On Anselm’s Ontological Argument in *Proslogion* II*

Paul E. Oppenheimer
Philosophy Department, University of Adelaide
CSLI, Stanford University

and

Edward N. Zalta
Center for the Study of Language and Information
Stanford University

**Abstract**

Formulations of Anselm’s ontological argument have been the subject of a number of recent studies. After examining these studies, the authors respond to criticisms that have surfaced in reaction to their earlier papers, identify a more refined representation of Anselm’s argument on the basis of new research, and compare their representation, which analyzes *that than which none greater can be conceived* as a definite description, to a representation that analyzes it as an arbitrary name.

In 1991, we argued that Anselm needed 2 premises, a minimal condition on the *greater than* relation, and a definition of *God*, to give a valid argument for God’s existence. In 2011, we showed how computational investigations established that one of the premises and the definition of *God* from the 1991 paper, were sufficient, just by themselves, to validly conclude God’s existence. In the analysis section of our 2011 paper, we offered a number of observations about the differences between the original 1991 formulation and the simplified 2011 representation of the argument.

The goal in what follows is to: (1) respond to criticisms that have surfaced in reaction to our papers of 1991 and 2011, (2) identify a somewhat improved representation of Anselm’s argument, and (3) show that this new version is immune to the criticisms of the earlier version (even if one were to grant that the criticisms are valid) and (4) show that Anselm’s term *that than which nothing greater can be conceived* is more accurately analyzed as a definite description than as an arbitrary name. In connection with (1), we address the criticism that our 2011 version is question-begging by investigating this notion a bit more carefully. Along the way, we point out how certain criticisms of our work insufficiently attend to the analysis and observations we made in 2011 concerning the simplified version of the argument.

1 Review

In preparation for the discussion of (1) – (3), here is a brief summary of our 1991 and 2011 papers. In our 1991 paper, we proposed a reading of the ontological argument on which Anselm needed two nonlogical premises and a meaning postulate about the *greater than* relation. To state these premises we used a first-order logic that is classical with respect to its treatment of constants but free for its primitive definite descriptions. In this logic, the quantifier ‘∃’ was not assumed to be existentially loaded (it simply means “there is”). To this logic we added an existence predicate ‘E!’, a conceivability predicate ‘C’, and a 2-place *greater than* predicate ‘G’. Let us again use φ₁ to represent the formula:

\[(φ₁)\ Cx \& \neg\exists y(Gyx \& Cy)\]

This formula asserts: x is conceivable and such that nothing greater is conceivable. We then formulated the two nonlogical premises and meaning postulate as follows:

Premise 1: \(\exists xφ₁\)

Meaning Postulate: \(\forall x\forall y(Gxy \lor Gyx \lor x = y)\)

Premise 2: \(\neg E!xφ₁ \rightarrow \exists y(Gyxφ₁ \& Cy)\)

Premise 1 asserts merely that there is a conceivable object such that nothing greater is conceivable. The Meaning Postulate for *greater than* asserts merely that *greater than* is a connected relation, i.e., for any two

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1 As noted above, \(\exists xφ\) does not imply \(\exists x(φ \& E!x)\).
greater than the Meaning Postulate for title and focus of our 2011 paper had to be on (a) how Premise 1 and 2 in the 1991 paper. This justified Anselm’s use of the definite description that than which nothing greater can be conceived (id quo majus cogitari non potest) in his reasoning in Proslogion II. Moreover, from this conclusion and Premise 2, the ontological argument easily proceeds to the conclusion that God exists, if given the definition that God (‘g’) is, by definition, ∃xφ. The proof cites logical theorems governing definite descriptions, which are explained in the 1991 paper.2

We subsequently discovered (2011), by computational means, that Premise 2 alone is sufficient to derive the conclusion that God exists. Again, the proof cites logical theorems governing description, which are explained in the 2011 paper.3 From this brief review, it is clear why the title and focus of our 2011 paper had to be on (a) how Premise 1 and the Meaning Postulate for greater than are redundant given the strength of Premise 2, and (b) the discovery of this redundancy by computational means. But our paper contained in addition a number of observations and some discussion of the soundness of the argument; this analysis, and in particular, our doubts about Premise 2 and our view about how one might tweak the 1991 argument to avoid the redundancies, seem to have been overlooked by the critics. In particular, in Section 4 of our 2011 paper, we compared the original 1991 version with the simplified 2011 version and in Section 5, we concluded our paper with reasons for objecting to Premise 2.

We show, in what follows, that these observations and reasons already anticipate and address some of the criticisms that have been raised about our work.4 This is not to say that no valid criticisms have been raised. We agree with the section of Garbacz’s 2012 paper (Section 3), where he raises some genuine methodological issues about our translation of the 1991 argument into Prover9 notation. He noted, for example, that our use of the constant ‘g’ implicitly validated Premise 1 simply by representing the definition that g is identical to the conceivable x such that none greater can be conceived.

We are happy to acknowledge these points from Section 3 of Garbacz’s paper. In 2009–2010, while doing research for our 2011 paper, we were in the early stages of learning how to use resolution-based automated reasoning systems to represent schematic statements that were cast in a first-order, natural deduction logic extended with definite descriptions. There are a number of subtleties that we had not recognized and so our methodology was not as refined as it might have been. We’ve therefore revised our webpage on the computational investigation of the ontological argument to reflect our enhanced understanding of how to implement the 1991 premises in a first-order reasoning environment. The new representation no longer defines ‘g’ by deploying a creative definition. We’ve acknowledged Garbacz’s concerns on that webpage.5 Nevertheless, despite these flaws, we deployed the automated reasoning engine with enough sophistication to discover that Premise 2 makes Premise 1 and the Meaning Postulate redundant.6 This hadn’t been noticed in the 20

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4 A notable exception is Parent 2015, who recognized that we were suspicious and, indeed, critical of the strength of Premise 2 in our 2011 paper.

5 See http://mally.stanford.edu/cm/ontological-argument/.

6 It is worth mentioning work by Blumson (2017), who implemented our 1991 and 2011 representations of the argument in the higher-order reasoning system Isabelle/HOL.
years subsequent to the publication of our 1991 paper. As we shall see, however, other criticisms raised in Garbacz’s paper are not as effective and we shall address these below. Before we examine these criticisms, we first address the charge of begging the question.

2 Does the Argument Beg the Question?

Rushby (2018, 1475–78, 1484) claims that our 1991 version of the argument begs the question. He introduces his claim by saying (1475):

In the next section, I introduce a strict definition of “begging the question” and show that a rendition of the Argument due to Oppenheimer and Zalta[19] is vulnerable to this charge.

His subsequent argument has two parts: first there is a definition of begging the question and then there is the claim that an analogue of Premise 2 begs the question according to that definition. We challenge both the definition and the claim that Premise 2 begs the question.

Here is Rushby’s definition of begging the question, where he uses the equality sign = as a biconditional sign (2018, 1476):

... if $C$ is our conclusion, $Q$ our “questionable” premise (which may be a conjunction of simpler premises) and $P$ our other premises, then $Q$ begs the question if $C$ is equivalent to $Q$, assuming $P$: i.e., $P \vdash C = Q$.

Note here that Rushby has defined what it is for a premise to beg the question. Rushby then says (2018, 1478):

...the premise $\text{Greater1}$ begs the question under the other assumptions of the formalization ... . Given that we have proved $\text{God}_r\text{e}$ from $\text{Greater1}$ and vice-versa, we can easily prove they are equivalent. Thus, in the definition of “begging the question” given earlier, $C$ here is $\text{God}_r\text{e}$, $Q$ is $\text{Greater1}$ and $P$ is the rest of the formalization (i.e., $\text{ExUnd}$, the definition of $\text{God}$?, and the predicate subtype $\text{trichotomous}$? asserted for $>$).

In this passage $\text{Greater1}$ refers to the sentence (2018, 1477):

$\text{Greater1}$: $\text{AXIOM}$ $\forall$ $x$: $(\text{NOT} \text{re?}(x) \Rightarrow \exists$ $y$: $y > x)$

where “The uninterpreted predicate $\text{re?}$ identifies those beings that exist ‘in reality’” (2018, 1477). This is just a more general version of our Premise 2, since Premise 2 which asserts specifically that if $\exists x \phi_1$ doesn’t exist, then there is a conceivable thing that is greater. So Rushby’s argument, if good, would imply that Premise 2 is question-begging.

Let’s turn first to his definition of begging the question. Of course, it should be observed that the fallacy of begging the question traditionally applies to arguments, not to premises, yet for some reason, Rushby takes it to apply to premises. Let’s put this aside for the moment, since we’ll later argue that begging the question is a charge that can be leveled only against an argument relative to a dialogical context. Let’s focus for now solely on Rushby’s definition of begging the question. We think it is easy to show that his definition categorizes premises that clearly don’t beg the question as ones that do. We can undermine his formal definition using a purely formal example. For consider the following valid argument:

$Fa \equiv Gb$

$Gb \equiv Rd$

$\therefore Fa \equiv Rd$

In this argument a biconditional conclusion follows from two biconditional premises. The argument is non-question begging—both premises play a role in the derivation of the conclusion. Moreover, the following, related argument is also valid and non-question begging:

$Fa \equiv Gb$

$Gb \equiv Rd$

$Fa$

$\therefore Rd$

But according to Rushby’s definition, the premise $Fa$ is question-begging in this last argument. That’s because from the first two premises $Fa \equiv Gb$ and $Gb \equiv Rd$, we can derive $Fa \equiv Rd$. That is, $Fa \equiv Gb, Gb \equiv Rd \vdash Fa \equiv Rd$. Now, in Rushby’s definition, let $C$ be the conclusion $Rd$.
let $Q$ be the premise $Fa$, and let $P$ (the other premises) stand for the first two premises. Then the derivation that we just identified, namely, $Fa \equiv Gb, Gb \equiv Rd \vdash Fa \equiv Rd$, would become represented in Rushby’s notation as $P \vdash C \equiv Q$. Thus, according to Rushby, $Fa$ begs the question. But it clearly doesn’t. Nor does $Rd$ beg the question in the argument:

$$
\begin{align*}
Fa & \equiv Gb \\
Gb & \equiv Rd \\
Rd & \\
\therefore Fa
\end{align*}
$$

This is not question-begging even though $Fa \equiv Gb, Gb \equiv Rd \vdash Fa \equiv Rd$. Again, Rushby’s schema, when applied, wrongly entails that the premise $Rd$ begs the question. One needn’t provide an interpretation of the formal claims to see that they constitute a counterexample to Rushby’s definition—any non-trivial interpretation will do.

It should also be noted here that Rushby ignores the fact that the definition we use in developing the 1991 argument, i.e., $g =_{df} \exists x \phi_1$, is crucial in deriving the conclusion $Eg$. This definite description doesn’t play a role in Rushby’s representation of our 1991 argument. So Rushby’s formalization isn’t faithful to our 1991 version. It may be that his formalization is subject to the charge of begging the question, but our 1991 version is not.

A better definition of question-begging is one that applies to arguments as a whole. Indeed, it applies to arguments relative to a dialogical situation.\(^8\) Though we don’t have a replacement definition to offer, we take it that, to a first approximation, an argument $A$ begs the question if and only if (1) $A$ is valid, and (2) in the dialogical situation in which $A$ is presented, the arguer is not entitled to one of the premises (even if only for the sake of the argument). This, at least, clearly explains why the valid argument $P \therefore P$ is question-begging: in any dialogical situation in which the arguer is presenting an argument for $P$, they are not entitled to the premise $P$ even if only for the sake of the argument. For if one were entitled to $P$ (even if only for the sake of the argument), one wouldn’t need an argument, since the premise is sufficient and no further argument is called for.

Using this rough definition, we see no reason to think that the 1991 version is question-begging. Moreover, a proponent of the 2011 version, with its single Premise 2, does not beg the question. For the conclusion additionally requires the definition $g =_{df} \exists x \phi_1$, which some might consider as a second premise. Moreover, it should be noted that even though the definition $g =_{df} \exists x \phi_1$ introduces a conservative extension of the premises used in both the 1991 and 2011 versions, the fact that the definition is conservative is justified by the corresponding premises—and requires such justification. Without such a justification, the definition would either be empty (in a free logic for individual constants) or creative (in a logic free for definite descriptions but not for individual constants)! These points have been insufficiently appreciated in the debate surrounding the argument and they are key to the understanding of the logic underlying the argument.

Finally, we think it is unlikely that Anselm was presenting the ontological argument in a dialogical situation in which he was confronting an atheist. So, if we were to suppose that the interlocutor is a theist, there is reason to think that the argument would be entitled to all the premises in all the versions of the argument we’ve presented. We emphasize here that we are not defending the soundness of the argument. Let’s suppose, for the purposes of this paper, that the argument is being presented in a dialogical situation where the interlocutor is an atheist or agnostic. We still think that one is entitled to use the premises of all the versions of the argument we’ve put forward, if only for the sake of the argument. This doesn’t require that the premises be true, but only that it doesn’t defeat the arguer’s purpose in the situation of arguing for a conclusion. So, one can continue the debate after the argument is presented by considering whether the premises are true or false. And that is where we think the focus should be with respect to the ontological argument as we’ve presented it, not on whether the argument begs the question. Indeed, we think that this way of looking at the matter acknowledges that there is, in the background, a question about the significance of the definition of God ($g =_{df} \exists x \phi_1$) in the argument. This is crucial to the conclusion (and given some theories of definition, has the status of a premise), since it stands in need of justification. So we take it to be a mistake to focus just on the premises when alleging that our argument is question begging.

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\(^8\)See the literature on the dialectical/dialogical approach to fallacies, for instance Hamblin 1970 and Walton 1994.
3 Other Criticisms of Premise 2

Since Premise 2 makes Premise 1 and the Meaning Postulate for greater than otiose, the revised version of the argument presented in 2011 has a single premise (Premise 2) and a single conclusion (Elxφ). In fact, it also crucially requires a definition. But let’s put this aside for the moment. Our 2011 version was criticized in Garbacz 2012, who claims, in Section 2 of his paper, that Premise 2 is equivalent to the conclusion that God exists and so using it in this argument is ‘epistemically inefficient’ (2012, 588). His criticism is developed on the basis of a certain understanding of our paper that he puts forward, namely, “The main contribution of Oppenheimer and Zalta 2011 is a discovery that premises Premise 1 and Connectedness are obsolete, i.e., that you can reach the same conclusion assuming only Premise 2” (Garbacz 2012, 586). While this is true, we hope to show that by focusing on this result, Garbacz turned his attention away from certain crucial facts about our argument and from points that we’ve made previously that address his concerns about Premise 2.

Let’s first look at exactly what Garbacz says. He notes, on pp. 586–587, that Premise 2 is equivalent to the claim that Elxφ. That is,

\[(A) \quad Elxφ ≡ (¬Elxφ → ∃y(Gy ∧ xφ & Cy))\]

Clearly, Garbacz is correct about this. The right-to-left direction of the above biconditional was established by our 2011 version of the argument. And the left-to-right direction follows by propositional logic: from p it follows that ¬p → q.

However, Garbacz uses this fact as part of his argument to the conclusion that “Premise 2 cannot be used in any proper argument for the existence of God” (2012, 586). His first reason is that “It [Premise 2] not only implies that God exists but it is (logically) equivalent to the latter claim”, where the ‘latter claim’ refers to ‘God exists’. But the equivalence (A) above doesn’t allow one to infer that Premise 2 is logically equivalent to the claim that God exists. For Garbacz’s claim to be true, Premise 2 would have to be logically equivalent to the claim Elg. It is not; that is, the claim Elg is not equivalent to the claim on the left side of equivalence (A): it is not equivalent to ‘the x such that φ1 exists’. Rather, for Elg and Elxφ to be equivalent, you need the definition g =_df ixφ1. And this is a non-trivial part of the argument. One can’t introduce this definition into the argument until it is justified. And the justification can be given by noting that the intermediate conclusion Elxφ1 implies that the definite description is well-defined. Specifically, if you examine the 2011 argument reproduced in footnote 3 above, you will see that the intermediate conclusion Elxφ1 occurs on line 8; this implies, by Russell’s theory of descriptions, that the definite description is well-defined (i.e., has a unique denotation). So to reach the conclusion that God exists from Premise 2, you need the non-trivial step of justifying and introducing the definition of God as the x such that φ1.

So, it is just a mistake to claim that Premise 2 is (logically) equivalent to the claim that God exists. Thus far, though, we’ve only shown that Premise 2 doesn’t imply that God exists. But the ‘converse’ holds as well: the claim that God exists doesn’t imply Premise 2. Again, to draw the inference from the claim that God exists to Premise 2, one must first convert Elg to Elxφ by the definition of God. Then one can correctly claim, as Garbacz does, that the latter is equivalent to Premise 2. But one can’t thereby conclude that God exists (Elg) implies Premise 2 without appealing to, and justifying, the definition of God. And that can’t be done in this direction; there is no justification for stipulating that g =_df ixφ1 simply on the basis of Elg.

This observation undermines Garbacz’s first reason for suggesting that our 2011 version is ‘epistemically inefficient’. For if the question is ‘Does God exist?’ and the formal representation of that claim is Elg, then Premise 2, despite its power, does not imply Elg without the definition of God.

But Garbacz gives two other reasons why Premise 2 shouldn’t be used in the ontological argument (587–58), namely, that the consequent of Premise 2 is logically inconsistent and so the claim Elxφ becomes equivalent to a tautology. But clearly, the logical inconsistency of the consequent of Premise 2 is not obvious and it takes some reasoning (indeed a diagonal argument) and the logic of definite descriptions to show that one can derive a contradiction from the assumption that ∃y(Gy ∧ xφ & Cy). This is precisely how Anselm’s argument gets its purchase—he coupled a consequent that subtly implies a contradiction to the antecedent ¬Elxφ1.

We suggest that these reasons Garbacz offers against Premise 2 don’t go beyond the reservations we already expressed about this premise in 2011. The fourth observation in Section 4 (2011, 346) included the lines:

\[
\ldots \text{it is interesting to note that one can (i) abandon the definition of God as } ixφ1, \text{ (ii) generalize Premise 2 to the claim that}
\]


And in Section 5 (2011, 348), we gave an extended argument against Premise 2. The last paragraph of our paper (2011, 348–349) included the following lines:

Thus, arguments ... above show that the defender of the ontological argument needs independent support for two claims: that the definite description denotes and that Premise 2 is true. Our 1991 analysis of the argument is still relevant, since it shows how the ontological arguer could justify Anselm’s use of the definite description. [Footnote 14: Given the argument outlined above against Premise 2, a defender of Anselm might consider whether the ontological argument can be strengthened by using our original formulation as in 1991, but with the general form of Premise 2 discussed earlier: \( \neg \exists x (Gyx \land Cy) \). The justification of this more general premise may not be subject to the same circularity that infects the justification of Premise 2 (though, of course, it may have problems of its own).] The present analysis shows why the use of the definite description needs independent justification. Consequently, though the simplified ontological argument is valid, Premise 2 is questionable and to the extent that it lacks independent justification, the simplified argument fails to demonstrate that God exists. The use of computational techniques in systematic metaphysics has illuminated the relationship between Premise 2 of the ontological argument and the conclusion that God exists.

It is clear from these passages not only that we gave reasons why Premise 1 and the connectedness of greater than are not obsolete, but also that we questioned Premise 2. Thus, we anticipated the conclusion of the extended argument that Garbacz develops in Section 2 of his paper, where he concludes that (589):

...Thus, it is little wonder that one can dispense with Premise 1 and Connectedness in Oppenheimer and Zalta’s [1991] ontological argument as their logical contribution (to this argument) is covered by Premise 2. Alas, this third premise seems to be too strong to be an acceptable basis for any argument for the existence of God.

Garbacz here makes it seem that we were advocating for Premise 2 in our paper. While the specific criticisms he puts forward do offer some additional reasons for not accepting Premise 2 (though see below), we did not argue in our paper that, in a proper reconstruction of the argument, Premise 2 should replace Premise 1 and the Meaning Postulate.

Indeed, in the fourth observation and the final paragraph (footnote 14) quoted above from our 2011 paper, we suggested that the 1991 paper should have weakened Premise 2 so that it becomes the following universal claim:

Premise 2': \( \forall x(\neg \exists y (Gyx \land Cy)) \)

This asserts, in essence, that if a thing fails to exist then something greater than it can be conceived. We were pointing out here that Premise 1, the Meaning Postulate, and Premise 2' yield a perfectly good argument for God’s existence without appealing to Premise 2. The resulting argument is a variant of the argument in the 1991 paper—once one establishes that the description \( tz \phi_1 \) denotes, one can instantiate it into Premise 2' to obtain the original Premise 2. And then the reductio proceeds in the same manner as in the 1991 paper. We lay out the revised argument with the weaker Premise 2' in a footnote.\(^9\) This revised version of the argument goes exactly like the original 1991 argument except at the point where Premise 2 is invoked.

We think it is easy to see that Premise 2' alone does not yield the intermediate conclusion that \( E\exists x \phi_1 \), nor does any pairwise combination of Premise 1, Premise 2' and the Meaning Postulate for greater than. We invite the reader to use automated reasoning tools to establish these last claims, as we shall not argue for them further here.\(^{10}\)

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10See Parent 2015 (478) for another way of weakening Premise 2.
Indeed, to forestall an objection to Premise 2′, we can weaken it even further, so that it asserts: for any x, if x is a conceivable thing such that nothing greater is conceivable and x fails to exist, then something greater than x can be conceived. Formally:

Premise 2′: ∀x((φ1 & ¬∃!x) → ∃y(Gyx & Cy))

This would forestall the objection that Premise 2′ is consistent with, and even licenses, the view that an existing conceivable evil thing is greater than a nonexistent one. A variant of the 1991 ontological argument still goes through with Premise 2″:

1. ∃xφ1
2. ∃!xφ1
3. ∃y(y = ixφ1)
4. C1xφ1 & ¬∃y(Gyxφ1 & Cy)
5. (C1xφ1 & ¬∃y(Gyxφ1 & Cy) & ¬∃!xφ1) → ∃y(Gyxφ1 & Cy) from (3), by Desc. Thm. 2 (1991)
6. ¬∃!xφ1
7. C1xφ1 & ¬∃y(Gyxφ1 & Cy) & ¬∃!xφ1
8. ∃y(Gyxφ1 & Cy)
9. ¬∃y(Gyxφ1 & Cy)
10. ∃!xφ1
11. ∃!y

This represents our best analysis of the argument; had we seen the issue with Premise 2 in 1991, we would have eliminated Premise 2 in favor of Premise 2″. Note here how Premise 1 and the Meaning Postulate for Greater than still play a role. The latter is needed for the derivation of Lemma 2, which moves us from the former to the claim that there is a unique x such that φ1. These are key to the justification of the use of the description ixφ1, which is, in turn, needed to instantiate Premise 2″ to obtain line 5. In free logic, one may not instantiate a term into a universal claim until one has established that the term has a denotation.

Consequently, by substituting either Premise 2′ or, preferably, Premise 2″ for Premise 2 in the 1991 argument, we obtain a valid argument in which all of the premises are needed to derive the intermediate conclusion E1xφ1.11 In either case, none of the premises are redundant and the argument doesn’t beg the question.12

4 Does Anselm Use a Definite Description?

In a recent paper, Eder & Ramharter (2015) also recognize that question-begging is not an accurate charge to bring against any of the versions of the ontological argument of the kind that we’ve been discussing. But they suggest that the regimentation of the argument using a definite description doesn’t properly capture the reasoning in Proslogion II. While our versions of the argument represent Anselm’s term that than which nothing greater can be conceived as a definite description, Eder & Ramharter argue that it is not. They introduce two abbreviations and put them in boldface: id quo abbreviates ‘id quo maius cogitari non potest’ (‘that than which nothing greater can be conceived’), and aliquid quo abbreviates ‘aliquid quo nihil maius cogitari potest’ (‘something than which nothing greater can be conceived’). Then they say (p. 2802):

Whether or not a reconstruction of Anselm’s argument is valid may crucially depend on whether id quo has to be understood as

weakening of Premise 2 (2012, 591). His proposed weakening asserts that, for any x, if it is not the case that both φ1 and E!x, then there is something y such that (a) y is greater than x, (b) φ1, and (c) x is conceivable. Formally, ∀x(¬(φ1 & E!x) → ∃y(Gyxφ1 & φ1 & Cy)). We think Premise 2″ is a more elegant weakening of Premise 2; φ1 is not required in both the antecedent and the consequent to derive the desired conclusion. And the antecedent of Premise 2″ considers only the case of a φ1 object that doesn’t exist (the antecedent of Garbacz’s proposed weakening considers the cases where either x isn’t such that none greater can be conceived or x doesn’t exist). Modulo the definition of God (see below), Premise 2″ is sufficient to yield the existence of God given Premise 1 and the connectedness of greater than.

12To see this with respect to the argument that cites Premise 2′, notice two things: (1) line 5 depends on Premise 1 and the Meaning Postulate for greater than even though those weren’t cited as the justification. But these two principles are needed to conclude that the description has a denotation and can thereby be instantiated, via the free logic version of ∀E, into Premise 2′. (2) When one assumes, for reductio, at line 6, that ¬E1xφ1, one cannot then reach line 7 from Premise 2″ alone; one must additionally have Premise 1 and the Meaning Postulate for greater than in order to establish the second and third conjuncts of line 7, which come from line 4.

13They say (2015, fn 4, 2797):

Sometimes, proofs of the existence of God are accused of being question-begging, but this critique is untenable. It is odd to ask for a deductive argument whose conclusion is not contained in the premises. Logic cannot pull a rabbit out of the hat.
a definite description. But we think that it is not just that we do not have to understand *id quo* as a definite description, but that we should not.\[18\] For one thing, if *id quo* had to be read as a definite description, Anselm would be committed to presupposing the *uniqueness*\[19\] of *aliquid quo* already in Chap. II, which seems to be in conflict with the fact that only in Chap. III does Anselm mention God’s uniqueness for the first time.\[20\] Rather, it seems to us that Anselm is using this diction only as a device to refer back to *something* “than which nothing greater can be conceived”. In other words, we think that Anselm’s *id quo* is best understood as an *auxiliary name*, which is used to prove something from an existence assumption.\[21\]

[In this quote, their footnote numbers are preserved in square brackets.]

Eder and Ramharter are suggesting here that the use of “*id quo maius cogitari non potest*” is not a definite description in *Proslogion II* but rather an *arbitrary name*.\[14\] So, even though *id quo* appears to be a definite description, they prefer to interpret this locution as an arbitrary constant (which they label ‘g’ later in their paper). On the face of it, there is a huge difference between a simple constant and an expression like *id quo maius cogitari non potest*. According to standard logical practice, an arbitrary name is a simple constant that has no descriptive content, whereas the phrase “*id quo maius cogitari non potest*” is a complex expression that has descriptive content. The descriptive content of “*id quo maius cogitari non potest*” is expressed by the arrangement of the negation and quantifier (nothing = not something), conjunction, and the predicates *greater than* and *conceives*. So the claim Eder & Ramharter make here seems inconsistent with current logical practice. They would have been better off if they had represented *id quo* as a Hilbert-style $\epsilon$-term, that is, as a term of the form $\epsilon x \phi_1$, which might be read: an $x$ such that $\phi_1$. It may be that they intended to avoid this suggestion, however, because technically speaking, it semantically requires a *choice function* to interpret the $\epsilon$-term. It may be that Anselm was using language that requires choice functions for its interpretation, but then it may be at least as likely that he was not.

But more importantly, the suggestion that *id quo* is being used by Anselm as an arbitrary name can’t be sustained given their representation of the argument. To see the structure of our objection to this suggestion, note that when a number theorist introduces an arbitrary name for a prime number by saying ‘let $\alpha$ be an arbitrary prime’, they may go on to reason about $\alpha$ using the known principles of number theory, including known principles about primes. But what they cannot do is, at some point in the argument, assert new principles (i.e., new axioms or premises) governing $\alpha$. This understanding is made clear by the classical rule for using arbitrary names within an argument to derive the conclusion from some premises. The classical rule requires that one choose an arbitrary name that doesn’t already occur in the premises (or in the existentially quantified premise or in the conclusion). In a standard logic text, e.g., Enderton (2001, 124), Rule EI is stated as follows, where $\Gamma$ is a set of premises, $\exists x \phi$ is an existentially quantified premise, $\psi$ is the conclusion, and $\phi^c$ is the result of substituting $c$ for the free occurrences of $x$ in $\phi$:

**Rule EI**

Assume that the constant symbol $c$ does not occur in $\phi$, $\psi$ or $\Gamma$, and that $\Gamma, \phi^c \vdash \psi$. Then $\Gamma, \exists x \phi \vdash \psi$, and there is a deduction of $\psi$ from $\Gamma$ and $\exists x \phi$ in which $c$ does not occur.

Thus, the rule says if you can derive $\psi$ from some premises $\Gamma$ in which $c$ doesn’t occur and from the fact that $c$ is such that $\phi$, then you can derive $\psi$ from $\Gamma$ and the claim $\exists x \phi$. One guarantees that the constant $c$ is arbitrary by requiring that it be new to the proof, i.e., that no other information about the particular name $c$ occurs in the premises. This implies that if one uses an arbitrary name $c$ as an instance of $\exists x \phi$, one can’t then introduce new premises or assumptions that govern $c$.

But Eder & Ramharter’s analysis of the argument seems to violate Rule EI in a number of ways. First, their conclusion contains the arbitrary name. In the formal proof in their paper (p. 2813), they introduce constant $g$ as the arbitrary constant (instead of the more cumbersome *id quo*). From the principles ExUnd ($\exists x \phi$) and Def C-God ($Gx : \equiv \neg \exists y (y > x)$) they conclude $\exists y \neg \exists x (x > y)$ and then say “let $g$ be such that $\neg \exists x (x > g)$.” Then they assume $\neg E! y$ for reductio, reach a contradiction, and so conclude $E! y$. But this conclusion of the argument contains the arbitrary name. The reasoning doesn’t really conform to Rule EI; the conclusion of the argument doesn’t seem to have the right form.

Let’s suppose that they can reformulate the argument to address this.

\[14\] Their position has recently been endorsed in Campbell 2018 (55).
issue. The second problem is it appears that they have used premises in their argument \textit{in which the arbitrary name occurs}, in violation of the rule that the arbitrary constant can’t occur in any premises used in the derivation. To match up their argument with the requirements of Rule EI, take $\exists x \phi$ to be \textit{aliquid quo} and take the arbitrary constant $c$ to be \textit{id quo} or $g$. Then Rule EI says that if you can derive God’s existence ($\psi$) from some premises in which \textit{id quo} doesn’t occur and from the claim that \textit{id quo} is an arbitrary \textit{aliquid pro}, you can derive God’s existence from just the premises and \textit{aliquid pro}.

But the argument put forward by Eder & Ramharter violates this rule because the premises they use to derive God’s existence are not free of the arbitrary name \textit{id quo}. Just consider the fact that on p. 2813, they introduce the arbitrary constant $g$, then define a new notion $(\mathcal{F}_{EI}(F))$ by means of a biconditional in which $g$ occurs, and then use that definition to derive facts about $g$. The definition in question is:

$$\mathcal{F}_{EI}(F) :\leftrightarrow Fg \lor F = E!$$

Let first note one problem that for which a repair will be suggested below: this definition will depend on the choice of $g$, yet this isn’t acknowledged by indexing the definiendum to $g$. But note second that a definition is equivalent to a biconditional axiom, and so an appeal to this definition in their proof becomes an appeal to a premise in which the arbitrary name occurs, something that is explicitly ruled out by Rule EI. Additionally, the definition uses the arbitrary name in the definiens! There is a real question here about the propriety of such a definition; inspection of the literature on the theory of definition suggests that such a definition hasn’t been well studied, if at all, as a legitimate logical method.\textsuperscript{15} Of course, it may be that the authors can discharge the definition so that no premise involving the arbitrary name is used in violation of Rule EI. For example, they might be able to (a) use a definition in which a variable replaces the arbitrary name, (b) prove theorems about the notion defined, and (c) appeal to those theorems when reasoning with the arbitrary name in the ontological argument.

For example, instead of the above, they could offer the following definition, indexing the definiendum to $x$ and $E!$:

$$\mathcal{F}_{x,E!}(F) :\leftrightarrow Fx \lor F = E!$$

With this definition, they might be able to prove theorems about the defined notion that hold for arbitrary $x$. Then, when reasoning in the ontological argument with respect to the arbitrarily chosen object $g$, they could instantiate these theorems to $g$ without the theorems counting as premises that would violate Rule EI.

We think, therefore, that the argument formulated in their paper has to be reformulated much more carefully, to make sure that their reasoning with arbitrary names is valid. But, again, for the sake of argument, let’s grant them that \textit{id quo} is an arbitrary name and that their reasoning with this name is valid according to the classical rule for reasoning with arbitrary names. The final problem for their representation is the fact that the name of God never makes an appearance in the argument. Examination of their formal representation shows that they introduce (p. 2808) the label \textit{God!} to stand for the formal claim $\exists x(Gx & E!x)$ (“there is a $x$ such that $x$ is a God and $x$ exists”), where $Gx$ is defined by the statement \textbf{Def C-God} identified above. Then on p. 2813, they say “Now that everything is in place, we are in a position to prove \textit{God!} as follows.”

Putting aside the fact that they use both second-order and third-order logic in the argument,\textsuperscript{16} the problem is that the formal representation doesn’t show that \textit{Proslogion II} has an argument for the existence of God. Nowhere is the name of God introduced into the argument. As we’ve seen, the constant $g$ is not a name of God, but rather an arbitrary name which they use to represent ‘\textit{id quo maius cogitari non potest}’. Thus, the conclusion of their argument, $E!g$, doesn’t use a name of God. So once you grant them that the phrase ‘that than which nothing greater can be conceived’ is an arbitrary name and not a description, they have only established a fact about an arbitrarily chosen object of the kind \textit{nothing greater is conceivable}, namely, that such an object exists. The conclusion of the argument should be that God exists, not that $g$ or \textit{id quo} exists.

\textsuperscript{15}See Frege 1879, §24; Padoa 1900; Frege 1903a, §§55–67, §§139–144, and §§146–147; Frege 1903b, Part I; Frege 1914, 224–225; Suppes 1957; Mates 1972 (197–203); Dudman 1973; Belnap 1993; Hodges 2008; and Gupta 2019.

\textsuperscript{16}The second-order quantifiers appear in (*) and (**), and third-order logic is used in the statement of \textbf{Realization}:

\begin{itemize}
  \item (*) $\forall p F((Fg \rightarrow Fa) \land (F = E! \rightarrow Fa))$
  \item (**) $\forall p F(Fa \rightarrow (Fg \lor F = E!))$
\end{itemize}

\textbf{Realization}: $\forall p, F \exists x \forall p, F \mathcal{F}(F) \leftrightarrow Fx$

We don’t see any textual justification for thinking that this higher-order machinery is used in Anselm’s argument.
We don’t think it would be a good response to suggest: since uniqueness isn’t discussed until _Proslogion III_, Anselm can’t conclude that God exists until that next chapter. Such a response won’t work for the following reason. In the opening of _Proslogion II_, Anselm directly uses the name ‘God’ (= ‘Deus’) and the vocative case for ‘Lord’ (= ‘Domine’ = vocative case of ‘Dominus’). So Anselm clearly takes the conclusion of the argument to apply to God. And that is how Eder & Ramharter understand _Proslogion II_. They say (2015, p. 2800):

Having established in Chap. II that God exists in reality from the assumption that God exists at least in the understanding, Anselm proceeds in Chap. III by proving it is inconceivable that God does not exist.[10]

So Eder & Ramharter themselves agree that in _Prologion II_, there is an argument that establishes something about God, and not just about some arbitrarily chosen object such that nothing greater can be conceived.

Indeed, we can’t accept the concluding clause of Eder & Ramharter’s claim (quoted earlier) that (p. 2802):

... if _id quo_ had to be read as a definite description, Anselm would be committed to presupposing the _uniqueness_ of _aliquid quo_ already in Chap. II, which seems to be in conflict with the fact that only in Chap. III does Anselm mention God’s uniqueness for the first time.[20]

Their evidence for the concluding clause, given in footnote 20, is to quote Anselm as saying, in _Proslogion III_, “In fact, everything else there is, except You alone, can be thought of as non existing. You, alone then, ...” (Anselm [MW, 2008, 88]). But this hardly counts as a statement that God is a unique thing such that nothing greater can be conceived. Here Anselm is saying only that God uniquely has existence in the highest degree, and this is claim that plays no role in the ontological argument, as far as we can tell. This is why we don’t accept their conclusion that our account, which uses a definite description for the argument in _Proslogion II_, “seems to be in conflict with the fact that only in Chap. III does Anselm mention God’s uniqueness for the first time.” By using ‘God’ as a proper name in _Proslogion II_, he is already presupposing uniqueness, and that presupposition, together with Premise 1 and the meaning postulate for _greater than_, justifies his move from _aliquid quo_ to _id quo_, as suggested by our representation of the argument.

We conclude that one can not so easily dismiss the suggestion that ‘_id quo maius cogitari non potest_’ is used as a definite description in _Proslogion II_. At present, Eder & Ramharter’s suggestion that _id quo_ is being used as an arbitrary name leads to the list of problems just outlined. They would have to make a much stronger case before one should be willing to accept this analysis.

5 Conclusion

The work we’ve examined in this paper leads us to conclude that the analysis of _Proslogion II_ using a definite description still has a lot to offer those trying to understand Anselm’s ontological argument for the existence of God. To our way of thinking, the interesting questions concern the truth of the premises and the justification of the definition of God. Given what we’ve now learned, the premises in question are Premise 1, Premise 2”, the Meaning Postulate for _greater than_, and the definition of God: _g =df xφG_.

In our paper of 2007, we argued that Premise 1 is the real culprit in the argument. We tried to show that Premise 1 is too strong because it yields the existence of an object that _exemplifies_ the property of being a conceivable thing such that nothing greater is conceivable. We argued that Anselm’s subsidiary argument for Premise 1 involves two assumptions: (1) that the mere understanding of the phrase ‘conceivable thing such that nothing greater is conceivable’ requires one to grasp an _intensional_ object, and (2) any such intensional object has to exemplify the property _being a conceivable thing such that nothing greater is conceivable_. We then challenged the second assumption, on the grounds that the intentionality [with-an-s] involved in understanding the phrase only requires that the intensional [with-a-t] object (which is thereby grasped) _encode_ the property _being a conceivable thing such that nothing greater is conceivable_. Here we appealed to the notion of _encoding_ used in the theory of abstract objects (Zalta 1983, 1988).

Interestingly, this is a point of contact with the work of Eder & Ramharter’s paper, since their principle _Realization_ (reported in footnote 16) is a kind of comprehension principle that underlies Anselm’s assertion that there is something in the understanding such that nothing greater is conceivable. Eder & Ramharter write:
So, bearing in mind that first-order quantifiers are ranging over objects existing in the understanding, Realization seems plausible. It appears to be an analytic truth that any consistent set of (primitive, positive) conditions is realized by some object in the understanding. This seems to be confirmed by passages like (II.8), where Anselm claims that ‘whatever is understood is in the understanding’.[41] Bearing in mind that by ‘understanding something’ Anselm means understanding what its properties are, we can see that whenever we conceive of a certain set of (non-contradictory) properties, this set gives rise to an object that exists in the understanding—and this is just what Realization says. So even though Anselm does not state Realization explicitly, we think that it is implicit in how Anselm thinks about objects.

The authors here are placing a lot of weight on this third-order principle (Realization) and it isn’t clear to us that we should accept that Anselm is committed to this principle simply on the basis of the fact that he takes whatever is understood to be in the understanding. Nor is it clear to us that “by ‘understanding something’ Anselm means understanding what its properties are”. We don’t see any textual evidence for this claim. But more importantly, they take Realization to be an integral premise of the ontological argument.

By contrast, our 2007 paper shows that whereas the comprehension principle for intensional objects might help us to see why Anselm thought Premise 1 is true, such a principle doesn’t need to be added as a premise in the ontological argument itself. It might be needed to justify Premise 1, but it doesn’t make an appearance in Proslogion II. To formulate the ontological argument, one shouldn’t need, as a premise, that for any primitive condition on properties, there is an object that exemplifies just the properties satisfying that condition. But this is what Realization intuitively asserts. Thus, the work in Oppenheimer & Zalta 2007 bears on this question, and we suggest that a question for further study is to focus on Premise 1 and the implicit comprehension principle that Anselm must be relying upon to conclude that it is true.

References


