Non-Symmetric Relations and Their Converses∗

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Abstract
MacBride (2014) argues that unwelcome consequences arise for any theory committed to Russell’s (1903, §219) principle that every non-symmetric relation has a distinct converse. And in a more recent work, he (a) offers a semantic argument to the effect that 2nd-order logic is committed to this principle, and (b) goes on to argue that the 2nd-order quantifiers don’t range over properties and relations. In this paper, I strengthen his argument in (a) by showing that 2nd-order logic with identity yields the Russell principle as a formal theorem; no semantic argument is necessary. But I then show that object theory (Zalta 1983, 1988), which is an extension of 2nd-order logic with identity (and so has the Russell principle as a theorem), has definitions for the identity of properties, relations, and states of affairs that forestall the unwelcome consequences. Then I show that MacBride’s argument in (b) doesn’t get any purchase on object theory; its 2nd-order quantifiers do range over properties, relations, and states of affairs.

1 Setting Up The Problems
Russell (1903, §218–219) argued that:†

Every (2-place) non-symmetric relation R has a converse \( R^* \) that is distinct from R. (1)

But MacBride (2014) develops unwelcome consequences for any theory that endorses (1). Then, in a forthcoming work, he:

(i) develops a (semantic) argument to the conclusion that 2nd-order logic is committed to (1), and

(ii) argues further that 2nd-order quantifiers don’t range over properties and relations.

In what follows, I strengthen MacBride’s argument in (i) by showing that 2nd-order logic with identity has a formal representation of (1) that is provable as a theorem; no semantic argument is needed to show that 2nd-order logic endorses (1). Thus, any theory expressed in, or extending, 2nd-order logic with identity will have (1) as a theorem and should inherit the unwelcome consequences that MacBride raises for (1). But I also plan to show that although object theory (Zalta 1983, 1988, and elsewhere) is such an extension of 2nd-order logic with identity, it has the resources to forestall the concerns and consequences that MacBride develops for (1). I then show that his argument in (ii) doesn’t apply to the conception of properties, relations, and states of affairs that is systematized in object theory.

To represent (1) in 2nd-order logic with identity, let’s consider only 2-place relations. Of course, things will become more complicated when we consider relations of arity 3 and greater, for there are multiple pairs of argument places in these relations that can be converted. But let’s put this complication aside for now. So where \( F \) and \( G \), are 2-place relation variables, we may define:

- \( F \) is non-symmetric if and only if it is not the case that for any objects \( x \) and \( y \), if \( x \) bears \( F \) to \( y \) then \( y \) bears \( F \) to \( x \), i.e.,²

\[
\text{Non-symmetric}(F) \equiv_{df} \forall x \forall y (Fxy \rightarrow Fyx)
\]

1Russell actually talked about ‘asymmetric’ relations, but see below where we define non-symmetric relations. I don’t think anything hangs on the difference.

2This is to be contrasted with:
• \( G \) is a converse of \( F \) if and only if for any objects \( x \) and \( y \), \( x \) bears \( G \) to \( y \) iff \( y \) bears \( F \) to \( x \), i.e.,

\[
\text{ConverseOf}(G,F) \equiv_{df} \forall x \forall y (Gxy \equiv Fyx)
\]

(3)

Given these definitions, we may represent (1) as follows:

\[
\forall F (\text{Non-symmetric}(F) \rightarrow \exists G (\text{ConverseOf}(G,F) & G \equiv F))
\]

(4)

Now to show that (4) is a theorem of 2nd-order logic with identity, we need note only one more fact, namely, that the following is an axiom or theorem of this logic:

Comprehension Principle for Relations (CP)

\[
\exists F^n \forall x_1 \ldots \forall x_n (F^n x_1 \ldots x_n \equiv \varphi), \text{ provided } F^n \text{ doesn’t occur free in } \varphi
\]

We can now show, without any analysis of the form of predication \( Fxy \) and without any particular semantic interpretation of the domain over which the relation variables range, that (4) is a theorem of 2nd-order logic with identity:

Proof: Pick an arbitrary relation \( R \) and assume \( R \) is non-symmetric. Then, by definition (2) and predicate logic, there are objects, say \( a \) and \( b \), such that both \( Rab \) & \( \neg Rba \). Note independently that CP implies that every relation has a converse, as follows. If we let \( \varphi \) be \( Gyx \), where \( G \) is a free variable, then \( \exists F \forall x \forall y (Fxy \equiv Gyx) \) is a 2-place instance of CP. It follows by universal generalization that:

\[
\forall G \exists F \forall x \forall y (Fxy \equiv Gyx)
\]

By instantiating to \( R \), it follows that \( \exists F \forall x \forall y (Fxy \equiv Ryx) \). Pick an arbitrary relation as a witness to this claim, say \( S \), so that we know:

\[
(A) \ \forall x \forall y (Sxy \equiv Ryx)
\]

(4) implies, by definition (3), that ConverseOf\((S,R)\). But we already know \( Rab \), since it’s the first conjunct of \( Rab \& \neg Rba \). Hence, \( Sba \), by instantiating \( b \) for \( x \) and \( a \) for \( y \) in (A). Now for reductio, assume \( S = R \). Then it follows that \( Rba \), by substitution of identicals. But this contradicts \( \neg Rba \), which is the second conjunct of \( Rab \& \neg Rba \). Hence \( S \neq R \), by reductio. We’ve therefore established ConverseOf\((S,R) \& S \neq R \). So by existential introduction, \( \exists G (\text{ConverseOf}(G,R) \& G \equiv R) \).

3 Strictly speaking, you also have to discharge the introduction of the witness \( S \) by existential elimination, but this is routine.

3 By conditional proof, then, it follows that Non-symmetric\((R) \rightarrow \exists G (\text{ConverseOf}(G,R) \& G \equiv R) \). But since \( R \) was arbitrary, universally generalizing on \( R \) yields (4).

So the formal representation of (1), namely (4), is a theorem of 2nd-order logic with identity. We didn’t need any semantic notions or a semantic argument to establish this.

It is relevant, and of significant interest, that (1) can be represented, and its proof developed, much more elegantly if we add \( \lambda \)-expressions to 2nd-order logic with identity. It is worth a short digression on the logic that results when we add these expressions; then afterwards, we’ll see how they simplify our definitions and theorems about converses. So assume that we have added complex, \( n \)-place relation terms of the form \([\lambda x_1 \ldots x_n \varphi] \) to our language \((n \geq 0) \). When \( n \geq 1 \), we read \([\lambda x_1 \ldots x_n \varphi] \) as: being objects \( x_1, \ldots, x_n \) such that \( \varphi \). Thus, \( \lambda \)-expressions do not denote functions, as in the functional \( \lambda \)-calculus, but rather relations.4 A simple predication like \([\lambda x \neg Px]y \) asserts: \( y \) exemplifies being an object \( x \) that fails to exemplify \( P \).

By adding \( \lambda \)-expressions to 2nd-order logic, we can replace CP by:

\( \lambda \)-Conversion

\[
[\lambda x_1 \ldots x_n \varphi]x_1 \ldots x_n \equiv \varphi
\]

This asserts: objects \( x_1, \ldots, x_n \) exemplify being an \( x_1, \ldots, x_n \) such that \( \varphi \) if and only if \( \varphi \). For example, \([\lambda xy \neg Fxy]xy \equiv \neg Fxy \) is an instance, and by universal generalization, it is a theorem of the relational \( \lambda \)-calculus that:

\[
\forall F \forall x \forall y ([\lambda xy \neg Fxy]xy \equiv \neg Fxy)
\]

To see how this works, instantiate this theorem to an arbitrary 2-place relation \( R \), and then to arbitrary objects \( a \) and \( b \). The result is the instance:

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we have established, the principle of λ-Conversion eliminates the need for CP since the latter becomes derivable. The proof is left to a footnote.\(^6\) Note that the 0-place case of CP asserts: 3p(p ≡ q), i.e., there exists a state of affairs p such that p obtains if and only if q (where q is any formula in which p doesn’t occur free).\(^7\)

With these general facts about the relational λ-calculus in hand, we can return to our topic of converses, for one can introduce a well-behaved converse operator ‘ by taking advantage of λ-expressions. Let us define the converse of F, i.e., F*, as being an x and y such that y bears F to x, i.e.,

\[ F^* = \lambda xy [\lambda xy Fyx] \]  

(5)

Though the standard notation for the converse of F is F\(^{-1}\), we use F* here to conform with the notation that MacBride uses and thereby prevent confusion.

We’re now in a position to see that by adding λ-expressions to 2nd-order logic with identity and explicitly defining the converse operator, we don’t need CP or definition (3) to establish that every relation has a converse; it is easy to show that ∀F∃G(G = F\(^*\)) is a theorem in such a logic.\(^8\) A fortiori, every non-symmetric relation has a converse. Thus, we can represent and prove (1) more elegantly as the claim that for any 2-place relation F, if F is non-symmetric, then its converse F\(^*\) is distinct.\(^9\)

\[ ∀F(\text{Non-symmetric}(F) \rightarrow F^* \neq F) \]  

(6)

Proof. Let R be an arbitrary relation and assume Non-symmetric(R). Then, ¬∀xy(Rxy → Ryx), i.e., for some objects, say a and b, we know Rab & ¬Rba. Now for reductio, assume R * = R. Then by symmetry of identity, R = R*, and from Rab, if follows that R*ab, by substitution of identicals. So by definition (5) of R*, we know [λxy Ryx]ab. But by λ-Conversion, this implies Rba. Contradiction. Hence R * = R. So by conditional proof, Non-symmetric(R) → R * = R. Since R was arbitrary, we may universally generalize to get (6).

So when we add λ-expressions to 2nd order logic with identity, the concepts and claims simplify and clarify. I’ll therefore use (6) as the clearer representation of (1) in what follows. But nothing hangs on this; my analysis will apply to (4) as well. Both (4) and (6) can be established as formal theorems, without any semantic argument.

We’ve previously mentioned that whereas Russell (1903, §219) endorsed (6), MacBride (2014) argues that (6) has a number of unwelcome consequences. Suppose for the moment that MacBride is right. Since 2nd-order logic with identity implies (6) as a theorem, any theory expressed in, or extending, such a logic, should inherit the unwelcome consequences. Object theory is just such a theory, for though it is an extension of 2nd-order quantified modal logic without identity, it includes definitions of identity for objects, properties, relations, and states of affairs that preserve the classical theorems of identity provable in 2nd-order logic with identity. So, in what follows, we’ll first confirm that object theory has (6) as a theorem and then spend the remainder of the
paper defending the theorem, and object theory, against the unwelcome consequences we’ve only alluded to thus far.

To confirm that object theory has (6) as a theorem, I’ll assume familiarity with one of the works that lay out the basics of the theory, such as Zalta 1983, 1988, 1993; Bueno, Menzel, & Zalta 2014, or Menzel & Zalta 2014, though there are others that do so as well. The principles of this theory that will play the most important role in this paper are its theory of identity for individuals and its theory of relations. First, the theory of identity for individuals includes a definition stipulating that x and y are identical if and only if they are ordinary objects that necessarily exemplify the same properties or they are both abstract objects that necessarily encode the same properties:

\[ x = y \equiv_{df} (O!x \& O!y \& \Box F(xF \equiv yF)) \vee (A!x \& A!y \& \Box F(xF \equiv yF)) \] (7)

Second, the theory of relations consists of existence and identity conditions for relations. The existence conditions are derived, since object theory includes the resources of the relational λ-calculus; λ-expressions of the form \[ \lambda x_1 \ldots x_n \phi \] are well-formed, but only if \( \phi \) doesn’t contain any encoding subformulas.\(^{11}\) So λ-Conversion, as stated above, is the main axiom governing λ-expressions. From this, a modal version of CP, which we call □CP, is derivable as a theorem schema that asserts existence conditions for relations as follows:\(^{12}\)

\[ \exists x'y \equiv_{df} \Box \forall x(Fx \equiv xG) \] (8)

- Properties F and G are identical if and only F and G are necessarily encoded by the same objects, i.e.,

\[ F = G \equiv_{df} \Box \forall x(Fx \equiv xG) \] (9)

- n-place relations F and G are identical just in case, for any n − 1 objects, every way of applying F and G to those n − 1 objects results in identical properties, i.e.,

\[ \forall y_1 \ldots \forall y_{n-1} ([\lambda x F_{y_1} \ldots y_{n-1}] = [\lambda x G_{y_1} \ldots y_{n-1}] & [\lambda x F_{y_1} y_2 \ldots y_{n-1}] = [\lambda x G_{y_1} y_2 \ldots y_{n-1}] & \ldots & [\lambda x F_{y_1} \ldots y_{n-1}] = [\lambda x G_{y_1} \ldots y_{n-1}]) \]

except that you use the Rule of Necessitation after universally generalizing on \( x_1, \ldots, x_n \), and just before existentially generalizing on the λ-expression.

\(^{10}\)For those completely unfamiliar with it, object theory may be sketched briefly by saying that it extends 2nd-order logic without identity by adding a new atomic formulas of the form \( FX \), which represent a new mode of predication that can be read as x encodes F, where F is a 1-place relation, i.e., property, variable. In its more powerful, 2nd-order form, object theory includes a necessity operator □. It also uses a primitive 1-place relation term \( E! \) for being concrete, and defines ordinary objects (O!x) as objects x that could be concrete, and defines abstract objects (A!x) as objects x couldn’t be concrete. It is axiomatic that ordinary objects necessarily fail to encode properties, though the theory allows that abstract objects can both exemplify and encode properties. It is also axiomatic that if x encodes a property, it necessarily does so. But the key principle about abstract objects is the comprehension principle that asserts, for any condition (formula) \( \phi \) in which x doesn’t occur free, that \( \exists x (A!x \& \forall F(xF \equiv \phi)) \).

\(^{11}\)In the latest version of object theory, currently under development (Zalta m.s.), every formula \( \phi \) becomes a permissible matrix of a λ-expression, but not every λ-expression has a denotation. Those in which the \( \lambda \) doesn’t bind a variable in an encoding formula subterm of \( \phi \) are guaranteed to denote relations. So in this new version of the theory, λ-expressions are governed by free logic. But for this paper, the published versions of the theory suffice; the logic of well-formed λ-expressions is classical.

\(^{12}\)The proof of this principle from λ-Conversion is analogous to the proof in footnote 6, Modal Comprehension for Relations (□CP)

\[ \exists F^n \Box \forall x_1 \ldots \forall x_n (F^n x_1 \ldots x_n \equiv \varphi) \] (□CP), provided F doesn’t occur free in \( \varphi \) and \( \varphi \) doesn’t contain any encoding subformulas.

When \( n = 1 \) and \( n = 0 \), respectively, this principle asserts existence conditions for properties and states of affairs:

\[ \exists F \Box (Fx \equiv \varphi) \] (□CP), provided F doesn’t occur free in \( \varphi \) and \( \varphi \) doesn’t contain any encoding subformulas.

\[ \exists p \Box (p \equiv \varphi) \] (□CP), provided p doesn’t occur free in \( \varphi \) and \( \varphi \) doesn’t contain any encoding subformulas.

In other words, any formula free of encoding conditions (and this excludes \( x = y \), which is defined in terms of encoding subformulas) can be used to produce a well-formed instance of □CP. Thus, relations, properties, and states of affairs exist under the same conditions as in classical 2nd-order logic without identity.\(^{13}\)

The identity conditions for relations are stated by cases: first for properties F and G, then for n-place relations F and G (n ≥ 2), and finally for states of affairs p and q. Identity for relation and states of affairs is defined in terms of identity for properties. The definitions are as follows:

- Properties F and G are identical if and only F and G are necessarily encoded by the same objects, i.e.,

\[ F = G \equiv_{df} \Box \forall x(Fx \equiv xG) \] (9)

- n-place relations F and G are identical just in case, for any n − 1 objects, every way of applying F and G to those n − 1 objects results in identical properties, i.e.,

\[ \forall y_1 \ldots \forall y_{n-1} ([\lambda x F_{y_1} \ldots y_{n-1}] = [\lambda x G_{y_1} \ldots y_{n-1}] & [\lambda x F_{y_1} y_2 \ldots y_{n-1}] = [\lambda x G_{y_1} y_2 \ldots y_{n-1}] & \ldots & [\lambda x F_{y_1} \ldots y_{n-1}] = [\lambda x G_{y_1} \ldots y_{n-1}]) \]

\(^{13}\)Again, in the latest version of object theory, under development in Zalta m.s. the ‘no encoding subformulas’ restriction on comprehension for states of affairs is eliminated. Every formula \( \varphi \) in which p doesn’t occur free can be used as a matrix to assert the existence of a state of affairs.
• States of affairs \( p \) and \( q \) are identical whenever (the property) being an individual \( z \) such that \( p \) is identical to (the property) being an individual \( z \) such that \( q \), i.e.,

\[
p = q \iff [\lambda z \ p] = [\lambda z \ q] \tag{10}
\]

From these definitions, it can be shown that the reflexivity of identity holds universally, i.e., that \( x = x \) is derivable from (7), that \( F = F \) is derivable from each of (8) and (9), and that \( p = p \) is derivable from (10). So object theory asserts only the substitution of identities as an axiom. It therefore has all the theorems about identity that are derivable in 2nd-order logic with identity. Identity is provably symmetric, transitive, etc., and since every term of the theory is interpreted rigidly, substitution of identities holds in any (modal) context whatsoever.

The foregoing facts make it clear that (6) is a theorem of object theory, by the same reasoning used in the proofs given earlier in the paper. So if MacBride is right, object theory inherits the unwelcome consequences that attend to Russell’s principle. It is therefore time to examine these consequences and determine whether they apply. I shall focus only on what I take to be the main concerns and argue that the theory has resources that forestall them. Then we’ll be in a position to turn to the conclusion MacBride draws in his later work (namely, that the 2nd-order quantifiers don’t range over properties and relations) and show that it, too, doesn’t apply.

2 Superfluities of Relations and States?

MacBride begins his analysis by distinguishing three degrees of relatedness, where “[r]elatedness to the second degree involves the notion of a converse” and “to embrace the second degree is to make the existential quantifiers don’t range over properties and relations) and show that it, as well as ‘greater’, ‘loves’ as well as ‘is loved by’. But we shouldn’t think that converse relations are familiar old friends. Each ternary non-symmetric relation has five mutual converses, and we don’t have names for any of them. Things only get worse when we consider that each quaternary non-symmetric relation has 23 converses, etc.

There are two issues being raised: that non-symmetric relations of arity greater than 2 have many distinct converses and that we don’t have any names for them.

The first issue doesn’t seem problematic in and of itself; it is more like a statement of fact. We can sharpen the claim by proving it in 2nd-order logic with identity (and so in object theory). For any \( n \)-place relation \( F \) \((n \geq 3)\) and for any \( i \) and \( j \) such that \( 1 \leq i < j \leq n \), we may define the \( i,j \)-th converse of \( F \), written \( F^{ij} \), as follows:

\[
F^{ij} =_{df} [\lambda x_1 \ldots x_i \ldots x_j \ldots x_n \ F x_1 \ldots x_j \ldots x_i \ldots x_n] \tag{11}
\]

And we can define \( F \) is non-symmetric with respect to its \( i \)-th and \( j \)-th places:

\[
\text{Non-symmetric}^{ij}(F) \equiv_{df} \neg \forall x_1 \ldots \forall x_i \ldots \forall x_j \ldots \forall x_n (F x_1 \ldots x_j \ldots x_i \ldots x_n \rightarrow F x_1 \ldots x_i \ldots x_j \ldots x_n) \tag{12}
\]

Then for any \( n \)-place relation \( F \) \((n \geq 3)\) and \( i,j \) \((1 \leq i < j \leq n)\), it is provable that:

\[
\forall F (\text{Non-symmetric}^{ij}(F) \rightarrow F \neq F^{ij}) \tag{13}
\]

The proof is just a generalization of the one given for (6). 14

It should be observed that even though 2nd-order logic with identity and object theory both have (6) and (13) as theorems, and both imply that every \( n \)-place relation \((n \geq 2)\) has the converses described above, neither is committed to the existence of non-symmetric relations. The following are not, respectively, instances of CP or □CP:

\[\exists F \forall x \forall y (Fxy \equiv \text{Non-symmetric}(F))\]

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14 Fix \( n, i, j \) and \( F \). Assume \( \text{Non-symmetric}^{ij}(F) \). Then by (12) there are objects \( x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n \), say \( a_1, \ldots, a_i, \ldots, a_j, \ldots, a_n \), such that \( Fa_1 \ldots a_i \ldots a_j \ldots a_n \) and \( \neg Fa_1 \ldots a_j \ldots a_i \ldots a_n \). Assume, for reductio, that \( F \equiv F^{ij} \). Then it follows by the substitution of identities that \( F^{ij}a_1 \ldots a_i \ldots a_j \ldots a_n \). So by definition (11), it follows that:

\[
[\lambda x_1 \ldots x_i \ldots x_j \ldots x_n \ F x_1 \ldots x_j \ldots x_i \ldots x_n] a_1 \ldots a_i \ldots a_j \ldots a_n 
\]

Hence, by \( \lambda \)-Conversion: \( Fa_1 \ldots a_j \ldots a_i \ldots a_n \). Contradiction.
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What about the second issue, namely, the lack of names for converses? Though it is true that we may not always have familiar names of converses in the vernacular, we can use natural language to delineate them by using nominalizations. For a non-symmetric 3-place relation $S$, we have as converses:

- being a first, second, and third thing such that $S$ relates the first and third to the second
- being a first, second, and third thing such that $S$ relates the second and first to the third
- being a first, second, and third thing such that $S$ relates the second and third to the first
- being a first, second, and third thing such that $S$ relates the third and first to the second
- being a first, second, and third thing such that $S$ relates the third and second to the first

These nominalizations are available in ordinary (perfectly understandable) natural language free of technical vocabulary. In addition, these nominalizations can be regimented using the $\lambda$-expressions introduced earlier. The first is represented as $[\lambda xyz Sxzy]$, the second as $[\lambda xyz Syxz]$, the third as $[\lambda xyz Syxz]$, etc. So we can systematically distinguish among the converses of a non-symmetric relation, no matter what arity it has. And, as we saw above, if $S$ is non-symmetric with respect to a pair of arguments, its converses with respect to those arguments are provably distinct from one another. 2nd-order logic with identity simply allows us to represent and demonstrate the validity of reasoning to such conclusions, as we’ve seen above.

The real problem, rather, seems to be a superfluity of states, not of relations. MacBride (2014, 4) rehearses this problem for converses by considering the relations on and under, both of which are asymmetric (and hence non-symmetric if there are objects that stand in those relations):

It’s one kind of undertaking to put the cat on the mat, something else to put the mat under the cat, but however we go about it we end up with the same state. To bring the cat to the forefront of our

---

$\exists F \Box \forall x \forall y (Fx y \equiv \text{Non-symmetric}(F))$

Nor are the following, for any choice of $n$ ($n \geq 3$), $i$, and $j$ ($1 \leq i < j \leq n$):

$\exists F \Box \forall x \forall y (Fx y \equiv \text{Non-symmetric}^{i,j}(F))$

$\exists F \Box \forall x \forall y (Fx y \equiv \text{Non-symmetric}^{i,j}(F))$

The formulas $\text{Non-symmetric}(F)$ and $\text{Non-symmetric}^{i,j}(F)$ contain a free occurrence of the variable $F$ and so violate the restrictions on $\varphi$ in CP and $\Box \text{CP}$. Intuitively, CP and $\Box \text{CP}$ only guarantee that for every condition on objects there exists a relation that relates the objects satisfying the condition; it doesn’t assert that for every condition on relations, there is a relation that satisfies that condition. Similarly, if one attempts to put $\text{Non-symmetric}(F)$ into a $\lambda$-expression, the result, $[\lambda xy \text{Non-symmetric}(F)]$, is one where the matrix doesn’t have any free occurrences of $x$ and $y$ to be bound by $\lambda xy$. So the $\lambda$-expression simply denotes a vacuous 2-place relation that obtains between $x$ and $y$ whenever $F$ is non-symmetric. That relation is symmetric! And the same considerations apply to the defined notion $\text{Non-symmetric}^{i,j}(F)$.

So although these systems are committed to converses, they are not committed to non-symmetric relations of any arity. (4), (6), and (13) are universal claims and their truth is consistent with the failure of the antecedent of the embedded conditional. So, these systems don’t automatically engender a ‘superfluity’ of relations. To engender the superfluity, one has to assert the existence of non-symmetric relations, and this assertion requires the resources of 2nd-order logic and its quantifiers over relations. So what MacBride sees as a problem of superfluity, we can view as just a neutral fact about the consequences of this assertion, consequences that can be investigated only if one uses second-order logic. I don’t yet see that this is any problematic kind of superfluity.

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15Indeed, these systems don’t commit one to many relations at all. In the smallest model of pure, unapplied 2nd-order logic (or pure, unapplied object theory), the domain of individuals has just one ordinary object (say $a$); the domain of properties has just two properties (one property is exemplified by $a$ and the other isn’t); the domain of 2-place relations has just two relations (one relation is such that $a$ bears it to itself, and the other is such that $a$ doesn’t bear it to itself); the the domain of 3-place relations has just two relations; etc. Interestingly, CP is nevertheless true in these models, since the smallest models represent relations extensionally and, as a result, all of the instances of CP that assert the existence of a complex $n$-place relation can be modeled by one or the other of the two relations in the relevant domain. Indeed, even 2nd-order quantified modal logic with identity doesn’t require more, since the modal principles remain true even if there is only one world in the domain of possible worlds.
audience's attention we describe this state by saying that the cat is on the mat; to bring the mat into the conversational foreground we say that the mat is under the cat. But whether it’s the cat we mention first, or the mat, what we succeed in describing is the very same cat-mat orientation. That’s intuitive but if—as the second degree describes—a non-symmetric relation and its converse are distinct, we must be demanding something different from the world, a different state, when we describe the application of the above relation to the cat and the mat from when we describe the application of the below relation to the mat and the cat.

So MacBride worries that converse relations commit us to the principle that if \( F \) is non-symmetric, then the state of affairs \( Fxy \) is distinct from the state of affairs \( F^\ast yx \). Since both \( Fxy \) and \( F^\ast yx \) are 0-place relation terms (they denote states of affairs relative to the values of the variables \( x, y \), and \( F \)) as well as formulas, we can represent the principle that concerns MacBride formally as follows:

\[
\forall F(Non\text{-}\text{symmetric}(F) \rightarrow \forall x \forall y(Fxy \neq F^\ast yx)) \tag{14}
\]

This, he claims, is counterintuitive and he cites support in Fine 2000.\(^{16}\)

If this is the concern, why not adopt the following principle instead:

- If 2-place relation \( F \) is non-symmetric, then for any \( x \) and \( y \), the state of affairs \( x \) bears \( F \) to \( y \) is identical to the state of affairs \( y \) bears \( F^\ast \) to \( x \), i.e.,

\[
\forall F(Non\text{-}\text{symmetric}(F) \rightarrow \forall x \forall y(Fxy \equiv F^\ast yx)) \tag{15}
\]

The answer MacBride gives is (2014, 4):

We might attempt to defend the second degree by maintaining that the application of \( R \) and \( R^\ast \) does not give rise to different states with respect to the same relata but different decompositions of the same state. So whilst \( above \) and \( below \) are distinct, the relational configuration \( cat\text{-}above\text{-}mat \) is a decomposition of the same state as the configuration \( mat\text{-}below\text{-}cat \). But these decompositions comprise what are ultimately different constituents—a non-symmetric relation and its converse are supposed to be distinct existences. But now we have the difficulty of explaining how such different decompositions can give rise to a single state. And admitting such polymorphous states still does nothing to reduce the superfluity of converses.

If we can give an clear account of why the two states are identical, the problem of the superfluity of states goes away. In the account I shall give, we don’t need to appeal to ‘decompositions’.

The problem to be solved here is that 2nd-order logic doesn’t have the resources to supply a good definition of the conditions under which states of affairs are identical, even if we add modality to the logic. For neither of the following definitions are good ones:

\[
p = q \equiv df \ p \equiv q
\]

\[
p = q \equiv df \ \square(p \equiv q)
\]

It is reasonable to suppose that the state of affairs there is a barber who shaves all and only those who don’t shave themselves (\( \exists x(Bx \& \forall y(Sxy \equiv \neg Syy)) \)) is distinct from the state of affairs there is a brown and colorless dog (\( \exists x(Dx \& Bx \& \neg Cx) \)), yet these are not just equivalent but necessarily equivalent (since both are necessarily false).

So whereas both of the above definitions might be used to explain why \( Fxy = F^\ast yx \) (e.g., “they are identical because necessarily equivalent”), the definitions aren’t good ones — the explanation would come at an overly burdensome cost. It is not true, as a matter of complete generality, that states of affairs are identical when materially equivalent or when necessarily equivalent; rather they are hyperintensional. But then, there is no obvious way to explain their identity by appealing to some no-
tion of ‘constituents’ and combining the distinct constituents \( F, F^*, a \) and \( b \) so as to build the states of affairs \( Fab \) and \( F^*ba \) for which \( Fab = F^*ba \).\(^{18}\)

But, as we’ve seen, object theory offers a theory of identity for states of affairs and so offers a theoretical framework for asserting (15), should one wish to do so. For one could then appeal to definition (10) to say that the identity \( Fxy = F^*yx \) implies that the properties \( \lambda z Fxy \) and \( \lambda z F^*yx \) are identical. And, in turn, one can then appeal to definition (8) to infer, from this last fact, that necessarily, all and only the objects that encode \( \lambda z Fxy \) also encode \( \lambda z F^*yx \).

Why does this address the difficulty in the last quote from MacBride? The answer is: because we can explain how ‘distinct existences’ (i.e., a relation \( F \), its converse \( F^* \), and objects \( x \) and \( y \)) can ‘give rise’ to the same state, without appealing to ‘decompositions’, ‘constituents’, etc. The definitions of identity for abstract objects (7) and for properties (8) place reciprocal bounds on the existence of these entities. The theory’s comprehension principle and identity conditions for abstract objects tell us that any (expressible) condition on properties can be used to define an abstract object. If we think of abstract objects as objects of thought or as logical objects, then the theory implies that if properties \( F \) and \( G \) are distinct, then there is a logical, abstract object of thought that encodes \( F \) and not \( G \) (and vice versa). And if \( F \) and \( G \) are identical, then no logical object of thought encodes \( F \) without encoding \( G \). So if the properties \( \lambda z Fxy \) and \( \lambda z F^*yx \) are identical, then no logical, abstract object of thought encodes \( F \) without encoding the other. One practical consequence of this identification is this: it prevents one from telling a consistent story about a fictional object, say \( c \), which exemplifies \( \lambda z Fxy \) without \( c \) exemplifying \( \lambda z F^*yx \). So if you believe \( Fxy = F^*yx \), then this seems intuitively true: if you tell a story about something in a situation in which the state cat-on-mat obtains, then if you think cat-on-mat is just mat-under-cat, you’ve told a story about something in which mat-under-cat obtains.

So, if one believes that \( Fxy = F^*yx \), one can use the apparatus embodied by object theory (and its theory of relations and states of affairs) to give a precise, theoretical answer to a philosophical question that, if left unanswered, would otherwise pose a ‘superfluity if states’ problem for converses. By doing so, we undermine the problem.

Note how this addresses MacBride’s next concern, namely, that endorsing distinct converses for non-symmetric relations requires a commitment to a ‘substantive linguistic doctrine’, namely, that when we switch from the active ‘Antony loves Cleopatra’ to the passive ‘Cleopatra is loved by Antony’, we “introduce a novel subject matter” (MacBride 2014, 5). But if the subject matter is defined by the state of affairs being referenced, there is no change – one can move from ‘Antony loves Cleopatra’ to ‘Cleopatra is loved by Antony’ without changing the subject matter, since those sentences designate, on this view, the same state of affairs.

Let me be clear, however. I’m not suggesting that object theory is committed to (15). I’m suggesting only that one can consistently add (15), if you think it is true, and offer a theoretical response to MacBride’s concern. Nor am I endorsing (15) as the only way to respond to MacBride. As it turns out, there is an alternative way to respond to the unwelcome consequences that he is raising for the converses of non-symmetric relations. It may be of interest to some readers to consider what one can say to undermine the unwelcome consequences if one accepts that \( Fxy \neq F^*yx \) when \( F \) is non-symmetric. Readers who accept \( Fxy = F^*yx \) in these examples can jump straight to Section 5, where we address MacBride’s argument that 2nd-order quantifiers don’t range over relations.

In the next two sections, then, I’ll try to show that, in the background ontology of object theory:

- one may also consistently assert, in cases of this type, that the states of affairs \( Fxy, \lambda z Fzx, \lambda z Fxy \), \( F^*yx \), \( \lambda z F^*zx \), and \( \lambda z F^*yz \) are all pairwise distinct but necessarily equivalent, and

- the intuition that there is one part of the world underlying these states (e.g., one cat-mat configuration, etc.) is explained by (a) the fact that there is a unique situation \( s \) that in which all of these states of affairs obtain, and (b) the fact that \( s \) is a part of the actual world whenever all of the states of affairs that obtain in \( s \) obtain simpliciter.

\(^{18}\)You can’t assert the principle \( Fxy = Gzw \equiv (F = G & x = z & y = w) \), for the scenario in which cat-on-mat \( (Ocm) \) and mat-under-cat \( (O’mc) \) are identical constitutes a counterexample. For the principle would imply the instance \( Ocm = O’mc \equiv (O = O’ & c = m & m’ = c) \). And from the fact that \( O \neq O’ \), or the fact that \( c \neq m \), it would follow that \( Ocm \neq O’mc \). So this is no help, since we’re trying to explain how we can have, simultaneously, \( O = O’ \) and \( c \neq m \) and yet \( Ocm = O’mc \).

\(^{19}\)I’m indebted to Daniel Kirchner, who was able to use his implementation of object theory in Isabelle/HOL (Kirchner 2017 [2021]) to confirm the consistency of adding (15).
In Section 3 we discuss the first of these bulleted points, and in Section 4, we discuss the second.

3 Another Option

MacBride begins the argument to the conclusion that $Fxy = F^*yx$ with the claim:

It’s one kind of undertaking to put the cat on the mat, something else to put the mat under the cat, but however we go about it we end up with the same state.

So MacBride agrees there there are two kinds of undertakings. Since to undertake to do something is to attempt to bring about a state of affairs, one might suggest that there are two distinct undertakings precisely because there are two distinct states of affairs to be brought about. But MacBride concludes that there is only one state. We’re now going to investigate how we can deny his conclusion without sacrificing the driving intuition; object theory not only allows us to represent and distinguish two (distinct) states, but also to assert that there is only one situation in the case described. We’ll therefore divide the following discussion into two parts: one concerning the distinct states of affairs and one concerning the single situation being described.

If we let $c$ stand for the cat, $m$ for the mat, $O$ for on top of, and $O^*$ for beneath, then one could argue that in the first undertaking, you are making it the case that $Ocm$ while in the second undertaking, you are making it the case that $O^*mc$. Moreover, one can consistently assert that none of the following states are pairwise identical: $Ocm$, $[\lambda x Oxm]c$, $[\lambda x Ocx]m$, $O^*mc$, $[\lambda x O^*xc]m$, and $[\lambda x O^*xc]m$. Though these are all (provably) necessarily equivalent states of affairs, in the sense that $\square(p \equiv q)$, it doesn’t follow that they are identical.

Though an argument that these are all necessarily equivalent probably isn’t needed, it is worth showing that the necessary equivalences are all provable. To see this, one need only appeal to $\lambda$-Conversion, which has the following instances:

\[
\begin{align*}
[\lambda x Fxy]x & \equiv Fxy \\
[\lambda x Fyx]x & \equiv Fyx \\
[\lambda xy Fyx]xy & \equiv Fyx
\end{align*}
\]

So by the Rule of Necessitation and Universal Generalization, we can conclude, respectively:

\[
\begin{align*}
\forall \forall \forall \forall \square([\lambda x Fxy]x \equiv Fxy) \quad (16) \\
\forall \forall \forall \forall \square([\lambda x Fyx]x \equiv Fyx) \quad (17) \\
\forall \forall \forall \forall \square([\lambda xy Fyx]xy \equiv Fyx) \quad (18)
\end{align*}
\]

If we then instantiate (16) to $O, c$, and $m$, and (17) to $O, m$, and $c$, we get, respectively:

\[
\begin{align*}
\square([\lambda x Ocm]c \equiv Ocm) \quad (19) \\
\square([\lambda x Ocx]m \equiv Ocm) \quad (20)
\end{align*}
\]

And if we instantiate (16) to $O^*, m$, and $c$, and (17) to $O^*, c$, and $m$, we get, respectively:

\[
\begin{align*}
\square([\lambda x O^*mc]c \equiv O^*mc) \quad (21) \\
\square([\lambda x O^*mx]c \equiv O^*mc) \quad (22)
\end{align*}
\]

Then if we instantiate (18) to $O$, we get:

\[
\forall \forall \forall \forall ([\lambda xy Oyx]xy \equiv Oyx)
\]

Given definition (5), i.e., $O^* = df [\lambda xy Oyx]$, it follows from this last fact that:

\[
\forall \forall \forall \forall (O^*xy \equiv Oyx)
\]

So instantiating this to $m$ and $c$, we obtain:

\[
\square(O^*mc \equiv Ocm) \quad (23)
\]

Consequently, simple modal biconditional reasoning from the lines numbered (19) – (23) implies that the following are all (pairwise) necessarily equivalent: $Ocm$, $[\lambda x Oxm]c$, $[\lambda x Ocx]m$, $O^*mc$, $[\lambda x O^*xc]m$, and $[\lambda x O^*xc]m$. But from definition (10) for the identity of states of affairs, we can’t infer that any of the above states of affairs are pairwise identical. Indeed, one can consistently assert that none of the states of affairs, taken pairwise, are identical.\(^{20}\)

So if one asserts the states of affairs are distinct, is there a problem of superfluity? Not if we can account for the intuition that leads one to think that there is only one state of affairs in these cases. MacBride says (quoted above): \(^{20}\)

\[^{20}\] Again, I’m indebted to Daniel Kirchner, whose implementation of object theory in Isabelle/HOL (2017 [2021]) confirms this claim.
But whether it’s the cat we mention first, or the mat, what we succeed in describing is the very same cat-mat orientation.

And recall Fine’s claim (quoted in footnote 17 above):

Yet surely the states $s_1$ and $s_2$ are the same. There is a single state of affairs $s$ “out there” in reality, consisting of the blocks $a$ and $b$ having the relative positions that they do…

Clearly, we have to accommodate the intuition that there is one underlying fact, one underlying piece of the world, in these cases.

This intuition is addressed by appealing to the notion of a situation and defining the conditions under which a state of affairs $p$ obtains in a situation $s$. Once these notions are defined, we can identify, for any state of affairs $p$, a canonical situation $s$ in which all and only the states of affairs necessarily equivalent to $p$ obtain. Then it will follow, for example, that the canonical situation in which the states necessarily equivalent to $Ocm$ obtain is identical to the canonical situation in which the states necessarily equivalent to $Omc$ obtain. I’ll develop this analysis in the next section, and to make it clear that the analysis generalizes, I’ll sometimes use an arbitrary 2-place relation $R$ and its converse $R^*$ instead of the on top of relation $O$ and its converse $O^*$. And I’ll sometimes use arbitrary objects $a$ and $b$ instead of $c$ and $m$.

4 Situations and Their Parts

In object theory (Zalta 1993, 410), situations are defined as abstract objects that encode only properties constructed out of states of affairs, i.e., encode only properties $F$ of the form $[\lambda z p]$, where $p$ ranges over states of affairs:

$$Situation(x) =_{df} A!x \& \forall F(xF \equiv \exists p(\Box(Rab \rightarrow p) \& F = [\lambda z p]))$$

(24)

In addition (1993, 411), a state of affairs $p$ obtains in a situation $s$ ($s \models p$) just in case $s$ encodes being a $z$ such that $p$:

$$s \models p =_{df} s[\lambda z p]$$

(25)

In what follows, therefore, we sometimes extend the notion of encoding by saying that $s$ encodes a state of affairs $p$ whenever $p$ obtains in $s$. That is, when $s \models p$, then instead of always saying that $s$ encodes $[\lambda z p]$, we may say instead that $s$ encodes $p$.

Now consider some state of affairs, say $Rab$. Given the foregoing definitions, object theory implies that there exists a situation $s$ such that a state of affairs $p$ obtains in $s$ if and only if $p$ is necessarily equivalent to $Rab$. To see this, note that the comprehension principle for abstract objects asserts that there is an abstract object that encodes exactly those properties $F$ such that $F$ is a property of the form $[\lambda z p]$ when $p$ is some state of affairs necessarily implied by $Rab$:

$$\exists x(A!x \& \forall F(xF \equiv \exists p(\Box(Rab \rightarrow p) \& F = [\lambda z p])))$$

(26)

Let $s_1$ be such an object, so that we know:

$$A!s_1 \& \forall F(s_1F \equiv \exists p(\Box(Rab \rightarrow p) \& F = [\lambda z p]))$$

(27)

Since $s_1$ is abstract and every property it encodes is a property of the form $[\lambda z p]$, it follows that $s_1$ is a situation, by definition (24). Moreover, the theory implies that $s_1$ is unique, i.e., that any abstract object that encodes all and only those states of affairs implied by $Rab$ is identical to $s_1$. Since situations are abstract objects, they are identical whenever they encode the same properties.\(^{21}\) And since situations encode only properties $F$ such that $\exists p(F = [\lambda z p])$, by (24), they obey the principle: $s$ and $s'$ are identical just in case the same states of affairs obtain in $s$ and $s'$ (Zalta 1993, 412, Theorem 2). So there can’t be two distinct abstract objects that encode all and only the states of affairs necessarily implied by $Rab$. Since (26) has a unique witness, we may treat $s_1$ as a name of this witness (introduced by definition) and treat (27) as a provable fact about $s_1$.

Two modal facts about $s_1$ become immediately relevant:

- A state of affairs obtains in $s_1$ if and only if it is necessarily implied by $Rab$, i.e.,

$$\forall p(s_1 \models p \equiv \Box(Rab \rightarrow p)).$$

(28)

\(^{21}\)Strictly speaking, the definition of identity (7) implies that abstract objects $x$ and $y$ are identical if and only if necessarily, they encode the same properties. But since $xF \rightarrow \Box xF$ is an axiom of object theory, it follows that if $x$ and $y$ encode the same properties, they necessarily encode the same properties, and so it is sufficient to show $\forall F(xF \equiv yF)$ to establish that $x = y$, for abstract $x$ and $y$. 

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• $s_1$ is modally closed in the following sense: for any states of affairs $p$ and $q$, if $p$ obtains in $s_1$ and $p$ necessarily implies $q$, then $q$ obtains in $s_1$, i.e.,

$$
\forall p \forall q ((s_1 \models p) \& [\Box (p \rightarrow q) \rightarrow (s_1 \models q)).
$$

(29)

The proof of (28) is straightforward and, interestingly, relies on the object-theoretic definition for the identity for states of affairs (10). 22 Note that it immediately follows from (28) that $\Box b$ obtains in $s_1$, since $[\Box (\Box b \rightarrow \Box b)]$ is an instance of the modal principle $\forall p [\Box (p \rightarrow p)]$. The proof of (29) relies on both the definition of identity for states of affairs and the transitivity of necessary implication, i.e., the fact that $\forall p \forall q \forall r ([\Box (p \rightarrow q) \& \Box(q \rightarrow r) \rightarrow [\Box (p \rightarrow r)])$. 23

It is an immediate consequence of (28) that all of the following states of affairs obtain in $s_1$, for they are all necessarily implied by $\Box b$:

$$
[\lambda x R x b]a, [\lambda x R x b]a, R^* ba, [\lambda x R^* x a]b, \text{and } [\lambda x R^* b x]a
$$

So it is provable that $s_1 \models [\lambda x R x b]a$, that $s_1 \models [\lambda x R x b]a$, etc. If we add the assumption that $R$ is non-symmetric, it follows that none of $R b a, R^* a b, [\lambda x R x a]b, \text{etc.}$, obtain in $s_1$, since none of these is necessarily implied by $\Box b$ when $R$ is non-symmetric.

At this point, it should be clear that $R, a,$ and $b$ can be any relation and objects of the kind described in the MacBride and Fine examples, such as $O, m$, and $c$. But it is interesting to observe that any one of the necessarily equivalent states of affairs in question can be used to define a unique situation in which they all obtain. All of the resulting situations become identified, since it is a theorem of modal logic that necessarily equivalent states of affairs imply the same states of affairs, i.e.,

$$
\forall p \forall q ([\Box (p \equiv q) \rightarrow \forall r ([\Box (p \rightarrow r) \equiv [\Box (q \rightarrow r)]))
$$

(30)

To see why this facts helps us to show that the resulting situations are all identified, consider the following situation, which is introduced in a manner similar to $s_1$, but with $R^* b a$ instead of $R b a$:

$$
\exists x (A ! x \& \forall F (x F \equiv 3 p ([\Box (R^* b a \rightarrow p) \& F = [\lambda z p])])
$$

This is the (provably unique) situation that makes true all and only the states of affairs necessarily implied by $R^* b a$. Call this $s_2$. Clearly, facts analogous to (28) and (29) holds for $s_2$; a state of affairs $p$ obtains in $s_2$ if and only if $R^* b a$ necessarily implies $p$, and $s_2$ is modally closed.

But the key fact for our purposes is that $s_1 = s_2$. 24 Moreover, the reasoning in the proof applies to all the other canonical situations definable in terms of the necessarily equivalent states of affairs mentioned above: these canonical situations are pairwise identical. Thus, there is a single canonical situation in which all of the states of affairs mentioned in the example obtain.

So I suggest, therefore, that this canonical situation validates the following claims:

• there is a single cat-mat orientation (when $R = O$, $R^* = O^*$, $a = c$, $b = m$, and the states $R b a, R^* b a$, etc., all obtain), and

22Proof. To show $s_1 = s_2$, it suffices to show that when they encode the same properties, for as we noted earlier in footnote 21, the object-theoretic principle $x F \rightarrow [\Box x F]$ implies that if $s_1$ and $s_2$ encode the same properties, then necessarily they encode the same properties. To show $s_1$ and $s_2$ encode the same properties, we show, for an arbitrarily chosen property, say $P$, that $s_1 P \equiv s_2 P$. Without loss of generality, we show only $s_1 P \equiv s_2 P$, since the proof of the converse is analogous. So, assume $s_1 P$. Then, by definition of $s_1$,

$$
3 p ([\Box (R b a \rightarrow p) \& P = [\lambda y q_1])
$$

Let $q_1$ be such a state of affairs, so that we know $[\Box (R b a \rightarrow q_1)]$ and $P = [\lambda y q_1]$. Now by the same reasoning we used to establish (23), we know $[\Box (R^* b a \rightarrow R^* b a)$. So by an appropriate instance of (30), it follows that $\forall r ([\Box (R^* b a \rightarrow r) \equiv (\Box (R b a \rightarrow r))]$. Instantiating this last result to $q_1$, it follows that $[\Box (R^* b a \rightarrow q_1) \equiv (\Box (R b a \rightarrow q_1)]$. But we already know $[\Box (R b a \rightarrow q_1)]$. Hence $[\Box (R b a \rightarrow q_1)]$. So we have established:

$$
[\lambda x R b a \rightarrow q_1] \& P = [\lambda y q_1]
$$

By existential generalization:

$$
3 p ([\Box (R^* b a \rightarrow p) \& P = [\lambda y q_1])
$$

But then, by definition of $s_2$, it follows that $s_2 P$. 23
• there is one fact responsible for, or grounding, or making true \(Rab, R'ba\), etc.

It is the canonical situation that is the witness to, and preserves, these intuitions. It may be of interest that this solution doesn’t attempt to ground (the truth of) a state of affairs (something with a logical structure) in a physical chunk of the world (something without a logical structure).

Finally, to account for the intuition that the situation in which the necessarily equivalent states obtain is part of the actual world, we turn to the principles (theorems and definitions) governing part of, actual situations, and possible worlds. Since \(x\) is a part of \(y\) as defined as \(\forall F(xF \rightarrow yF)\), it follows that a situation \(s\) is part of a situation \(s' (s \subseteq s')\) just in case every state of affairs that obtains in \(s\) also obtains in \(s'\) (Zalta 1993, 412, Theorem 4). Moreover, an actual situation is a situation \(s\) such that every state of affairs that obtains in \(s\) obtains simpliciter (1993, 413). And a possible world is a situation \(s\) that might be such that it makes true all and only the truths (1993, 414). Formally:

\[
\begin{align*}
  s \subseteq s' & \equiv \forall p (s \models p \rightarrow s' \models p) \\
  Actual(s) & \equiv \forall p (s \models p \rightarrow p) \\
  PossibleWorld(s) & \equiv \forall p (s \models p \equiv p)
\end{align*}
\]

Object theory then yields, as a theorem (1993, Theorem 18):

There is a unique actual world, i.e.,

\[
\exists! s (PossibleWorld(s) \& Actual(s))
\]

The proof of this theorem rests on the fact that there is a unique situation that encodes all and only the states of affairs that obtain, i.e., there is a unique situation \(s\) such that all and only the states that obtain in \(s\) are states that obtain simpliciter.\(^{25}\)

So reconsider the case that MacBride describes, where the cat is on the mat, i.e., \(Ocm\). Then consider the situation that encodes all the states of affairs necessarily implied by \(Ocm\). Let this now be \(s_1\). So every state necessarily implied by \(Ocm\) obtains in \(s_1\). If \(Ocm\) obtains, then (a) every state that obtains in \(s_1\) obtains simpliciter, and (b) \(s_1\) is an actual situation.

It also follows that if we define a situation by starting with any of the other states of affairs necessarily equivalent to \(Ocm\), then the situation so defined is an actual situation that is identical to \(s_1\).

Finally, note that it is a theorem of object theory (1993, Theorem 19) that:

\[
\forall s (Actual(s) \rightarrow s \subseteq w_a)
\]

It therefore becomes provable that if \(Ocm\) obtains, the actual situation \(s_1\) that encodes just the states necessarily implied by \(Ocm\) is part of \(w_a\).

And something analogous holds for any relation \(R\) and its converse \(R'\).

This addresses the intuition that there is a single piece of the world in which all of the following states of affairs obtain: \(Ocm\), \([\lambda x Oxm]c\), \([\lambda x Ocm]m\), \([\lambda x O'mc]c\), and \([\lambda x O'cm]m\). We need not suppose that there is exactly one state of affairs.

The foregoing analysis therefore preserves the conclusion that Russell develops, when he says (1903, §219) regarding the terms greater and less:

These two words have certainly each a meaning, even when no terms are mentioned as related by them. And they certainly have different meanings, and are certainly relations. Hence if we are to hold that “\(a\) is greater than \(b\)” and “\(b\) is less than \(a\)” are the same proposition, we shall have to maintain that both greater and less enter into each of these propositions, which seems obviously false.

One might reframe Russell’s point by noting that if non-synonymous relational expressions signify or denote different relations, then the simple statements we can make using those expressions signify different states of affairs. That principle has been preserved, without sacrificing any contrary intuitions. And so when MacBride says (in discussing this passage) “But Russell also felt the deep intuitive force of another answer: that we are saying just the same whether we employ ‘\(a\) is greater than \(b\)’ or ‘\(b\) is less than \(a\)’ to do so” (2012, 157), I take it that the foregoing analysis accommodates the intuitive force of the other answer.
Note that if one accepts that states of affairs \( Fxy \) and \( F^*yx \) are distinct, one can still say that the active-passive transformation doesn’t introduce a novel subject matter. The analysis here can account for sameness of subject matter by proposing that subject matter be identified with the canonical situation in which the states expressed by active and passive claims both obtain. If one views subject matter that way, then there isn’t a change of subject matter when we switch from active to passive. And on this understanding, we need not conclude, from the fact that there is a change of the sentence subject in an active/passive transformation, that there is a change of subject matter.

Before we turn to our final question, of whether the quantifiers of 2nd-order logic range over relations, it should be observed that the key to the foregoing object-theoretic analysis is to (a) distinguish possible worlds and situations, on the one hand, from states of affairs, on the other, and (b) use encoding to understand Wittgenstein’s claim (1921) that the world is all that is the case. As we’ve seen, the actual world encodes all that is the case by encoding all and only states of affairs that obtain. And possible worlds generally are defined in terms of what obtains in them, not by a physical, mereological part-whole relation (cf. the worlds defined in Lewis 1986). Situations become parts of a possible world by encoding some of the states that obtain in that world and nothing else.26

5 The 2nd-Order Quantifiers

Macbride (forthcoming) argues that:

... We cannot interpret second-order quantifiers as ranging over relations without our either lapsing into unintelligibility or else having

26 This analysis of possible worlds, as encoding states of affairs, avoids the objection that arises for those who take possible worlds to be maximal and possible states of affairs and say that \( q \) obtains at \( p \) if \( p \) necessarily implies \( q \). On this view, one can’t maintain that there is a unique actual world and that states are hyperintensional. For suppose that \( p_1 \) is a world, i.e., a state of affairs that is possible and such that for every \( q \), either \( p_1 \) necessarily implies \( q \) or \( p_1 \) necessarily implies \( \neg q \). And suppose \( p_1 \) is an actual world (i.e., \( p \) is a possible world that obtains). Then \( p_1 \) & (\( r \lor \neg r \)) will also be an actual world, since the same states are necessarily implied by \( p_1 \) and \( p_1 \) & (\( r \lor \neg r \)). But on a hyperintensional view of states, \( p_1 \) and \( p_1 \) & (\( r \lor \neg r \)) would be distinct actual worlds. See Zalta 1988 (73–74), Zalta 1993 (393–394), and also McNamara (1993), who makes a similar point. Such a consequence doesn’t affect the present view.

I plan to discuss this argument in two parts, filling in some details along the way. First I suggest that the argument is based on presuppositions that are misconceptions from the point of view of 2nd-order logic and object theory, and that once the misconceptions are identified, the argument has a clear rejoinder. Then, once we eliminate the misconceptions, I’ll respond to the version of the argument that can be reconstructed in terms of a proper understanding of the concepts involved.

Macbride’s argument is based on the following presupposition, namely, that ‘\( Fab \)’ is a predicate that stands for a property and that ‘\( \exists F(Fab) \)’ as-

His argument goes by way of a dilemma, the horns of which are derived from the disjunction “[e]ither pairs of mutually converse predicates, such as ‘\( x \) is on top of \( z \)’ and ‘\( x \) is underneath \( z \)’, refer to the same underlying relation or they refer to distinct converse relations” (forthcoming, 1). Since the first disjunct is provably false in 2nd-order logic, we focus only on the argument for the horn that is derived from the second. At the highest level of abstraction, the key premises of this argument are:

- If a quantification such as \( \exists F(Fab) \) is understandable, then ‘higher-order predicates’ of the form \( Fab \) have a determinable significance.
- But ‘higher-order predicates’ of the form \( Fab \) don’t have a determinable significance.

MacBride concludes that since such quantifications aren’t understandable, the 2nd-order quantifiers don’t range over relations.

This general argument emerges first in the following passage:

... But even if pairs of mutually converse relations are admitted, thus avoiding the difficulties that arose from dispensing with them, higher-order predicates of the form ‘\( a \Phi b \)’ are still required for the intelligibility of quantification into the positions of converse predicates, i.e. higher-order predicates capable of being true or false of a relation belonging to the domain independently of how that relation is specified. ...

... do we understand higher-order predicates of the form ‘\( a \Phi b \)’?

I will argue that we do not because of the semantically dubious consequences of thinking so. (forthcoming, 15–16)
sists that a relation has that property. But, it is important to recognize
that, in the formalism of 2nd-order logic and object theory, the expres-
sion ‘Fab’ is not a higher-order predicate; it doesn’t stand for a property
and a statement of the form ‘∃F(Fab)’ doesn’t say that some relation has
a property.

A clearer view of the matter goes back to the precise definition of the
formalism for 2nd-order logic and object theory. Since the formalism for
2nd-order logic is simpler and suffices to make the point, we’ll focus on
that. If you examine any definition of the language of 2nd-order logic,
it is immediately clear that ‘Fab’ isn’t a predicate. Rather, ‘Fab’ an open
formula and, if the 2nd-order language allows the arity of relations to go
to zero, as we have done, ‘Fab’ is also an open 0-place relation term. So
the following facts hold:

- As an open formula, ‘Fab’ has truth conditions and so its deter-
minable significance can be precisely specified if we can precisely
specify its truth (or satisfaction) conditions. We’ll discuss these be-
low.

- As an open, 0-place relation term, ‘Fab’ doesn’t stand for a prop-
erty of relations, for 2 reasons: (a) such higher-order properties are
distinguishable in 2nd-order logic, and (b) even if they were
expressible, ‘Fab’ is the wrong kind of expression to stand for a prop-
erty – only a 1-place relation term can stand for a property. Rather,
as an open, 0-place relation term, ‘Fab’ is a kind of com-
plex variable ranging over states of affairs; the identity of the state
of affairs that it denotes depends on the value assigned to the vari-
able F. So the determinable significance of the term ‘Fab’ can be
precisely specified if we can precisely specify the state of affairs
denoted.

Moreover, what MacBride says on p. 18 of his paper (quoted above in
footnote 27) is simply incorrect: a statement of the form ‘∃F(Fab)’ does not
say that some relation has the higher-order property that a relation
has when it applies to a first and b second. That isn’t expressible in 2nd-
order logic. Rather, it says only that some relation is such that a bears it to b.

So the presuppositions of MacBride’s argument don’t hold for either
2nd-order logic or object theory. Therefore, it would seem that he can’t
draw any conclusions about the 2nd-order quantifiers on the basis of
these presuppositions. Later, we’ll consider a higher-order logic and a
higher-order version of object theory; these are systems in which ‘Fab’
can be used to form the 1-place term ‘[λF Fab]’ (i.e., in which the λ binds
the free variable F). This expression does stand for a property of rela-
tions, namely, the property: being a relation F such that a bears F to b.
We’ll then consider whether there is a version of MacBride’s argument
that holds when adapted to such systems. But for now, let’s consider
whether we can address the concern MacBride is trying to develop with-
out going higher-order.

The concern seems to be that the expression ‘Fab’ doesn’t have a de-
terminable significance. I won’t go into the details of the argument at
this point because it is cast in terms of ‘order-indifferent’ higher-order
relations and, as we’ve just learned, higher-order relations aren’t express-
ible in these systems. So all we have to do to address the argument is

MacBride uses Φ as a 2nd-order variable (not a metavariable) in infix notation and
says:

We can avoid this problem by interpreting ‘Alexander Φ Bucephalus’ as
standing for a property sensitive to the order in which Alexander and Bu-
cephalus are related by whatever relation has this property. (17)

Then later:

If a higher-order predicate of the form ‘a Φ b’ expresses the higher-order
property that a relation has when it applies to a first and b second, then
what a statement of the form ‘∃Φ(aΦb)’ says is that some relation has that
property. (18)

And then later:

So prima facie interpreting higher-order predicates of the form ‘a Φ b’ as
standing for a property that a relation has if it applies to a first and b second
imports ordinal notions...

The argument MacBride gives (forthcoming, 18–19) is a kind of reductio and has vari-
ous versions. Here is a paraphrase of the version that concerns symmetric relations:

Assume that ‘Fab’ stands for a property that a relation has if it applies to a
first and b second. Then such properties of relations import ordinal notions,
such as ‘first’ and ‘second’. But the following claims (involving symmetric
relations) have the form ‘Fab’:


and claims of this form, according to cited experts (Langacker; Dowty; and
Rappaport, Hovav, & Levin), don’t import ordinal notions because they don’t
say different things – there is no principled way to determine whether differs from
applies to Darius first and Alexander second, or Alexander first and

Footnotes:
27 27
28 28
to specify the determinable significance of the expression ‘Fab’. That is, we need to say precisely what it denotes in so far as it is a term and to say precisely what its truth conditions are in so far as it is a formula. If we let \( R \) be some relation assigned to the variable ‘\( R \)’ and let \( a \) and \( b \) be the objects denoted by the constants ‘\( a \)’ and ‘\( b \)’, then we can say precisely that the open, 0-place relation term ‘\( Fab \)’ denotes the state of affairs \( Rab \).

And we can say precisely (in metaphysical terms, rather than in Tarski-style semantics) that the formula ‘\( Fab \)’ is true just in case the state of affairs \( Rab \) obtains. These answers are available in object theory because we have independently stated a theory (with existence and identity conditions) of relations and states of affairs, using notation that is logically precise. No special notion of order (other than the implicit one needed to assert \( a \) bears \( F \) to \( b \)) or of higher-order properties is needed to assign the expression ‘\( Fab \)’ a determine significance, either in its guise as a 0-place term or in its guise as a formula.

But let’s now use higher-order logic to restate MacBride’s argument. I plan to show that the higher-order theory of relations and states of affairs undermines the resulting argument. So suppose we allow ourselves the resources of 3rd-order logic with identity, with higher-order \( \lambda \)-expressions and a principle of \( \lambda \)-Conversion that applies to those higher-order expressions. This logic let’s us quantify over properties of relations, and form complex names like \([\lambda F \land xFx]x\) (“being a relation \( F \) that is reflexive”) and \([\lambda F \land \neg \forall x \forall y (Fxy \rightarrow Fyx)]x\) (“being a relation that is non-symmetric”). Since object theory has been developed in a type-theoretic framework, it has these resources; indeed, we need only the fragment of typed object theory that includes such expressions and principles (Zalta 2020). Then we can recast the sub-argument MacBride uses to establish that there is a ‘semantically dubious consequence’ of supposing that ‘\( Fab \)’ is false.

In order for a predicate of the form ‘\( a \Phi b \)’ to have this kind of self-determinable significance it must stand for a higher-order property which relations have independently of how they are picked out. This requirement is fulfilled if relations hold between the things they relate in an order, where the notion of order in play is absolute in the following sense: for any relation \( R \) which holds between any two things \( a \) and \( b \), either \( R \) applies to \( a \) first and \( b \) second or \( b \) first and \( a \) second.

Though this is just a part of the argument MacBride develops, we can stop with this premise; it doesn’t allow for the present method for assigning an expression a ‘self-standing significance’. We can see this by recasting the claim in terms of the expression ‘\( [\lambda F Fab] \)’, which does indeed stand for a higher-order property. Then the claim seems to be that for this expression to have a determinate significance, some notion of order is required to explain how first-order relations apply to their relata.

But a reply to this conclusion is ready to hand. To establish that ‘\( [\lambda F Fab] \)’ has a clear and determinate significance in which order is not essential, we need only appeal to the following instance of higher-order \( \lambda \)-Conversion, which is an axiom that asserts:

\[ [\lambda F Fab]R \equiv Rab \] (31)

We may read this as: \( R \) exemplifies the property being a relation \( F \) such that \( a \) bears \( F \) to \( b \) if and only if \( a \) bears \( R \) to \( b \), i.e., if and only if the state of affairs \( Rab \) obtains. This doesn’t explicitly import ordinal notions. The type-theoretic principles governing relations in object theory hold throughout the type hierarchy and so its theory of (i.e., existence and identity conditions for) higher-order properties is a generalization of 2nd-order object theory. So there is a theory of properties that offers determinate application conditions of the higher-order property that the expression ‘\( [\lambda F Fab] \)’ stands for.

Consequently, (31) provides exactly the application conditions that MacBride says we lack when he says (forthcoming, 23):

\[ I \text{ conclude that we lack a facility with higher-order predicates of the form ‘}a \Phi b\text{’ interpreted as standing for a higher-order property of relations because we lack a principled grasp of their application conditions.} \]

Instead, I conclude that the higher-order, 1-place property denoted by \([\lambda F Fab] \) is intelligible, given both the 3rd-order version of \( \lambda \)-Conversion

\[ \text{Darius second. So the assumption, that ‘} Fab\text{’ stands for a property that a relation has if it applies to } a \text{ first and } b \text{ second, is false.} \]

MacBride gives related arguments for some non-symmetric relations and then generalizes (23) to conclude: if there is a higher-order property of \( F \) (such as applying to \( a \) first and \( b \) second), then there should be a fact of the matter as to the order in which \( F \) applies to \( a \) and \( b \), but since there is no fact of the matter as to the order in which \( F \) applies, we can’t make sense of higher-order predicates of the form ‘\( Fab \)’.
and the precise theory of (higher-order) properties available typed object theory. So, even if higher-order properties were needed to make sense of open formulas such as $F ab$ in 2nd-order logic, there would be some that do the job.

Let me conclude by forestalling an objection. Even though higher-order, 1-place expressions like $[\lambda F F ab]$ don’t require a notion of order, it might be thought that a notion of order might still play a role in our understanding of $\lambda$-expressions like those discussed in Section 2, where we described and formally represented the various converses of a 3-place relation $S$. Recall that one of those converses is:

being a first, second, and third thing such that $S$ relates the first and third to the second, i.e., $[\lambda xyz Sxzy]$

Doesn’t some notion of order play a role here? The answer is: yes, but the notion of order can be analyzed away. The expression $[\lambda xyz Sxzy]$ is governed by the following instance of $\lambda$-Conversion:

$[\lambda xyz Sxzy]xyz \equiv Sxzy$

The order required for understanding the left-side of the biconditional is thereby analyzed away by the equivalent right-side of the biconditional. So this notion of order is unproblematic.

6 Conclusion

We’ve seen two ways to undermine the main unwelcome consequences that are alleged to arise for (4), (6), and (13). Both solutions extend object theory, though they extend object theory in incompatible ways. As yet, I don’t see a definitive reason for preferring one solution to the other. The data don’t seem to require it, though it may be that future research will determine that one of the solutions is to be preferred.

We’ve also undermined arguments used to conclude that 2nd-order quantifiers don’t range over relations. The arguments are based on questionable presuppositions and once these are identified and eliminated, the conclusion can’t be sustained. Object theory provides existence and identity conditions for the relations over which 2nd-order quantifiers range, and even if we think about the system in a higher-order context, we have an extendable foundation from which we can theorize about relations as entities in their own right.

Bibliography


