properties. I think the foregoing work shows that the arguments for dialetheism are inconclusive to anyone who adopts the logic of encoding as an analytic method for resolving the inclosure contradictions. This logic offers uniform, classical analyses of those puzzles in which the law of noncontradiction has seemed to fail. The case against this law, therefore, remains unpersuasive to an encoding logician. Whenever a dialetheic logician concludes that some contradiction is true (and false), the data that drives this conclusion can be explained in terms of abstract objects that encode contradictory properties. An encoding logician might therefore conclude that no 'noumenal' object, so to speak, whether ordinary or abstract, exemplifies contradictory properties. However, contradictory properties may characterize both 'phenomenal' objects and our conceptions of objects. Once these phenomenal objects and conceptions are analyzed in terms of abstract individuals or abstract properties, the contradictory behavior can be explained in terms of incompatible encoded properties.

I think the main argument for preserving the classical law of noncontradiction is that it allows us to preserve our pretheoretic understanding of what it is to exemplify or instantiate a property. Since dialetheic logicians conceive (exemplification) predication to be such that there are objects x and properties F such that $Fx \,\&\, \neg Fx$, they force us to abandon our pretheoretic understanding of what it is to instantiate or exemplify a property. Our pretheoretic understanding of ordinary predication is grounded in such basic cases as the exemplification of simple and complex properties. Even if we don't have exact analyses for the simple or complex properties in question, we have a pretheoretic understanding of what it is for something to exemplify being red, being round, being straight, being triangular, being a detective, etc. Part of that understanding is that if an object x exemplifies a property P, then it is not the case that x fails to exemplify P. How are we to understand ordinary predication, or understand the idea of an object *exemplifying* such properties as having a color, having a shape, etc., if an object's exemplifying such properties doesn't exclude its failure to exemplify such properties? Of course, a dialetheic logician may counter that they only abandon our ordinary notion of predication in certain special cases. But my claim is that the ordinary cases ground our understanding of *what it is* to exemplify a property—what exemplification is excludes something's both exemplifying and failing to exemplify a property. If the special cases force us to abandon this, then it is unclear whether we really understand what exemplification is.

If logic is the study of the forms and consequences of predication, as I think it is, then it is legitimate to investigate a logical system which preserves this pretheoretic understanding of ordinary predication, especially if that logic has the capability to address the problems posed by impossibilia and contradictory (limit) objects of thought. The logic of encoding is such a logic; it doesn't tamper with our notion of ordinary predication, but rather appeals to a second form of predication to handle the problematic cases.

The technical development of the logic of encoding suggests that there is one constraint which places a limit on thought. This constraint is motivated by the paradoxes of encoding. As discussed in Zalta (1983) (Appendix), and elsewhere, the paradoxes derive from the interplay of unrestricted comprehension over abstract objects and unrestricted comprehension over relations (the latter includes properties and propositions). A single solution solves *both* paradoxes. The principal paradox is that if the expression '$[\lambda x\, \exists F(xF \,\&\, \neg Fx)]$' were to denote a genuine property, one could produce a contradiction by considering the abstract object that encodes just the property in question and noting that such an object exemplifies that property iff it does not. This is the 'Clark Paradox'.[13] We have formulated our system so as to preclude this result by banishing encoding subformulas from λ-expressions. The formation rules for the expression $[\lambda x_1 \ldots x_n\, \phi]$ require that the formula ϕ not contain encoding subformulas. This constraint also solves the other paradox of encoding, namely, the 'McMichael Paradox'.[14]

The comprehension principle for relations that is derivable from λ-abstraction therefore does not guarantee the existence of relations corresponding to arbitrary formulas ϕ. Instead, it only guarantees that for any formula ϕ free of encoding subformulas (and for which F^n isn't free), there is a relation F^n that objects $x_1, \ldots, x_n$ exemplify iff $x_1, \ldots, x_n$ are such that ϕ.[15] Of course, one may freely add new axioms which assert

[13]See Clark (1978) and Rapaport (1978).

[14]See McMichael and Zalta (1980), and Zalta (1983) (Appendix). A paradox arises if general identity is assumed to be a relation on individuals. On such an assumption, one could assert the existence of the paradoxical abstract object which encodes all and only the properties F such that $\exists y(F = [\lambda z\, z = y] \,\&\, \neg yF)$.

But a solution to this paradox falls immediately out of the solution to the Clark paradox if we define '=' for abstract objects in terms of encoding subformulas as we have done (abstract objects x and y are 'identical' whenever they encode the same properties). The condition stating the identity of abstract objects can not be used in relation comprehension to assert the existence of the relation of identity. Thus, '$[\lambda z\, z = y]$' is not well-formed, though the condition '$z = y$' is nevertheless well-defined and assertable. Moreover, the existence of a relation of identity on *ordinary* objects is not affected. Ordinary objects x and y are identical whenever they (necessarily) exemplify the same properties, and this identity condition does constitute a relation.

[15]The λ-abstraction principle, which is part of the logic of encoding, asserts:

$$[\lambda y_1 \ldots y_n\, \phi]x_1 \ldots x_n \equiv \phi^{x_1,\ldots,x_n}_{y_1,\ldots,y_n}$$

This asserts: objects $x_1, \ldots, x_n$ exemplify the relation $[\lambda y_1 \ldots y_n\, \phi]$ iff $x_1, \ldots, x_n$ are the existence of properties, relations, and propositions defined in terms of encoding subformulas, but one has to prove that the resulting theory is consistent when one adds such axioms. So the limit on thought is that one may not always assume that arbitrary open (closed) formulas ϕ correspond to relations (propositions).

Now some dialetheists will no doubt charge that the 'no encoding subformulas' restriction on relation comprehension is *ad hoc*! They might argue that if you take paraconsistent logic as your background logic, no such restrictions are required. I think one can actually meet this charge head on, both by motivating the 'no encoding subformulas' constraint on comprehension for relations and by noting that we are free to formulate our theories in the simplest way that explains the data.[16] But since such a defense probably wouldn't convince a dialetheist, I shall respond (and bring the paper to a close) simply by pointing out that dialetheism and the theory of abstract objects are on equal footing, as far as this charge goes. While theorists on both sides can justify their approach by saying that by adopting their system they can explain a wide range of data, nevertheless both systems make a certain theoretical moves which will seem *ad hoc* from the point of view of the other. While our constraints on comprehension for relations in the logic of encoding may seem *ad hoc* from the point of view of dialetheism, the subversion of our pretheoretic understanding of exemplification predication and the law of noncontradiction by dialetheists seems equally *ad hoc* from the point of view of the logic of encoding. Nor can a dialetheist claim that they are in a superior position, on the grounds that they can 'accept' the theory of abstract objects without having to place any restrictions on comprehension. Abstract object theorists can argue equally well that they can 'accept' (the paraconsistent logic underlying) dialetheism without giving up the law of noncontradiction. In Zalta (1997), I tried to show that within the logic of encoding, one could develop a *classically-based* conception of impossible worlds, i.e., worlds where contradictions are true. I used those worlds to interpret a special consequence relation on propositions $\overset{R}{\Rightarrow}$ that is axiomatized along the lines of a paraconsistent logic.[17]

If there is parity, then, between the logic of encoding and the dialetheist's paraconsistent logic on this charge, then I've established that the arguments of the dialetheists for rejecting the law of noncontradiction are not yet conclusive. The adequacy, applicability, and fruitfulness of the two systems should be compared along other lines before any decision is to be made as to which offers the deeper insight into the explanation of important philosophical puzzles.

such that ϕ. Since ϕ in $[\lambda y_1 \ldots y_n \phi]$ may not contain encoding formulas, the application of Universal Generalization n times (on the x_i) followed by Existential Generalization on the λ-expression yields the following comprehension principle for relations:

$$\exists F^n \forall x_1 \ldots \forall x_n (F^n x_1 \ldots x_n \equiv \phi),$$
where ϕ has no free F^ns and no encoding subformulas

Note that the comprehension principle for properties falls out as a special case:

$$\exists F \forall x (Fx \equiv \phi), \text{ where } \phi \text{ has no free } F\text{s and no encoding subformulas}$$

Finally, note that the following is the 0-place instance of λ-abstraction:

$$[\lambda\, \phi] \equiv \phi$$

This asserts: that-ϕ is true iff ϕ. (The notion of truth is what remains of the notion of exemplification in the 0-place, degenerate instance of λ-abstraction.) From this 0-place instance, the following comprehension principle for propositions follows immediately:

$$\exists p (p \equiv \phi), \text{ where } \phi \text{ has no free } p\text{s, and no encoding subformulas}$$

Readers unfamiliar with the logic of encoding should be aware that identity conditions for properties, relations, and propositions have also been formulated, and these are consistent with the idea that necessarily equivalent properties, relations, and propositions may be distinct.

[16] Indeed, the theory in question can be stated informally as follows: hold the domain of properties fixed and suppose that for every condition on properties, there is an abstract object that encodes the properties satisfying the condition. If that is the theory we are trying to formalize, then it is not *ad hoc* to place restrictions on relation comprehension. The addition of such a restriction is just the way one goes about holding the domain of properties fixed.

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[17] Ultimately, the legitimacy of the 'no encoding subformulas' restriction depends on whether the logic of encoding and theory of abstract objects offer a better framework both for analyzing the relevant, pretheoretical data and for asserting (and, sometimes, proving) all of the important philosophical claims we have good reason to assert. I think the present theory will fare well in any comparison and cost-counting with dialetheism, given the variety of other applications described in the works cited in the Bibliography. Even if the two theories were on a par application-wise, it strikes me that the cost of the employing the 'no encoding subformulas' restriction is certainly no more expensive than the cost of accepting certain contradictions as true (and the havoc that plays on the notion of exemplification).

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