Non-Symmetric Relations and Their Converses*

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Abstract

In this paper, I investigate two arguments that draw startling and puzzling conclusions about 2nd-order logic (2OL). The first argument concludes that 2nd-order quantifiers can't be interpreted as ranging over relations. This is puzzling given the traditional understanding of 2OL. The second argument, which concludes that unwelcome consequences arise if relations and relatedness are analyzed rather than taken as primitive, utilizes premises that imply that 2OL faces the very same consequences. This is puzzling because relations and predication are taken as primitive in 2OL, and so the latter should be immune to the problems raised for an analysis. I consider these two arguments in light of a precise theory of relations. In particular, I show that object theory (Zalta 1983, 1988), which is an extension of 2OL with identity, provides systematic existence and identity conditions for relations, properties, and states of affairs that forestall the two arguments.

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1 Setting Up the Problems

Two recent recent arguments draw somewhat puzzling conclusions about second-order logic (2OL) and its philosophical interpretation. The first conclusion appears in a recent paper by MacBride (forthcoming, p. 1), where he argues, by way of a dilemma, that "we cannot interpret 2nd-order quantifiers as ranging over relations without our either lapsing into unintelligibility or else having to embrace incredible consequences". This conclusion is startling because MacBride is not claiming that relations don't exist or that some other (e.g., ontologically more neutral) interpretation of the 2nd-order quantifiers is to be preferred, but rather that the 2nd-order quantifiers *can't* be interpreted unproblematically as ranging over relations.[1] The conclusion is also puzzling given a traditional understanding of 2OL. Philosophers and logicians since Russell have supposed that relational statements of natural language of the form 'a loves b', 'a gives b to c', etc., can be uniformly rendered in the predicate calculus as statements of the form $Ra_1 \ldots a_n$, where the arity of R is n for any $n \geq 1$ and where $Ra_1 \ldots a_n$ expresses the claim that $a_1,\ldots,a_n$ exemplify (or stand in or instantiate) n-place relation R. For example, Väänänen (2020, Section 2) notes that "The intuitive meaning of $X(t_1,\ldots,t_n)$ is that the elements $t_1,\ldots,t_n$ are in the relation X or are *predicated* by X". So it is puzzling to be informed that when we existentially generalize on the statement $Ra_1 \ldots a_n$ to derive a claim of the form $\exists F(Fa_1 \ldots a_n)$, we can't regard this latter claim as quantifying over relations.

The second puzzling conclusion appears in Macbride 2014. On the one hand, Macbride argues that relations, predication (relation application), and relatedness should be taken as primitive (2014, pp. 1, 2, 15), on the grounds that any analysis leads to unwelcome consequences. But, on the other hand, the unwelcome consequences he describes for the analysis of relations are already present in the 2OL, where relations, predication and relatedness are primitive. He endorses the primitive nature of relatedness when he writes:

[1]Thus, I have no objection to philosophers attempting to look for other ways to interpret the second-order quantifiers, either by using plural (Boolos 1984, 1985), denominalized (Rayo & Yablo 2001), or neutral (Wright 2007) interpretations of the second-order quantifiers. Rather, what I'm reacting to is an argument that says such quantifiers can't be successfully interpreted as ranging over relations.

> I will argue that the capacity of a non-symmetric relation R to apply to the objects a and b it relates so that aRb rather than bRa must be taken as ultimate and irreducible. ...
>
> It's a familiar thought that we cannot account for the fact that one thing bears a relation R to another by appealing to a further relation relating R to them—that way Bradley's regress beckons. To avoid the regress we must recognize that a relation is not related to the things it relates, however language may mislead us to think otherwise. We simply have to accept as primitive, in the sense that it cannot be further explained, the fact that one thing bears a relation to another [citations omitted]. But it is not only the fact *that* one thing bears a (non-symmetric) relation R to another that needs to be recognized as ultimate and irreducible. *How* R applies—whether the aRb way or the bRa way—needs to be taken as primitive too. (2014, 2; italics in original)

While this seems correct, the argument that MacBride gives for this conclusion ensnares 2OL with identity (i.e., $2OL^=$), where relatedness is primitive. His argument revolves around the following claim (Russell 1903, §218–219):[2]

> Every (2-place) non-symmetric relation R has a converse R^* that is distinct from R. (1)

This claim plays a role in MacBride's forthcoming paper as well, but in 2014, he argues any analysis of relations and relation application that endorses (1) gives rise to 'unwelcome consequences', namely (a) a multiplicity of converse relations,[3] and (b) "the profusion of states that arise from the application of these relations" (2014, 4). The worry about (a) is puzzling because $2OL^=$, in which relations, predication, and relatedness are primitive, has a formal representation of (1) as a theorem. So it seems we face a multiplicity of relations no matter whether we endorse (1) by way of an analysis or by way of taking relations and predication as primitive in $2OL^=$. As part of our investigation, we'll also examine both (b) and MacBride's conclusion that there is no good analysis of the identity and distinctness of states of affairs. He says:

[2]Russell actually talked about 'asymmetric' relations, but we'll discuss the differences below, where we formally define non-symmetric relations. I don't think anything hangs on the difference.

[3]For example, ternary non-symmetric relations have 5 converses and quarternary non-symmetric relations have 23.

> What vexes the understanding is ... an analysis of the fundamental fact that $aRb \neq bRa$ for non-symmetric R. ... Anyone who wishes to give an analysis of the fact that $aRb \neq bRa$ faces a dilemma. ... Since neither ... analyses are satisfactory, this recommends our taking the fact that $aRb \neq bRa$ to be primitive. (2014, 8)

[We provide the full quote later in the paper.] But when we examine this (second) dilemma, we'll see that there is an analysis which is immune to the dilemma and which MacBride doesn't consider. One can unproblematically analyze the *identity* of states of affairs within a theory on which the fact that a state of affairs *obtains* is primitive.

My plan is as follows. In Section 2, I lay out the dilemma used to establish that the 2nd order quantifiers don't range over relations (i.e., the first puzzling conclusion) and show that the first horn doesn't apply. In Section 3, I examine the second horn and narrow our focus to an issue on which the conclusion rests, namely, a question about the identity of certain states of affairs. In Section 4, I examine the second puzzling conclusion, from MacBride's 2014 paper, and connect the argument there with the issue we reached in Section 3. Then in Section 5, I review a theory of relations and states of affairs that MacBride doesn't consider but which defines identity conditions for states of affairs. In Sections 6 and 7, I use the theory in Section 5 to ground two answers to the question (about the identity of states of affairs) on which both of MacBride's puzzling conclusions rest. I show that these answers undermine the main lines of argument that MacBride uses to establish his conclusions.

My strategy throughout will be to show how one can deploy and extend 2OL to formulate a theory of relations, predication, and states of affairs that forestalls the puzzling conclusions. In the first part of the paper, we extend 2OL in (known) ways that systematize the language that MacBride uses in his arguments. Later in the paper, starting in Section 5, I'll appeal to the theory of abstract objects developed in Zalta 1983, 1988, and 1993, which I henceforth refer to as 'object theory' ('OT').[4] OT extends 2nd-order logic in ways that allow us to state unproblematic identity conditions for relations and states of affairs.

Before we begin, however, it is important to review some terminology and notation. '2OL' refers only to a formal, axiomatic system un-

[4]This theory has been applied and developed in a number of more recent publications, including Linsky & Zalta 1995, Zalta 2006, Nodelman & Zalta 2014, Menzel & Zalta 2014, Zalta 2020, and elsewhere. These texts contain useful introductions to the theory.

der an objectual interpretation (i.e., where the quantifiers range over domains of entities). My arguments don't require that we interpret 2OL in terms of *full* models (where the domain of properties has to be as large as the full power set of the domain of individuals); instead, *general* models (where the domain of properties need only be as large as some proper subset of the power set of the domain of individuals) suffice. The only requirement is that the models validate the axioms of 2OL. In what follows, I'll represent a 2-place atomic predication as 'Rab' instead of as 'aRb' except when we're discussing *identity*, in which case I'll use '$a = b$' (i.e., infix notation). Though the atomic formula '$F^n x_1 \dots x_n$' can be generally read as "$x_1, \dots, x_n$ exemplify (or instantiate) F^n", I'll often read the 2-place predication 'Fxy' as "x bears F to y". Moreover, I'll use 'F' instead 'Φ' as a 2nd-order variable; Greek letters will be used as metavariables instead. So where MacBride talks about the 2nd-order quantified sentence '$\exists\Phi(a\Phi b)$', I'll represent this sentence as '$\exists F(Fab)$'.

In the next few sections, we shall extend 2OL in various ways, in part to systematize the language that MacBride uses in his arguments. We'll start with 2OL$^=$, in which identity claims of the form '$F^n = G^n$' (for any n) are primitive.[5] We'll also treat states of affairs as 0-place relations, and instead of using $F^0, G^0, \dots$ as 0-place relation variables, we'll use $p, q, \dots$. So we'll be using a language and logic in which identity claims such as '$p = q$', asserting the identity of states of affairs, are well-formed. Moreover, we'll also introduce n-place λ-expressions ($n \geq 0$), interpreted relationally; these are complex terms that denote relations and states of affairs.[6] And we'll let formulas be complex terms that denote states of affairs, so that when MacBride uses expressions like '$aRb = bRa$' and '$aRb \neq bRa$' (2014, 8), we can represent this talk precisely as identity and non-identity claims about the states of affairs denoted by the

[5]Though logic texts (e.g., Mendelson 1964 [1997], Enderton 1972 [2001]) often formulate 2OL instead of 2OL$^=$, Shapiro (1991, 64) and Väänänen (2020, §2) mention that 2OL$^=$, in which identity is taken as a primitive, is a simple extension of 2OL.

[6]The definitions of the language of 2OL are easily adapted when we let $n = 0$, thereby including constants and variables ranging over states of affairs or propositions (where these are taken to be 0-place relations). And there are extensions of 2OL in which n-place λ-expressions have been included as complex names for n-place relations ($n \geq 0$). This suggestion appears in Prior 1971 (Chapter 3, 43–44), though Prior subsequently questions ontological implications of λ-expressions (1971, 45). More recently, λ-expressions were adopted in Zalta 1983 (Chapters III, IV), 1993 (407–409), and in Menzel 1986 (7, 26) and 1993 (67ff), where they are used in an untyped setting.

formulas flanking the identity symbol.[7] When we extend 2OL to OT in Section 5, we'll add a new, primitive mode of predication and a primitive modal operator. Using OT, we'll define the primitive claims of the form '$F^n = G^n$' (for $n \geq 1$) and '$p = q$'; thus, we'll provide identity conditions for relations and states of affairs. I'll then be in a position to argue that OT thereby offers an analysis of '$aRb = bRa$' or '$aRb \neq bRa$' without facing any dilemmas.

It is also important to mention that we shall *not* follow MacBride (who in turn cites Dummett 1981, 38–9) in his use of the technical term *predicate*:

> ... what is a second-order predicate? A first-order predicate (say of the form '$F\xi$') results from the extraction of one or more names ('a') from a closed sentence ('Fa') in which it occurs. A second-order predicate (say of the form '$\exists x\Phi x$') results from the extraction of a first-order predicate ('$F\xi$') from a closed sentence ('$\exists xFx$')
> (forthcoming, 3)

What MacBride calls a 'predicate' I shall simply refer to as an open formula, and I shall instead use the term 'predicate' to refer to a relation term Π (constant or variable) that can be predicated of individual terms $\kappa_1, \dots, \kappa_n$, i.e., that can occur as Π in an atomic formula of the form $\Pi\kappa_1 \dots \kappa_n$. Thus, where '$S$' represents *sells*, then 'S' is a predicate be-

[7]Thus, the language of 2OL$^=$ that we'll need can be specified precisely in terms of a definition, by simultaneous recursion, of the notions of *formula* and *term*:

- Base clause for terms: Every simple constant and variable is a term (i.e., individual constants and variables are individual terms, and n-place relation constants and variables ($n \geq 0$) are n-place relation terms).
- Base clauses for formulas: (a) for any $n \geq 0$, whenever $\kappa_1, \dots, \kappa_n$ are any individuals term and Π^n is any n-place relation term, $\Pi^n\kappa_1 \dots \kappa_n$ is a formula, and (b) whenever κ and κ' are any individual terms, or Π and Π' are any n-place relation terms (for some n), $\kappa = \kappa'$ and $\Pi = \Pi'$ are formulas.
- Recursive clause for formulas: If φ and ψ are any formulas, and α is any variable, $\neg\varphi$, $\varphi \to \psi$, and $\forall\alpha\varphi$ are formulas.
- Recursive clauses for terms: where $\nu_1, \dots, \nu_n$ ($n \geq 0$) are distinct individual variables and φ is any formula, then $[\lambda\nu_1 \dots \nu_n\, \varphi]$ is an n-place relation term and φ itself is a 0-place relation term.

We define $\varphi \,\&\, \psi$, $\varphi \vee \psi$, $\varphi \equiv \psi$, and $\exists\alpha\varphi$ (α any variable) in the usual way. Note that by these definitions, formulas of the form $\exists p(p \equiv \varphi)$, where φ is any formula, are well-formed. Suitably restricted, this schema will serve as the 0-place case of the comprehension principle for relations.

cause we can represent 'a sells b to c' as '$Sabc$'. But if we replace 'b' by 'x' to obtain the open formula '$Saxc$', then this latter expression is not a predicate. Given an assignment to the variable x, the open formula '$Saxc$' has truth conditions and denotes a state of affairs. By contrast, we may regard the complex 1-place relation term '$[\lambda x\, Saxc]$' ('being an x such that a sells x to c') as a predicate; it denotes a property and we can combine it with 'b' to form the atomic predication '$[\lambda x\, Saxc]b$' ("b exemplifies being an x such that a sells x to c"). And '$[\lambda xy\, Sxyc]$' is a predicate because we can form the atomic statement '$[\lambda xy\, Sxyc]ab$'.

Thus, predicates are nominalized expressions that denote properties and relations. Variables such F, G, etc., are also predicates since the expression 'Fa', 'Gxy', etc. are well-formed atomic formulas; the variables F, G, etc., denote properties and relations relative to an assignment to the variables. Similarly, when we replace the constant '2' with 'x' in the complex closed sentence '$E2\,\&\,P2$' ("2 is even and 2 is prime"), we obtain '$Ex\,\&\,Px$'. This latter expression isn't a predicate – it can't be predicated of anything since it is a conjunction of two statements. Of course, relative to any variable assignment, (a) '$Ex\&Px$' has truth conditions, and (b) denotes a (complex) state of affairs. Semantically, one can define a sense in which an individual in the domain can *satisfy* this open formula, in Tarski's sense, but this is not to say that the open formula can be *predicated* of that individual or predicated of the individual term 'a'. By contrast, the complex 1-place relation term '$[\lambda x\, Ex\,\&\,Px]$' can be combined with an individual constant to form a predication; that is, we can form the predication '$[\lambda x\, Ex\,\&\,Px]2$', which predicates the property denoted by the λ-expression of an individual. And in 2OL$^{=}$, we can infer '$\exists F(Fa)$' from '$[\lambda x\, Ex\,\&\,Px]2$'. So whereas we call '$[\lambda x\, Ex\,\&\,Px]$' a (nominalized) predicate, we won't call '$Ex\,\&\,Px$' a predicate.

Similarly, we shall not say that the open formulas 'Fab' and '$Fa\,\&\,Qb$' (where 'F' is a free variable and the other letters are constants) are 2nd-order predicates. Again, these are open formulas that denote states of affairs relative to an assignment to the free variable F. As such, these expressions are 0-place relation terms, i.e., terms that denote states of affairs (relative to any variable assignment). Instead, our practice will be call the expressions '$[\lambda F\, Fab]$' and '$[\lambda F\, Fa\,\&\,Qb]$' (nominalized) predicates; these correspond, respectively, to the open formulas 'Fab' and '$Fa\,\&\,Qb$'. The predicates '$[\lambda F\, Fab]$' and '$[\lambda F\, Fa\,\&\,Qb]$' are part of the language of 3rd-order logic (3OL), for they denote properties of relations. These predicates can be used to form predications in 3OL such as '$[\lambda F\, Fab]R$', i.e., R exemplifies the property of being a relation F that a bears to b. We'll introduce these higher-order predicates later, at the point in the discussion when they become relevant.[8]

2 The First Horn

We can now outline and investigate Macbride's argument about the interpretation of the 2nd-order quantifiers. It proceeds under the reasonable assumption that 2nd-order quantification is a straightforward generalization of 1st-order quantification. So let's suppose that the 1st- and 2nd-order quantifiers range over (mutually exclusive) domains and that the axioms and inference rules of the 2nd-order quantifiers mirror those of the 1st-order quantifiers. MacBride's argument, to the conclusion that we cannot interpret 2nd-order quantifiers as ranging over relations, goes by way of a dilemma. Let's call this the Dilemma for Converses. We may summarize his sketch by way of the following outline (quoting, for the most part, Macbride forthcoming, pp. 1–2):

> **Dilemma for Converses** (Outline)
> Either pairs of mutually converse predicates, such as 'ξ is on top of ζ' and 'ξ is underneath ζ', refer to the same underlying relation or they refer to distinct converse relations. If they refer to the same relation then we lack

[8] It might be thought that such higher-order predicates are expressible in 2OL. One might point to the following passage in Shapiro 1991, 64–65:

> Second-order variables, as well as non-logical predicate, relation, and function names, may be called 'higher-order terms', items that 'denote' relations and functions. By way of analogy, this opens the possibility of relations of relations, functions on relations, etc. These may be called *higher-order non-logical terms*. An example would be a property TWO of properties such tyat TWO(P) 'asserts' that P applies to exactly two things. A relevant 'definition' would be:
>
> $$\mathrm{TWO}(P) \equiv \exists x \exists y[x \neq y \,\&\, \forall z(Pz \equiv (z = x \vee z = y))]$$

But here Shapiro is talking loosely and signals that he is talking loosely by putting the word 'denote' (and other terms) in quotation marks. The expression 'TWO(P)' can be *defined* in 2OL but it can't be interpreted as *denoting term*, or as a term that denotes a property of properties, since there is no domain of properties of properties in the interpretation of 2OL. It is simply an open formula that some properties satisfy and others don't. In 2OL, the predicate $[\lambda F\ \mathrm{TWO}(F)]$ wouldn't be well-formed; there is no domain of properties of properties that could provide a denotation for this expression.

an understanding of the higher-order predicates required to understand second-order quantifiers as ranging over a domain of relations. If, by contrast, 'ξ is on top of ζ' and 'ξ is underneath ζ', refer to distinct converse relations then whilst we can at least make a certain abstract sense of the higher-order predicates required to interpret quantifiers in this way, the consequences for our understanding of lower-order constructions render this an unappealing semantic hypothesis.

We need not state the full argument derived from each horn of the dilemma now, because it can be shown that, given the reasonable assumption that non-symmetric relations exist, the first horn of the Dilemma for Converses is provably false in $2OL^=$. We spend the remainder of this section showing this.

Since MacBride's argument in the Dilemma for Converses involves claims about converse relations, let us define:

- G is a *converse of* F if and only if for any objects x and y, x bears G to y iff y bears F to x, i.e.,

$$\mathit{ConverseOf}(G,F) \equiv_{df} \forall x \forall y (Gxy \equiv Fyx) \tag{2}$$

In addition, the horns of the Dilemma for Converses concern the identity and distinctness of converses and so involve statements of the form '$R = S$' and '$R \neq S$'. Thus, to see that the condition leading to the first horn of the Dilemma is false, i.e., to see that it is not the case that mutually converse predicates refer to the same underlying relation, we only need to show that there are distinct converses:

$$\exists F \exists G (\mathit{ConverseOf}(G,F) \,\&\, G \neq F) \tag{3}$$

Any predicates that witness this claim will show that not all predicates for converses denote the same underlying relation.

Though (3) is not a theorem of $2OL^=$, it is implied by two claims: that non-symmetric relations have distinct converses and that there are non-symmetric relations. To see how, let us first define:

- F is *non-symmetric* if and only if it is not the case that for any objects x and y, if x bears F to y then y bears F to x, i.e.,[9]

[9]This is to be contrasted with:

F is *asymmetric* if and only if for any objects x and y, if x bears F to y then it is not the case that y bears F to x, i.e.,

$$\mathit{Asymmetric}(F) \equiv_{df} \forall x \forall y (Fxy \rightarrow \neg Fyx)$$

Russell discusses *asymmetric* relations in 1903, §218. In what follows, however, we discuss the more general notion of non-symmetric relations now being defined in the main text.

$$\mathit{Non\text{-}symmetric}(F) \equiv_{df} \neg \forall x \forall y (Fxy \rightarrow Fyx) \tag{4}$$

Given this definition, the two claims needed to establish (3) may be represented as follows:

$$\exists F (\mathit{Non\text{-}symmetric}(F)) \tag{5}$$

$$\forall F (\mathit{Non\text{-}symmetric}(F) \rightarrow \exists G (\mathit{ConverseOf}(G,F) \,\&\, G \neq F)) \tag{6}$$

As mentioned above, (5) is a reasonable assumption that MacBride adopts in his paper. And (6) is a formal represntation of (1). Since (5) is an assumption, we now show that (6) is a theorem of $2OL^=$, and then it will be a simple matter to show that (3) follows from (5) and (6).

2.1 The Reasoning

Two facts about $2OL^=$ have to be mentioned before we begin. First, $2OL^=$ includes the standard two axioms that logic texts use to systematize identity claims, namely, the reflexivity of identity and the substitution of identicals.[10]

Second, where $n \geq 0$, $2OL^=$ includes the following axiom by virtue of being an extension of 2OL:

Comprehension Principle for Relations (CP)
$\exists F^n \forall x_1 \ldots \forall x_n (F^n x_1 \ldots x_n \equiv \varphi)$, provided F^n doesn't occur free in φ

[10]The reflexivity of identity can be expressed by the schema $\alpha = \alpha$, where α is either an individual variable or an n-place relation variable, for some n. So $F = F$ becomes an instance of the reflexivity of identity, where F is any relation variable of any arity. The substitution of identicals can be expressed by the schema $\alpha = \beta \rightarrow (\varphi \rightarrow \varphi')$, where α and β are both individual variables or both n-place relation variables (for some n) and φ' is the result of substituting the variable β for one or more occurrences of α in φ, provided that β is substitutable for α in φ (i.e., doesn't get 'captured' by a variable-binding operator when substituted). So as instances of the substitution of identicals, we have $F = G \rightarrow (\varphi \rightarrow \varphi')$, where φ' is the result of substituting the variable G for one or more occurrences of F in φ, providing G is substitutable for F in φ.

From these two principles, one can derive that identity for relations is symmetric and transitive. For example, to derive symmetry, i.e., $F = G \rightarrow G = F$, assume $F = G$. Then consider the instance of the substitution of identicals $F = G \rightarrow (F = F \rightarrow G = F)$. From this instance and our assumption, it follows that $F = F \rightarrow G = F$. But from this and reflexivity, it follows that $G = F$. Hence, by conditional proof, $F = G \rightarrow G = F$.

We read this as: there exists an n-place relation F such that any objects $x_1,\ldots,x_n$ exemplify F if and only if φ. In the case where $n = 0$ and 'p' is used as a 0-place variable instead of 'F^0', then **CP** asserts $\exists p(p \equiv \varphi)$, i.e., there exists a state of affairs p such that p obtains if and only if φ. Note that we read 'p' as it occurs in '$p \equiv \varphi$' as 'p obtains', since (a) 'p' occurs as a formula and (b) *obtains* for states of affairs is the 0-place case of *exemplification*. The 0-place case of **CP** will be of service later, but for now we focus on the cases of **CP** where $n \geq 1$.

Before we show how 2OL$^=$ yields (6) as a theorem, a few words about the role **CP** plays in 2OL$^=$ are in order. First, it is often thought that 2OL$^=$ requires a large ontology of relations simply in virtue of including **CP** as an axiom. After all, in the 1-place case, **CP** has instances such as the following:

- $\exists F\forall x(Fx \equiv \neg Gx)$
  (Any given property) G has a negation.

- $\exists F\forall x(Fx \equiv Gx \,\&\, Hx)$
  (Any given properties) G and H have a conjunction.

- $\exists F\forall x(Fx \equiv \exists y Hyx)$
  There is a property that objects exemplify whenever a 2-place relation H is projected into its first argument place.

And in the 2-place case, **CP** has the instances like the following:

- $\exists F\forall x\forall y(Fxy \equiv Gyx)$
  (Any given relation) G has a converse.

But in fact, the smallest models of 2OL$^=$ require only that the domain of n-place relations contains just two relations, for each n. How can that be? The answer is: the smallest models of 2OL$^=$ make **CP** true by identifying properties and relations with the same extension. More specifically, in the smallest models of 2OL$^=$, (a) the domain of individuals contains just a single element, say b, (b) the domain of 1-place relations contains just two properties—one exemplified by b and one exemplified by nothing; (c) the domain of 2-place relations contains just two relations—one that relates b to itself and one that is empty; and so on. For example, if we let P_1 be the property that is exemplified by b and P_2 be the empty property, then P_2 is the negation of P_1 and vice versa. Moreover, (i) the conjunction of P_1 with itself is just P_1, (ii) the conjunction of P_2 with itself is just P_2, and (iii) the conjunction of P_1 with P_2 (and the conjunction of P_2 with P_1) is just P_2, since nothing exemplifies both P_1 and P_2. And so on for the other 1-place instances of **CP**. Now for the case of 2-place relations, let R_1 be the relation that relates b to itself, and R_2 be the empty relation. Then R_1 is the negation of R_2 and vice versa. Moreover, R_1 and R_2 both have converses—each has itself as a converse. R_1 is a converse of itself because $R_1bb \equiv R_1bb$, and R_2 is a converse of itself because of the exact same reason, though in this second case, the biconditional is true because both sides are false. And so on for the other 2-place instances of **CP**.

So if we don't add any distinguished, theoretical properties, 2OL$^=$ doesn't commit us to much at all. But though 2OL$^=$ does commit us to the existence of converse relations, it does *not* commit us to the existence of non-symmetric relations. In the smallest models of 2OL$^=$, as we just saw, there are only two 2-place relations; we've called them R_1 and R_2. Note that both R_1 and R_2 are symmetric; they both satisfy the open formula $\forall x\forall y(Fxy \rightarrow Fyx)$. R_1 satisfies this formula because b is the only object that can instantiate the 1st-order quantifiers and $R_1bb \rightarrow R_1bb$ is a theorem of logic, derivable either as a tautology or from the truth of the conditional's consequent. R_2 is symmetric because, again, b is the only object that can instantiate the 1st-order quantifiers and $R_2bb \rightarrow R_2bb$ is again a theorem of logic, derivable either as a tautology or from the failure of the conditional's antecedent. We can consider this same point *proof-theoretically*: 2OL$^=$ doesn't force the domain to contain non-symmetric relations. The claim $\exists F(\textit{Non-symmetric}(F))$ is not a theorem of this logic.[11]

Of course, (6) can still be true even if there are no non-symmetric relations, by failure of the antecedent. But the key fact is not that (6) is true independently of the existence of non-symmetric relations, but that it is derivable as a theorem. The proof doesn't depend on the existence of non-symmetric relations, doesn't employ any analysis of predi-

[11] The claim that there are non-symmetric relations, i.e., $\exists F(\textit{Non-symmetric}(F))$, expands to the following, by definition (4):

$$\exists F\neg\forall x\forall y(Rxy \rightarrow Ryx)$$

Clearly, this claim is not an instance of **CP**, since it has the wrong form. Moreover, we can't derive the existence of non-symmetric relations from instances of **CP**, such as:

$$\exists F\forall x(Fx \equiv \textit{Non-symmetric}(F))$$

This is not a well-formed instance of **CP** either, but in this case, the problem is that the variable F is free in the formula $\textit{Non-symmetric}(F)$, violating the axiom's condition.

cation, and doesn't require any particular semantic interpretation of the domain over which the relation variables range. I've put the proof in a footnote.[12] So the formal representation of (1), namely (6), is a theorem of $2\mathrm{OL}^{=}$.

But the combination of (6) with the reasonable assumption (5) yields the conclusion that there are mutually converse predicates that don't refer to the same underlying relation. For let 'R' be a witness to assumption (5), so that we know *Non-symmetric*(R). Then by (6), we obtain the conclusion $\exists G(\mathit{ConverseOf}(G,R) \,\&\, G \neq R)$, which tells us only that R has a distinct converse. Note that we're not quite done, since the condition leading to the first horn of the Dilemma for Converses is about *predicates*, and to show that it is false, we need a bit more reasoning and semantic ascent. So let 'S' be a witness to our last result, so that we know $\mathit{ConverseOf}(S,R) \,\&\, S \neq R$. Then, by semantic ascent, we have established that the predicates 'R' and 'S' denote converse relations that are distinct. Thus, the first horn of MacBride's Dilemma for Converses doesn't hold with respect to $2\mathrm{OL}^{=}$ under any interpretation. The first horn starts from the falsehood that pairs of mutually converse predicates refer to the same underlying relation.

[12]*Proof*. Pick an arbitrary relation R and assume R is non-symmetric. Then, by definition (4) and predicate logic, there are objects, say a and b, such that both $Rab \,\&\, \neg Rba$. Note independently that **CP** implies that every relation has a converse, as follows. If we let φ be Gyx, where G is a free variable, then $\exists F\forall x\forall y(Fxy \equiv Gyx)$ is a 2-place instance of **CP**. It follows by universal generalization that:

$$\forall G\exists F\forall x\forall y(Fxy \equiv Gyx)$$

By instantiating to R, it follows that $\exists F\forall x\forall y(Fxy \equiv Ryx)$. Pick an arbitrary relation as a witness to this claim, say S, so that we know:

(A) $\forall x\forall y(Sxy \equiv Ryx)$

(A) implies, by definition (2), that $\mathit{ConverseOf}(S,R)$. But we already know Rab, since it's the first conjunct of $Rab \,\&\, \neg Rba$. Hence, Sba, by instantiating b for x and a for y in (A). Now for reductio, assume $S = R$. Then it follows that Rba, by substitution of identicals. But this contradicts $\neg Rba$, which is the second conjunct of $Rab \,\&\, \neg Rba$. Hence $S \neq R$, by reductio. We've therefore established $\mathit{ConverseOf}(S,R) \,\&\, S \neq R$. So by existential introduction, $\exists G(\mathit{ConverseOf}(G,R) \,\&\, G \neq R)$. By conditional proof, then, it follows that $\mathit{Non\text{-}symmetric}(R) \rightarrow \exists G(\mathit{ConverseOf}(G,R) \,\&\, G \neq R)$. But since R was arbitrary, universally generalizing on R yields (6).

2.2 Simplifying the Reasoning

Before we turn to the second horn of MacBride's Dilemma for Converses in Section 3, it is relevant, and of significant interest, that (1) can be represented, and its proof developed, much more elegantly if we add λ-expressions to $2\mathrm{OL}^{=}$. λ-expressions are complex expressions for naming relations and they will play an important role in what follows. So we begin the explanation of how λ-expressions simplify our definitions and theorems about converses by saying a few words about the logic that results when we add these expressions.[13] Assume, therefore, that we have added complex, n-place relation terms of the form $[\lambda x_1 \dots x_n\, \varphi]$ to our language ($n \geq 0$). When $n \geq 1$, we read $[\lambda x_1 \dots x_n\, \varphi]$ as *being objects* $x_1,\dots,x_n$ *such that* φ; when $n = 0$, we read $[\lambda\, \varphi]$ as *that*-φ. Thus, λ-expressions do not denote functions, as in the functional λ-calculus, but rather relations and, in the 0-place case, states of affairs. A simple predication like '$[\lambda x\, \neg Px]y$' asserts: y exemplifies *being an object x that fails to exemplify P*; and '$[\lambda\, \neg Rab]$' denotes the state of affairs *that a doesn't bear R to b*.

By adding λ-expressions to 2nd-order logic, we can replace **CP** by:

λ-**Conversion** (λC)
$$[\lambda x_1 \dots x_n\, \varphi]x_1 \dots x_n \equiv \varphi$$

This asserts: $x_1,\dots,x_n$ exemplify *being objects $x_1,\dots,x_n$ such that φ* if and only if φ. For example, $[\lambda xy\, \neg Fxy]xy \equiv \neg Fxy$ is an instance, and by universal generalization, it is a theorem of the relational λ-calculus that:

$$\forall F\forall x\forall y([\lambda xy\, \neg Fxy]xy \equiv \neg Fxy)$$

To see how this works, instantiate this theorem to an arbitrary 2-place relation R, and then to arbitrary objects a and b. The result is the instance: $[\lambda xy\, \neg Rxy]ab \equiv \neg Rab$.[14]

[13]In essence, we will be using the λ-calculus under the interpretation in which λ-expressions denote relations rather than functions. There is a nice discussion of this in Section 9.3 of Alama & Korbmacher 2021.

[14]In what follows, I also assume two other principles of the λ-calculus (understood relationally), namely η-Conversion, which asserts $[\lambda x_1 \dots x_n\, \Pi^n x_1 \dots x_n] = \Pi^n$, for any n-place relation term Π, and α-Conversion, namely, that alphabetically-variant λ-expressions denote the same relation. η-Conversion tells us that a λ-expression such as '$[\lambda xy\, Rxy]$', in which all the free variables in the atomic exemplification formula 'Rxy' are bound by the λ, denotes the same relation that 'R' denotes, i.e., the identity '$[\lambda xy\, Rxy] = R$' holds. As an example of α-Conversion, we have '$[\lambda xy\, Rxy] = [\lambda yz\, Ryz]$'.

As previously mentioned, (λC) eliminates the need for **CP** since the latter becomes derivable. The proof is left to a footnote.[15] This applies even to the 0-place case of (λC). When $n = 0$, (λC) asserts $[\lambda\, \varphi] \equiv \varphi$, i.e., that-$\varphi$ obtains if and only if φ. For example, '$[\lambda\, \neg Lmj] \equiv \neg Lmj$' might assert: (the state of affairs) that-Mary-doesn't-love-John obtains if and only if Mary doesn't love John. Note that the 0-place case of **CP** immediately follows from the 0-place case of (λC), by Existential Introduction.[16] Again, the 0-place case of (λC) will play a role later, but for now, lets focus on the cases where $n \geq 1$.

We can use λ-expressions to introduce a well-behaved *converse* operator * on predicates by taking advantage of λ-expressions. Let us define the converse of F, i.e., F^*, as *being an x and y such that y bears F to x*, i.e.,

$$F^* =_{df} [\lambda xy\, Fyx] \tag{7}$$

Note how this definition immediately implies that every relation has a converse, where this is expressible as $\forall F \exists G(G = F^*)$.[17] *A fortiori*, every non-symmetric relation has a converse. Thus, we can now represent and prove (1) more elegantly as the claim that for any 2-place relation F, if F is non-symmetric, then its converse F^* is distinct:[18]

[15]Just universally generalize on $x_1,\ldots,x_n$ in (λC) to conclude:

$$\forall x_1 \ldots \forall x_n([\lambda x_1 \ldots x_n\, \varphi]x_1 \ldots x_n \equiv \varphi)$$

Then, we can existentially generalize on the λ-expression (provided F doesn't occur free in φ) so that we obtain (**CP**):

$$\exists F \forall x_1 \ldots \forall x_n(Fx_1 \ldots x_n \equiv \varphi), \text{ provided } F \text{ doesn't occur free in } \varphi$$

If F were free in φ it would get 'captured' by the quantifier $\exists F$, and the resulting principle would be invalid, for it would have the contradictory instance $\exists F \forall x(Fx \equiv \neg Fx)$.

[16]In this case, the proviso is that p doesn't occur free in φ. Given that proviso, we can existentially generalize on the 0-place relation term $[\lambda\, \varphi]$ in $[\lambda\, \varphi] \equiv \varphi$ to obtain: $\exists p(p \equiv \varphi)$, i.e., there is a state of affairs p such that p obtains if and only if φ.

[17]Let R be an arbitrary relation. Then, in classical 2OL$^=$, in which every term (including every λ-expression) has a denotation, we have, as an instance of the reflexivity of identity, that $[\lambda xy\, Ryx] = [\lambda xy\, Ryx]$. So by Existential Introduction, $\exists G(G = [\lambda xy\, Ryx])$. And by definition of R^*, it then follows that $\exists G(G = R^*)$. Since R was arbitrary, we have established $\forall F \exists G(G = F^*)$.

[18]Of course, one could more stricly represent (1) as follows:

$$\forall F(\mathit{Non\text{-}symmetric}(F) \rightarrow \exists G(G = F^* \;\&\; G \neq F))$$

But the consequent of this quantified conditional, $\exists G(G = F^* \,\&\, G \neq F)$, is just equivalent to the consequent of claim (8) in the text, namely, $F^* \neq F$. The proof of both directions of the equivalence is straightforward. For the left-to-right direction, suppose $\exists G(G = F^* \;\&\; G \neq F)$. Then let H be such a relation, so that we know both $H = F^*$ and $H \neq F$. Then by substitution of identicals, $F^* \neq F$. For the right-to-left direction, assume $F^* \neq F$. Then by reflexivity of identity, $F^* = F^* \;\&\; F^* \neq F$. Hence by existential introduction, $\exists G(G = F^* \,\&\, G \neq F)$. Given the equivalence just established, we use the simpler $F^* \neq F$ as the consequent of when representing (1) as (8).

$$\forall F(\mathit{Non\text{-}symmetric}(F) \rightarrow F^* \neq F) \tag{8}$$

Again, I've put the proof in a footnote,[19] and I encourage the reader to compare the proof of (8) in footnote 19 with the proof of (6) in footnote 12, to confirm how λ-expressions simplify the reasoning. Thus, as soon as we instantiate the reasonable assumption (5) to an arbitrary predicate, say 'R', that denotes a non-symmetric relation, we can immediately cite the new predicate 'R^*' and (8) to conclude $R \neq R^*$, and further conclude, by semantic ascent, that the condition leading to the first horn of the Dilemma for Converses is false.

So when we add λ-expressions to 2OL$^=$, the concepts and claims simplify and clarify. I'll therefore use (8) as the clearer representation of (1) in what follows. But my analysis will apply to (6) as well. Both (6) and (8) have been established as formal theorems, without any analysis of predication or any semantic arguments about converses.

3 The Second Horn

MacBride's Dilemma for Converses concludes that the quantifiers of 2OL don't range over relations and we've now seen that the first horn of the dilemma fails in 2OL$^=$ (i.e., the logic needed to systematize talk about the identity or distinctness of relation converses). The argument in the second horn is outlined as follows:

> If we suppose they [mutually converse predicates] don't co-refer but pick out distinct converse relations then I argue that whilst we may have some understanding of the higher-order predicates [required for second-order quantification over a domain of relations] ... we do so only at undue semantic and metaphysical cost.
>
> (forthcoming, 1)

[19]*Proof*. Assume *Non-symmetric*(R), where R is arbitrary. Then, $\neg\forall x \forall y(Rxy \rightarrow Ryx)$, i.e., for some objects, say a and b, we know $Rab \;\&\; \neg Rba$. Now for reductio, assume $R^* = R$. Then by symmetry of identity, $R = R^*$, and from Rab, if follows that R^*ab, by substitution of identicals. So by definition (7) of R^*, we know $[\lambda xy\, Ryx]ab$. But by (λC), this implies Rba. Contradiction. Hence $R^* \neq R$. So by conditional proof, *Non-symmetric*(R) $\rightarrow R^* \neq R$. Since R was arbitrary, we may universally generalize to get (8).

A fuller outline of the argument emerges later in the paper, beginning in the following passage:

> ... But even if pairs of mutually converse relations are admitted, thus avoiding the difficulties that arose from dispensing with them, higher-order predicates of the form '$a\Phi b$' are still required for the intelligibility of quantification into the positions of converse predicates, *i.e.* higher-order predicates capable of being true or false of a relation belonging to the domain independently of how that relation is specified. ...
>
> ...do we understand higher-order predicates of the form '$a\Phi b$'? I will argue that we do not because of the semantically dubious consequences of thinking so. (forthcoming, 15–16)

Before we look at the specific way in which MacBride argues for this conclusion, let's first make the metalanguage that MacBride needs to present his argument a bit more precise.

3.1 Third-Order Language and Logic (3OL)

I shall suppose that MacBride's metalanguage is 3rd-order, since he wants to understand the properties of relations expressed by the open formula 'Fab'. If we use λ-expressions, we can formally represent the property expressed by the open formula 'Fab' as $[\lambda F\, Fab]$. We read this λ-expression as: being a relation F such that a bears F to b. So let us take on board the resources of a 3rd-order language and logic (3OL), including monadic, higher-order λ-expressions of the form $[\lambda F\, \varphi]$ for naming complex properties of relations. 3OL let's us quantify over, and name, such properties of relations as $[\lambda F\, \forall x Fxx]$ ("being a relation F that is reflexive") and $[\lambda F\, \neg\forall x\forall y(Fxy \rightarrow Fyx)]$ ("being a relation that is non-symmetric"), etc.

In 3OL, λ-expressions of the form $[\lambda F\, \varphi]$ are governed by the following schema:

(Monadic) Higher-Order λ-Conversion (HOλC)
$[\lambda F\, \varphi]F \equiv \varphi$

So by Universal Generalization, the following is a theorem schema of 3OL:

$$\forall F([\lambda F\, \varphi] \equiv \varphi) \tag{9}$$

With this formalization in mind, we can return to MacBride's argument.

MacBride argues that in order for '$\exists F(Fab)$' to be interpreted as quantifying over relations, we have to be able to grasp the expression 'Fab' as being true or false of relations independently of how such relations are named or picked out. But, he suggests, we lack such a grasp of 'Fab'. He then proceeds to consider and reject certain proposals for understanding 'Fab'.

3.2 The First Argument for the Second Horn

The first proposal that MacBride considers, and rejects, appeals to the determinate-determinable distinction. He defines (forthcoming, pp. 9–10): 'Alexander Φ Bucephalus' has determinable significance when it is true of a relation R just in case R relates Alexander to Bucephalus in *some manner or other* but without settling any determinate arrangement for them. He then argues that the supposition, 'Fab' has a determinable significance, gets the truth conditions wrong for non-symmetric relations:

> Consider,
>
> [1] Alexander is on top of Bucephalus
>
> and one of its consequences,
>
> [7] ¬Bucephalus is on top of Alexander.
>
> If 'Alexander Φ Bucephalus' has purely determinable significance then 'Bucephalus Φ Alexander' does too but they will mean the same. The latter will stand for a property that a relation has if it relates Bucephalus and Alexander in some manner or other. But a relation has the property of relating Bucephalus and Alexander in some manner or other iff it has the property of relating Alexander and Bucephalus in some manner or other—because the property of relating some things in some manner or other is order-indifferent. (forthcoming, 16)

He then draws the conclusion that we can't explain the valid inference from [1] to [7] given this analysis, for whereas [1] says that *on top of* has the order-indifferent property of relating Alexander and Bucephalus in some manner or other, [7] says that this relation doesn't have that property.

Though the conclusion follows, the premise and the proposal it reflects is a non-starter. 'Fab', 'Fba', and '$\neg Fba$' don't have a *determinable* significance, as MacBride defines this, because the higher-order properties of relations in question already have a *determinate* significance. We may represent the higher-order properties in question as $[\lambda F\, Fab]$, $[\lambda F\, Fba]$, and $[\lambda F\, \neg Fba]$. These higher-order properties are all well-defined. To see why, let φ in (9) be, successively, Fab, Fba, and $\neg Fba$, and instantiate the quantifier $\forall F$ to the relation R in each case. Then all of the following are theorems of 3OL derivable from (HOλC):

$$[\lambda F\, Fab]R \equiv Rab \tag{10}$$

$$[\lambda F\, Fba]R \equiv Rba \tag{11}$$

$$[\lambda F\, \neg Fba]R \equiv \neg Rba \tag{12}$$

These are *not* schemata. (10) says: relation R exemplifies *being a relation that a bears to b* just in case a bears R to b. (11) says: R exemplifies *being a relation that b bears to a* just in case b bears R to a. And (12) says: R exemplifies *being a relation that b fails to bear to a* just in case b fails to bear R to a.

Thus 'Alexander Φ Bucephalus' ('Fab') and 'Bucephalus Φ Alexander' ('Fba') have a determinate significance represented by the higher-order properties $[\lambda F\, Fab]$ and $[\lambda F\, Fba]$. Moreover, they clearly don't mean the same; they aren't even materially equivalent. $[\lambda F\, Fab]$ is exemplified by R, given the fact that Rab and (10), and $[\lambda F\, Fba]$ fails to be exemplified by R, given the fact that $\neg Rba$ and (11). So the proposal that "['Bucephalus Φ Alexander'] …stand[s] for a property that a relation has if it relates Bucephalus and Alexander in some manner or other" isn't a plausible option. The correct understanding of the higher-order property MacBride designates as 'Alexander Φ Bucephalus' is given by (10) and that gives us the means to understand the open formula 'Fab'.

3.3 The Second Argument for the Second Horn

The next proposal that MacBride considers and rejects is the suggestion that we understand 'Fab' in terms of a higher-order property of relations in which ordinal notions ('first', 'second') play some role. In particular, the proposal under consideration is that 'Fab' is to be understood in terms of the higher-order property that a relation has if it applies to a

first and b second. MacBride develops an extended argument (pp. 15–23) against this proposal, by considering symmetric relations and then non-symmetric relations. At the end, he says:

> I conclude that we lack a facility with higher-order predicates of the form '$a\Phi b$' interpreted as standing for a higher-order property of relations because we lack a principled grasp of their application conditions. (forthcoming, 23)

If this conclusion holds, MacBride will have established the claim that we can't understand the quantified formula '$\exists F(Fab)$' as quantifying over relations.

Fortunately, we don't have to go through the extended argument in detail because we already have the means to undermine the conclusion just quoted. MacBride says we don't have a 'principled' grasp of the application conditions of the higher-order properties that invoke ordinal concepts. But in reaching this conclusion, he failed to consider (HOλC) as providing the needed principle. Over the next few paragraphs, I (a) show why (HOλC) is the right principle, (b) defuse some reasons that might be suggested as to why it isn't, (c) show how (HOλC) helps us to undermine some of the claims MacBride makes during the course of his argument for the second horn, and (d) narrow our focus to a question that I think is really driving MacBride's concern about quantification over relations.

Clearly, (HOλC) is a logical principle and it states exemplification conditions (i.e., 'application' conditions) for the higher-order properties denoted by predicates of the form $[\lambda F\, \varphi]$. So, clearly, we do *not* lack a principled grasp of the higher-order predicate '$[\lambda F\, Fab]$' that is formulable from the open formula 'Fab' involved in MacBride's main example (i.e., which he writes as '$a\Phi b$'). As an instance of (HOλC), we have '$[\lambda F\, Fab]F \equiv Fab$'. This is a principled statement of the application conditions of the higher-order property $[\lambda F\, Fab]$.

One reason MacBride doesn't consider (HOλC) is that he doesn't distinguish the open formula 'Fab' from the closed predicate '$[\lambda F Fab]$'. But another reason he doesn't consider it is his requirement that we have to understand the open formula 'Fab' independently of how any relation satisfying it is specified. He says:

> …higher-order predicates of the form '$a\Phi b$' are still required for the intelligibility of quantification into the positions of converse predi-

> cates, i.e. higher-order predicates capable of being true or false of a relation belonging to the domain independently of how that relation is specified. (forthcoming, 15)

So it may be that he wouldn't accept (HOλC) as the necessary principle because it is a schema whose non-schematic instances, such as '$[\lambda F\, Fab]R \equiv Rab$', all require that we instantiate the principle to some predicate such as 'R'. Thus, the worry becomes that we don't understand how the open formula 'Fab' is true of a relation independently of how that relation is specified by a predicate.

But this wouldn't be a good reason for thinking that (HOλC) isn't a principle that yields an intelligble understanding of 'Fab'. For the instance '$[\lambda F\, Fab]F \equiv Fab$' of (HO$\lambda$C) directly governs the open formula 'Fab'. No 1st-order predicate constant is being used in this instance and so no 1st-order relation has been specified by this instance. The two free occurrences of 'F' in this instance refer to an arbitrary relation (i.e., whatever is assigned to the free variable 'F'), independent of how that relation is specified ('F' is a variable, after all). Moreover, as we saw earlier, the universally quantified formula (9), i.e., $\forall F([\lambda F\, Fab]F \equiv Fab)$, is an immediate consequent of (HOλC). It quantifies over every entity in the domain of the quantifier '$\forall F$', independently of how those entities are specified. So (HOλC) is just the right principle to explain the higher-order property that MacBride says must be in play in our understanding of the open formula 'Fab'.

Nor can one reject (HOλC) on the grounds that it is trivial. MacBride might argue that (HOλC) trivially recasts the open formula as a higher-order predicate and so doesn't help us understand 'Fab' or the higher-order property in question. But neither (λC) in 2OL nor (HOλC) in 3OL are trivial. (λC) in 2OL is a significant principle that is an integral part of the λ-calculus of relations and thus one of the key axioms for axiomatizing relations (Zalta 1983 (69), Menzel 1986 (38), Zalta 1993 (406), and Menzel 1993 (84)). It is stronger than **CP** (it implies **CP**, as we've seen, but **CP** doesn't imply it), and it is not plausible to suggest that **CP** is a trivial principle. (HOλC) has a similar significance in 3OL.

So by systematizing the distinction between an open formula such as 'Fab' and the higher-order predicate '$[\lambda F\, Fab]$' and connecting them via (HOλC), we forestall MacBride's conclusion that we lack a principled understanding of the application conditions of 'Fab'. None of his arguments undermine this answer, since they don't consider (HOλC).[20]

Nevertheless, one might ask whether the specific arguments that MacBride presents, to reach the conclusion that we lack a principled understanding of the application conditions of 'Fab', are cogent. We've read the corresponding predicate '$[\lambda F\, Fab]$' as "being an F such that a bears F to b", but let's grant that it can be read as "being an F such that F applies to a first and b second".[21] Under this reading, (HOλC) remains true. MacBride then considers symmetric and non-symmetric relational statements and, in each case, finds reasons to question the understanding of 'Fab' in terms of ordinal notions. For example, with respect to the symmetric relation *differs from*, he argues that 'Darius differs from Alexander' and 'Alexander differs from Darius' intuitively say the same thing, but that the understanding of the open formula 'Fda' and 'Fad' implies that they say different things. He argues:

[20]There is another way to forestall MacBride's conclusion, without appealing to 3OL, namely by developing a precise semantics for the (open) formulas of 2OL that is grounded in a theory of relations and states of affairs. For example, the language in Zalta 1983 provides truth conditions, relative to an assignment to the variables, for the open formula 'Fab'. These are stated in terms of the relation that serves as the denotation of 'F' relative to a variable assignment (the denotation of 'F' relative to a variable assignment f is just the entity assigned to 'F' by f). This semantics is grounded in the theory of relations that is expressible in the extended 2OL formalism developed in Zalta 1983. We'll discuss this theory later in the paper.

[21]Strictly speaking, it doesn't matter which of a and b that F 'applies to' first and second; it is not order of application but the position in the relation F that a and b have to occupy. So it might be better to read $[\lambda F\, Fab]$ as "being an F such that a occupies the first position and b the second position of F". Similarly, the open formula 'Fab' indicates, for an arbitrary relation F, that a occupies the first position and b the second position of F. At least this much ordinal talk is needed to make sense of a symmetric relation as any F such that $\forall x \forall y(Fxy \rightarrow Fyx)$. The ordinal concepts first, second, third, etc., are built into the primitive notion of a relation; they aren't listed among the primitives of the predicate calculus. Moreover, it doesn't follow from the fact that atomic formulas have the general form $F^n x_1 \ldots x_n$ that the numeral 'n' that serves as a superscript in 'F^n' and as a subscript in 'x_n' is a variable that can be bound by a quantifier. The superscript 'n' on 'F' can be inferred from the list of arguments to 'F' in the formula and the subscripts '1',…,'n' on the xs simply provide a way to have distinct variables. The concept *number* isn't a primitive of the predicate calculus.

So the expressions denoting relations involve at least some implicit notion of order, if only so that we can say that the way in which 'a', 'b', and 'c' are arranged in '$Fabc$' is different from the way they are arranged in '$Fbac$'. After all, to say "a, b, and c exemplify F" is different from saying that "b, a, and c exemplify F" and if the best explanation of this difference is that ordinal notions are implicit, then the theory of relations does indeed implicitly assume ordinal notions. But, in any case, we're granting that some notion of order is required.

> If a higher-order predicate of the form '$a\Phi b$' expresses the higher-order property that a relation has if it applies to a first and b second, then what a statement of the form $\exists\Phi\, a\Phi b$ says is that some relation has that property. . . . it follows . . . that atomic statements in which symmetric predicates occur attribute to symmetric relations the property of applying to the things they relate in an order. But it is far from plausible that they do. Consider, for example,
>
> [8] Darius differs from Alexander.
>
> [9] Alexander differs from Darius.
>
> If predicates of the form '$a\Phi b$' mean what they're proposed to mean then [8] says that the relation picked out by 'ξ differs from ζ' applies to Darius first and Alexander second, whereas [9] says that it applies to Alexander first and Darius second. But, as both linguists and philosophers have reflected, *prima facie* statements like [8] and [9] don't say different things but are distinguished solely by the linguistic arrangements of their terms. (forthcoming, 18–19)

Although MacBride cites a number of authorities for his last claim, he also mentions that Russell (1903, §94) argued against it and for the view that statements like [8] and [9] express distinct propositions.

Clearly, the crux of MacBride's argument in the above passage is the claim that [8] and [9] don't say different things. But surely there is a sense of 'says' on which [8] and [9] do say different things. If we ignore the particular symmetric relation involved and consider a non-symmetric relation, then to say 'John loves Mary' is not to say 'Mary loves John'. So MacBride's argument turns on a *semantic* notion of 'says' on which [8] and [9] say the same thing. That is, MacBride's argument turns on whether [8] and [9] denote the same state of affairs (or proposition). He is convinced that they do, whereas I think this isn't at all clear. The point at issue concerns the identity of states of affairs and if one allows, for example, that necessarily equivalent states of affairs may be distinct, it is by no means a fact that [8] and [9] say the same thing.[22] Indeed, I hope to show in what follows that as long as we have a clear theory of

[22] I don't think MacBride here is claiming that the state of affairs Fda is identical to Fad on the grounds that they are necessarily equivalent. That is, he does not give the following argument. Define $Symmetric(F) =_{df} \forall x \forall y(Fxy \rightarrow Fyx)$. Now if we assume the necessity of identity, we would know not only that $\forall x \forall y(x = y \rightarrow \Box x = y)$, but also that $\forall x \forall y(x \neq y \rightarrow \Box x \neq y)$. So from [8] ($d \neq a$) and [9] ($a \neq d$) it would follow that $\Box d \neq a$ and $\Box a \neq d$, respectively. But $(\Box\varphi \,\&\, \Box\psi) \rightarrow \Box(\varphi \equiv \psi)$ and so it would follow that $\Box(d \neq a \equiv a \neq d)$.

relations and states of affairs (something that can be developed *without* the resources of 3OL), one can both (a) challenge the suggestion that [8] and [9] denote the same state of affairs *and* (b) argue that even if we leave the question open, we can still understand the application conditions of 'Fab' and conclude that '$\exists F(Fab)$' quantifies over relations.[23]

But before we turn to the theory of relations and states of affairs that support this position, the second puzzling conclusion mentioned at the outset of the paper becomes relevant. For the argument to this conclusion partly turns on the question of the identity of states of affairs.

4 The Second Puzzling Conclusion

Recall that the second puzzling conclusion mentioned at the outset occurs in MacBride 2014. In this paper, MacBride distinguishes three degrees of relatedness and says, where R^* signifies the converse of R, that "to embrace the second degree is to make the existential assumption that

So if one subscribes to the view that necessarily equivalent states of affairs are identical, it would follow that $d \neq a = a \neq d$, thereby identifying the two states of affairs in question. This argument would hold for any symmetric relation like *differs from* that holds necessarily whenever it holds.

But MacBride doesn't argue this way and in what follows, we shall not suppose that necessarily equivalent states are identical. There are well-known counterexamples to the proposal that necessarily equivalent relations, properties, and states of affairs are identical. In what follows, we take them to be hyperintensional entities, i.e., entities that may be distinct even if necessarily equivalent.

[23] It should be mentioned that I've passed over the fact that MacBride reads '$\exists F(Fab)$' as *saying* that some relation has "the higher-order property that a relation has when it applies to a first and b second" (p. 18). But '$\exists F(Fab)$' doesn't say this, not even semantically. The claim that MacBride attributes to '$\exists F(Fab)$' is representable in 3OL by the formula: $\exists G([\lambda F\, Fab]G)$. This does indeed say, given MacBride's hypothesis about the ordinal notions involved: some relation G exemplifies the property of being a relation F such that applies to a first and b second. The semantics of 2OL doesn't explicitly require quantification over *properties of relations* when it assigns truth conditions to '$\exists F(Fab)$' and so one can interpret this claim in 2OL without invoking properties of relations.

Moreover, I've passed over the fact [8] does *not* say, nor can one *derive* that it says "the non-symmetric relation picked out by 'ξ differs from ζ' applies to Darius first and Alexander second", as MacBride suggests. For one thing, [8] doesn't say anything about predicates picking out, or denoting, relations. Instead, [8] simply says Darius differs from Alexander (or, when regimented as $d \neq a$, [8] says 'd and a exemplify being non-identical'). Of course, when we regiment [8] in 3OL as '$d \neq a$', we can instantiate (9) to $\neq$ to obtain $[\lambda F\, Fda]{\neq} \equiv d \neq a$, and infer from this last fact and the representation of [8] that $[\lambda F\, Fda]{\neq}$, i.e., that the relation *differs from* exemplifies the higher-order property of being a relation that Darius bears to Alexander. And something similar applies MacBride's sentence [9].

every non-symmetric relation has a distinct converse ($R \neq R^*$)" (2014, 3). He then argues that relatedness in the second degree 'spells trouble' and has 'unwelcome consequences', namely, that it commits us to a "superfluity of converse relations and states" (2014, 4). Let's consider these claims in turn, i.e., by focusing first on the superfluity of relations.

Let me begin by noting that I don't think that the superfluity of converse relations is the main objection, for recall that the conclusion in MacBride 2014 (as we saw at the outset) is that we should take relations and relation application as primitive. Since these notions are primitive in 2OL, the conclusion MacBride draws in 2014 doesn't eliminate the multiplicity of relations. Since (1), when represented as (6), is a theorem of $2OL^=$, the multiplicity of converse relations arises even when relations and relation application are primitive, as soon as one asserts that non-symmetric relations exist. And this holds not only for 2-place non-symmetric relations but also non-symmetric relations of higher arity.[24] Though MacBride also suggests that we can't name the relations given such a multiplicity, in fact we can if we use λ-expressions.[25] In any case,

[24]To see that the generalization of (6) remains a theorem for relations of higher arity, let F be any n-place relation ($n \geq 3$) and let i and j be such that $1 \leq i < j \leq n$. Then we may define the i,j^{th}-converse of F, written $F^*_{i,j}$, as follows:

$$F^*_{i,j} =_{df} [\lambda x_1 \ldots x_i \ldots x_j \ldots x_n\, Fx_1 \ldots x_j \ldots x_i \ldots x_n] \tag{ϑ}$$

And we can define F is *non-symmetric with respect to its* i^{th} and j^{th} places:

$$\begin{aligned} &\mathit{Non\text{-}symmetric}_{i,j}(F) \equiv_{df} \neg\forall x_1 \ldots \forall x_i \ldots \forall x_j \ldots \forall x_n(Fx_1 \ldots x_i \ldots x_j \ldots x_n \rightarrow \\ &\qquad Fx_1 \ldots x_j \ldots x_i \ldots x_n) \end{aligned} \tag{ξ}$$

Then for any n-place relation F ($n \geq 3$) and i,j ($1 \leq i < j \leq n$), it is provable that:

$$\forall F(\mathit{Non\text{-}symmetric}_{i,j}(F) \rightarrow F \neq F^*_{i,j})$$

The proof is just a generalization of the one given for (8) and goes as follows. Fix n, i, and j. Assume $\mathit{Non\text{-}symmetric}_{i,j}(F)$. Then by ($\xi$) there are objects $x_1,\ldots,x_i,\ldots,x_j,\ldots,x_n$, say $a_1,\ldots,a_i,\ldots,a_j,\ldots,a_n$, such that $Fa_1 \ldots a_i \ldots a_j \ldots a_n$ and $\neg Fa_1 \ldots a_j \ldots a_i \ldots a_n$. Assume, for reductio, that $F = F^*_{i,j}$. Then it follows by the substitution of identicals that $F^*_{i,j}a_1 \ldots a_i \ldots a_j \ldots a_n$. So by definition ($\vartheta$), it follows that:

$$[\lambda x_1 \ldots x_i \ldots x_j \ldots x_n\, Fx_1 \ldots x_j \ldots x_i \ldots x_n]a_1 \ldots a_i \ldots a_j \ldots a_n$$

Hence, by (λC): $Fa_1 \ldots a_j \ldots a_i \ldots a_n$. Contradiction.

[25]MacBride says "Each ternary non-symmetric relation has five mutual converses, and we don't have names for any of them" (2014, 4). But if S is a 3-place non-symmetric relation, we can name its converses as follows: $[\lambda xyz\ Sxzy]$, $[\lambda xyz\ Syxz]$, $[\lambda xyz\ Syzx]$, $[\lambda xyz\ Szxy]$, and $[\lambda xyz\ Szyx]$. The first of these can be read: being objects x, y, and z such that x, z, and y exemplify S; the second as: being objects x, y, and z such that y, x, and z exemplify S; etc.

MacBride's argument that relations and relation application should be taken as primitive doesn't avoid the conclusion that there are a multiplicity of converse relations

So the real problem about the fact that non-symmetric relations have distinct converses concerns the 'profusion' of states of affairs. MacBride (2014, 4) rehearses this problem by considering *on* and *under*, both of which are asymmetric (and hence non-symmetric if there are objects that stand in those relations):

> It's one kind of undertaking to put the cat on the mat, something else to put the mat under the cat, but however we go about it we end up with the same state. To bring the cat to the forefront of our audience's attention we describe this state by saying that the cat is on the mat; to bring the mat into the conversational foreground we say that the mat is under the cat. But whether it's the cat we mention first, or the mat, what we succeed in describing is the very same cat-mat orientation. That's intuitive but if—as the second degree describes—a non-symmetric relation and its converse are distinct, we must be demanding something different from the world, a different state, when we describe the application of the above relation to the cat and the mat from when we describe the application of the below relation to the mat and the cat.

The worry is that converse relations commit us to the principle that if R is non-symmetric, then for any x and y, the state of affairs Rxy is distinct from the state of affairs R^*yx. We can formally represent the allegedly problematic principle as follows:

$$\forall F\Box(\mathit{Non\text{-}symmetric}(F) \rightarrow \forall x\forall y(Fxy \neq F^*yx)) \tag{13}$$

This, it is claimed, is counterintuitive and support is adduced in Fine 2000.[26] If this is the concern, why not adopt the following principle instead:

[26]In Fine (2000, 3), we find:

> What makes this consequence so objectionable, from a metaphysical standpoint, is a certain view of how relations are implicated in states or facts. Suppose that a given block a is on top of another block b. Then there is a certain state of affairs s_1, that we may describe as the state of a's being on top of b. There is also a certain state of affairs s_2 that may be described as the state of b's being beneath a. Yet surely the states s_1 and s_2 are the same. There is a single state of affairs s "out there" in reality, consisting of the blocks a and b having the relative positions that they do; and the different descriptions as-

- For any 2-place relation F, necessarily, if F is non-symmetric, then for any x and y, the state of affairs x *bears* F *to* y is identical to the state of affairs y *bears* F^* *to* x, i.e.,

$$\forall F \Box(Non\text{-}symmetric(F) \rightarrow \forall x \forall y(Fxy = F^*yx)) \tag{14}$$

The answer MacBride gives is (2014, 4):

> We might attempt to defend the second degree by maintaining that the application of R and R^* does not give rise to different states with respect to the same relata but different decompositions of the same state. So whilst *above* and *below* are distinct, the relational configuration *cat-above-mat* is a decomposition of the same state as the configuration *mat-below-cat*. But these decompositions comprise what are ultimately different constituents—a non-symmetric relation and its converse are supposed to be distinct existences. But now we have the difficulty of explaining how such different decompositions can give rise to a *single* state.

So, again, the problem being raised is about the identity of states of affairs. In these cases, MacBride is confident that there is a single state involved.

Note that we've now connected up the issue on which MacBride's forthcoming paper turns with the main problem that he raises in his 2014 paper, namely, the identity of states of affairs. What gives rise to this problem is that the 2OL and 2OL$^=$ don't have the resources to supply a good definition of the conditions under which states of affairs are identical, even if we add modality to the logic. For neither of the following definitions are good ones:

$$p = q \equiv_{df} p \equiv q$$

$$p = q \equiv_{df} \Box(p \equiv q)$$

It is reasonable to suppose that the state of affairs *there is a barber who shaves all and only those who don't shave themselves* ($\exists x(Bx \,\&\, \forall y(Sxy \equiv \neg Syy))$) is distinct from the state of affairs *there is a brown and colorless dog* ($\exists x(Dx \,\&\, Bx \,\&\, \neg Cx)$), yet these are not just equivalent but necessarily equivalent (since both are necessarily false).

sociated with s_1 and s_2 would merely appear to provide two different ways at getting at this single state of affairs.

So whereas both of the above definitions might be used to explain why $Fxy = F^*yx$ (e.g., "they are identical because necessarily equivalent"), the definitions fail when states of affairs (or propositions) are regarded as hyperintensional entities. The identity conditions for states of affairs are more fine-grained than material or necessary equivalence. Furthermore, when F is non-symmetric, there is no obvious way to account for the identity of Fab and F^*ba by appealing to some notion of 'constituents'. On what grounds, expressible in 2OL, would one claim that the distinct constituents F, F^*, a and b can be combined so that the identity $Fab = F^*ba$ holds?[27] And how can one state hyperintensional identity conditions for state of affairs that also allow us to assert, in the case of a non-symmetric relation F, that $Fab = F^*ba$?

MacBride, as noted at the outset, finalizes this problem for any analysis of the identity (or non-identity) of states of affairs as a *dilemma*. We provided an edited down version of the argument, to give the reader the general idea. But the passage posing the dilemma goes as follows, in full:

> What vexes the understanding is the difficulty of disentangling one degree of relatedness from another when we try to provide an analysis of the fundamental fact that $aRb \neq bRa$ for non-symmetric R. We can usefully distinguish, albeit in a rough and ready sense, between two analytic strategies for explaining this fundamental fact—that the world exhibits relatedness in the first degree. Intrinsic analyses aim to account for the fact that $aRb \neq bRa$ by appealing to features of those states themselves; extrinsic analyses attempt to account for their difference by appealing to features that aren't wholly local to them. Anyone who wishes to give an analysis of the fact that $aRb \neq bRa$ faces a dilemma. If they adopt the intrinsic strategy then they will find it difficult to avoid a commitment to either R's converse or an inherent order in which R applies to the things it relates. Alternatively our would-be analyst can avoid entangling the first degree with the second and third by adopting the extrinsic strategy. But this approach embroils us in other unwelcome consequences.

[27] You can't assert the principle $Fxy = Gzw \equiv (F = G \,\&\, x = z \,\&\, y = w)$, for the scenario in which cat-on-mat (Ocm) and mat-under-cat (O^*mc) are identical constitutes a counterexample. For the principle would imply the instance $Ocm = O^*mc \equiv (O = O^* \,\&\, c = m \,\&\, m = c)$. And from the fact that $O \neq O^*$, or the fact that $c \neq m$, it would follow that $Ocm \neq O^*mc$. So this is no help, since we're trying to explain how we can have, simultaneously, $O \neq O^*$ and $c \neq m$ and yet $Ocm = O^*mc$.

> Since neither intrinsic nor extrinsic analyses are satisfactory, this recommends our taking the fact that $aRb \neq bRa$ to be primitive.
>
> (2014, 8)

In the remainder of the paper, I show how object theory (OT) takes n-place relations as primitive (including states of affairs, understood as 0-place relations), takes relation application (predication) as primitive, but defines identity for relations and states of affairs. These identity conditions don't appeal to 'decompositions' or 'constituents'. Nevertheless, they allow one to *consistently* assert that (some) necessarily equivalent relations and states may be distinct. Using this theory of relations and states, we can address the 'profusion of states' problem (in MacBride 2014) in either of two ways, and address the problem underlying first puzzling conclusion (in MacBride forthcoming) as well. As we shall see, a precise theory of relations and state may leave certain identity questions open, just as the precise theory of sets ZFC leaves open certain identity questions. The solution in ZFC is not to conclude that its quantifiers can't range over sets, but to find and justify axioms that help decide the open questions within the precise, but extendable, framework ZFC provides (i.e., one which clearly quantifies over sets). Something similar happens in OT.

5 The Theory of Relations and States of Affairs

This section can be skipped by those familiar with OT, since the material contained herein has been outlined and explained in a number of publications over the past 30+ years (e.g., 1983, 1988, 1993, Bueno, Menzel, & Zalta 2014, or Menzel & Zalta 2014, and others). For those completely unfamiliar with it, OT may be sketched briefly by saying that it extends 2OL, not $2OL^{=}$, since identity isn't taken as a primtive. Instead, OT adds new atomic formulas of the form 'xF', which represent a new mode of predication that can be read as "x *encodes* F", where 'F' can be replaced by any predicate or 1-place relation term. OT also includes a necessity operator ($\Box$) and uses a primitive 1-place relation term '$E!$' for *being concrete*, and defines *ordinary* objects ('$O!x$') as objects x that might be concrete, and defines *abstract* objects ('$A!x$') as objects x that couldn't be concrete. It is axiomatic that ordinary objects necessarily fail to encode properties ($O!x \rightarrow \Box\neg\exists F xF$), though the theory allows that abstract ob-

jects can both exemplify and encode properties. It is also axiomatic that if x encodes a property, it necessarily does so ($xF \rightarrow \Box xF$).

But the key principle for abstract objects is the comprehension schema that asserts, for any condition (formula) φ in which x doesn't occur free, that there exists an abstract object that encodes all and only the properties such that φ:

$$\exists x(A!x \,\&\, \forall F(xF \equiv \varphi)) \tag{15}$$

Here are some instances, expressed in technical English. There exists an abstract object that encodes all and only the properties that y exemplifies (let φ be 'Fy', where y is any object). There exists an abstract object that encodes just the property G, for any property G (let φ be '$F = G$'). There is an abstract object that encodes all the properties necessarily implied by G (let φ be '$\Box\forall x(Gx \rightarrow Fx)$'). There is an an abstract object that encodes all and only the propositional properties constructed out of true propositions (let φ be '$\exists p(p \,\&\, F = [\lambda x\, p])$'). And so on. Intuitively, for any group of properties you can specify to describe an abstract object, there is an abstract object that encodes just those properties and no others.

The other principles of this theory that will play an important role in what follows are the definitions of identity for individuals and the principles (existence and identity conditions) for relations. First, the theory of identity for individuals includes a definition stipulating that x and y are identical if and only if they are ordinary objects that necessarily exemplify the same properties or they are both abstract objects that necessarily encode the same properties:

$$x = y \equiv_{df} \\ (O!x \,\&\, O!y \,\&\, \Box\forall F(Fx \equiv Fy)) \vee (A!x \,\&\, A!y \,\&\, \Box\forall F(xF \equiv yF)) \tag{16}$$

Second, the theory of relations consists of *existence* and *identity* conditions for relations. The existence conditions are *derived*, since OT includes the resources of the relational λ-calculus; λ-expressions of the form $[\lambda x_1 \ldots x_n\, \varphi]$ are well-formed, but only if φ doesn't have any encoding subformulas.[28] So (λC), as stated above, is the main axiom gov-

[28] In the latest version of OT, currently under development (Zalta m.s.), every formula φ becomes a permissible matrix of a λ-expression, but not every λ-expression has a denotation. Those in which the λ doesn't bind a variable in an encoding formula subterm of φ are guaranteed to denote relations. So in this new version of the theory, λ-expressions are governed by a free logic. But for this paper, the published versions of the theory suffice; the logic of well-formed λ-expressions is classical.

erning λ-expressions. One can derive from (λC) a *modal* version of **CP**. This theorem schema, $\Box$**CP**, asserts existence conditions for relations as follows:[29]

Modal Comprehension for Relations ($\Box$**CP**)
$\exists F^n \Box \forall x_1 \ldots \forall x_n (F^n x_1 \ldots x_n \equiv \varphi)$, provided F doesn't occur free in φ and φ doesn't contain any encoding subformulas.

When $n = 1$ and $n = 0$, respectively, this principle asserts existence conditions for properties and states of affairs:

$\exists F \Box \forall x (Fx \equiv \varphi)$, provided F doesn't occur free in φ and φ doesn't contain any encoding subformulas.

$\exists p \Box (p \equiv \varphi)$, provided p doesn't occur free in φ and φ doesn't contain any encoding subformulas.

In other words, any formula free of encoding conditions can be used to produce a well-formed instance of $\Box$**CP**. It is of some interest that there are still very small models of OT; for example, the smallest model involves one possible world, one ordinary object, two 0-place relations, two 1-place relations, two 2-place relations, etc., and four abstract objects. Though this isn't a model of theory when it is applied, it does show that without further axioms, the theory doesn't commit one to much. Thus, relations, properties, and states of affairs exist under conditions analogous to those in classical, modal 2OL.[30]

The identity conditions for relations are stated by cases: (a) for properties F and G, (b) for n-place relations F and G ($n \geq 2$), and (c) for states of affairs p and q. Identity for relations and states of affairs is defined in terms of identity for properties. The definitions are as follows:

- Properties F and G are identical if and only F and G are necessarily encoded by the same objects, i.e.,

$$F = G \equiv_{df} \Box \forall x (xF \equiv xG) \tag{17}$$

[29]The proof of this principle from (λC) is analogous to the proof in footnote 15, except that you use the Rule of Necessitation after universally generalizing on $x_1, \ldots, x_n$, and just before existentially generalizing on the λ-expression.

[30]Again, in the latest version of OT, under development in Zalta m.s., the 'no encoding subformulas' restriction on comprehension for states of affairs is eliminated. Every formula φ in which p doesn't occur free can be used as a matrix to assert the existence of a state of affairs.

- n-place relations F and G ($n \geq 2$) are identical just in case, for any $n-1$ objects, every way of applying F and G to those $n-1$ objects results in identical properties, i.e.,

$$\begin{aligned} F = G \equiv_{df}\ & \forall y_1 \ldots \forall y_{n-1} ([\lambda x\, Fxy_1 \ldots y_{n-1}] = [\lambda x\, Gxy_1 \ldots y_{n-1}]\ \& \\ & [\lambda x\, Fy_1 xy_2 \ldots y_{n-1}] = [\lambda x\, Gy_1 xy_2 \ldots y_{n-1}]\ \& \ldots \& \\ & [\lambda x\, Fy_1 \ldots y_{n-1} x] = [\lambda x\, Gy_1 \ldots y_{n-1} x]) \end{aligned} \tag{18}$$

- States of affairs p and q are identical whenever (the property) *being an individual z such that* p is identical to (the property) *being an individual z such that* q, i.e.,

$$p = q \equiv_{df} [\lambda z\, p] = [\lambda z\, q] \tag{19}$$

From these definitions, it can be shown that the reflexivity of identity holds universally, i.e., that $x = x$ is derivable from (16), that $F = F$ is derivable from each of (17) and (18), and that $p = p$ is derivable from (19). So OT asserts only the substitution of identicals as an axiom governing identity. It therefore has all the theorems about identity that are derivable in $2OL^=$. Identity is provably symmetric, transitive, etc., and since every term of the theory is interpreted rigidly, substitution of identicals holds in any (modal) context whatsoever.

The foregoing facts make it clear that (8) is a theorem of OT, by the same reasoning used in the proofs given earlier in the paper. So as soon as one adds the hypothesis that a particular 2-place relation, say R, is non-symmetric, OT also implies that $R^* \neq R$. And so on for 3-place relations. The multiplicity of relations is just a fact about both 2OL and OT when extended with the claim that non-symmetric relations exist. So taking relations and relation application as primitive, as MacBride argues and concludes in his 2014 paper, doesn't provide immunity to the multiplicity of relations.[31] This multiplicity isn't egregious, in any case, for as we've seen, λ-expressions give us the expressive power to distinguish among the converses of non-symmetric relations. So let's return to the questions about the identity of states of affairs, to see how they fare with a precise theory of relations and states of affairs in hand.

[31]See the passages in MacBride 2014, where he says "We simply have to accept as primitive, in the sense that it cannot be further explained, the fact that one thing bears a relation to another" (2), "...we should just take the difference between *aRb* and *bRa* as primitive" (14), and "The difficulties that result from attempting to analyse the first degree suggest that that the operation of relational application should itself be taken as primitive" (15).

6 Asserting the Identity of States

Recall that the puzzling conclusion reached in MacBride's forthcoming paper turned on the question of whether the states of affairs denoted by [8] and [9] are the same or distinct. This question can now be posed without discussing the converses of relations and without invoking 3OL. Let R be *any* symmetric relation, and let a and b be two particular and distinct objects. Then consider the states of affairs Rab and Rba (or if you prefer $[\lambda\, Rab]$ and $[\lambda\, Rba]$). MacBride has no doubt they are the same state. So let's suppose they are, i.e., that $Rab = Rba$. What happens to his argument that if we understand 'Fab' in terms of ordinalized, higher-order properties, then 'Rab' and 'Rba' don't express the same state of affairs? Answer: it has no force against the theory of states of affairs in OT. For in OT, all that is relevant to the truth of '$Rab = Rba$' is principle (19), i.e., the question of whether the properties $[\lambda z\, Rab]$ and $[\lambda z\, Rba]$ are identical, i.e., by (17), whether there might be objects that encode $[\lambda z Rab]$ without encoding $[\lambda z Rba]$. In particular, OT allows one to assert that when R is symmetric, $[\lambda z\, Rab]$ and $[\lambda z\, Rba]$ are identical, i.e., that no abstract object encodes $[\lambda z\, Rab]$ without also encoding $[\lambda z\, Rba]$, and vice versa.

Does this mean we don't understand the open formula 'Fab' or the quantified claim '$\exists F(Fab)$'? Not at all. First, the semantics of OT is perfectly precise on this score. Let '$\boldsymbol{a}$' and '$\boldsymbol{b}$' be the semantic names of the objects assigned to 'a' and 'b'. Now consider some assignment f to the variables and suppose that '$\boldsymbol{R}$' is the semantic name of the relation assigned to the variable 'F' by f. Then the open formula 'Fab' is true relative to f if and only if the state of affairs $\boldsymbol{Rab}$ obtains.[32] And '$\exists F(Fab)$' is true just in case some relation in the domain satisfies the open formula 'Fab', no matter how that relation is specified. Second, OT is intelligible without its formal semantics, just as ZF is intelligible simply given the language and axioms alone. Thus, the identity conditions for states of affairs are understandable as expressed in (19). For the axioms and theorems of OT give us an understanding of the open formula 'xF', and in turn, give us an understanding of the defined condition $Rab = Rba$. To suggest otherwise would be like suggesting that we don't understand '$x \in y$'. This is a primitive of set theory and the more we understand the consequences of the axioms (i.e., the more theorems we prove in set theory), the better we understand '$x \in y$'. The same holds for OT.

[32]OT does have a formal semantics which primarily serves the purpose of showing that it is consistent and so makes use of set theory. In the formal semantics, 'Fab' is true relative to f if and only if the ordered pair $\langle \boldsymbol{a}, \boldsymbol{b} \langle$ is in the exemplification extension of the relation $\boldsymbol{R}$. And this latter holds if and only if the extension of the 0-place relation $\boldsymbol{Rab}$ is The True. But these semantic conditions only give us a set-theoretic representation of the truth conditions; they are not a substitute for the metaphysics of relation application and states of affairs.

So if one is inclined to accept MacBride's view that the states of affairs expressed by [8] and [9] are identical, one should then be inclined to accept the following general principle:

$$\forall F \Box (Symmetric(F) \rightarrow \forall x \forall y (Fxy = Fyx)) \tag{20}$$

(20) is consistent with OT. So we need not conclude that the open formula 'Fab' is unintelligble or that the second-order quantifiers don't range over relations. Instead, we make use of a theory of relations and states of affairs in which relation application is primitive but identity is defined. And we address the problem by asserting a principle, not concluding the language is unintelligible; indeed, it seems to be the principle that MacBride is relying upon to make his case.

This generalizes to non-symmetric relations. For recall the objection to (14), which is the claim:

$$\forall F \Box (Non\text{-}symmetric(F) \rightarrow \forall x \forall y (Fxy = F^{*}yx)) \tag{14}$$

The problem with (14) was, according to MacBride, to explain how different decompositions can give rise to the same state (2014, 4, quoted above). But no such explanation is needed, since the identity of states of affairs is not a matter of decompositions and constituents. If F is non-symmetric, then the above principle implies, by definition (19), that $[\lambda z\, Fxy] = [\lambda z\, F^{*}yx]$, for any objects x and y. That is consistent with OT.

Why does this address the difficulty in MacBride 2014 (p. 4)? The answer: because we're not attempting to *explain* how 'distinct existences' (i.e., a non-symmetric relation F, its converse F^{*}, and objects x and y) can 'give rise' to the same state; we're adopting a principle (indeed, a principle on which MacBride relies) that asserts that they do, without appealing to 'decompositions', 'constituents', etc. The definitions of identity for abstract objects (16) and for properties (17) place reciprocal bounds on the existence of these entities. The theory's comprehension principle and identity conditions for abstract objects tell us that *any* (expressible) condition on properties can be used to define an abstract object. If we think of abstract objects as objects of thought or as logical objects, then the theory implies that if properties F and G are distinct, then there is a logical, abstract object of thought that encodes F and not G (and vice versa). And if F and G are identical, then no logical, abstract object of thought encodes F without encoding G. So if the properties $[\lambda z\, Fxy]$ and $[\lambda z\, F^{*}yx]$ are identical, then no logical, abstract object of thought encodes the one without encoding the other.[33]

So, if one adopts (14), one can *use* OT's theory of identity for states of affairs to give a precise, theoretical answer to a philosophical question ("Under what conditions are states of affairs identical?") which, if left unanswered, would otherwise leave one open to MacBride's concerns about the intelligibility of 2OL.[34]

Before we turn, finally, to the intuition that states of affairs like those expressed by [8] and [9] are distinct, there is one final way to generalize the concern that MacBride has raised, given his understanding of the identity of states of affairs. We can formulate the concern as follows. Consider *any* relation F, whether symmetric or non-symmetric, and let a and b be any two particular objects. Then consider the property $[\lambda x Fxb]$, i.e., being an object that bears F to b. Now predicate that property of a, to obtain the state of affairs $[\lambda x\, Fxb]a$, i.e., a exemplifies the property of bearing F to b. Put this aside for the moment and now consider the property $[\lambda x\, Fax]$, i.e., being an object to which a bears F. Now predicate that property of b, to obtain the state of affairs $[\lambda x\, Fax]b$, i.e., b exemplifies

[33]One practical consequence of this identification is this: it prevents one from telling a consistent story about a fictional object, say c, in which Fxy is true in the story but $F^{*}yx$ is not, for some relation F and objects x and y. For example, if you believe *cat-on-mat* is identical *mat-under-cat*, then you can't tell a story in which one is true and the other is not, or describe a fictional object such that the one is true while the other is not. I'm not ruling out stories where some fictional character *believes* that Rab and doesn't believe that $R^{*}ba$, for in that case, we're not talking about the states denoted by 'Rab' and '$R^{*}ba$', but about the senses of these expressions. And OT represents these as *abstract* states of affairs, which requires the typed-version of OT. See Zalta 2020.

[34]This answer, if adopted, would put to rest another of MacBride's concerns, namely, that endorsing distinct converses for non-symmetric relations requires a commitment to a 'substantive linguistic doctrine', namely, that when we switch from the active 'Antony loves Cleopatra' to the passive 'Cleopatra is loved by Antony', we "introduce a novel subject matter" (MacBride 2014, 5). But our solution allows one to agree with MacBride that if the subject matter is defined by the state of affairs being referenced, there is no change – one can move from 'Antony loves Cleopatra' to 'Cleopatra is loved by Antony' without changing the subject matter, since those sentences designate, on this view, the same state of affairs.

the property of being an object to which a bears F. Now, we might ask:

> What is the relationship between the states of affairs Fab, $[\lambda x Fxb]a$, and $[\lambda x\, Fax]b$ — are they all the same or are they all pairwise distinct? (A)

If you accept MacBride's view about the identity of states of affairs, then you would answer (A) by adopting the following principles:

$$Fab = [\lambda x\, Fxb]a \tag{21}$$

$$[\lambda x\, Fxb]a = [\lambda x\, Fax]b \tag{22}$$

From these principles it easily follows by the transitivity of identity that $Fab = [\lambda x Fax]b$. I'm not *advocating* this answer to (A) because we haven't considered any contexts where one might wish to distinguish these states of affairs. If, for example, you think there are contexts (e.g., hyperintensional ones) which requires $Fab \neq [\lambda x\, Fxb]a$, then if you adopt (21), you won't be able to account for the hyperintensionality.

But the general point is clear. Some precise, axiomatized theories leave open certain questions of identity and those questions can be answered by looking for principles rather than questioning whether the quantifiers of the theory range over the entities being axiomatized. ZFC has precise identity conditions for sets but leaves open the Continuum Hypothesis ('CH'), and yet we can still interpret the quantifiers in set theory as ranging over sets. CH can be formulated as the claim $2^{\aleph_0} = \aleph_1$, and though CH and its negation are consistent with ZFC, we don't give up the interpretation of the quantifiers of ZFC as ranging over sets just because CH is an open question; instead, we look for axioms that will help decide the issue. The same applies to the theory of relations.[35]

As it turns out, there is an alternative way to respond to the problems MacBride has raised. It may be of interest to some readers to consider what happens to his arguments if one instead asserts that $Fxy \neq Fyx$ when F is symmetric, or accepts that $Fxy \neq F^{*}yx$ when F is non-symmetric, or generally accepts that $Fxy \neq [\lambda z\, Fxz]y \neq [\lambda z\, Fzy]x$. In the final section, then, I show that, with OT's theory of states of affairs:

- one may also consistently assert these non-identities,

[35]I'm indebted to Daniel Kirchner, who was able to use his implementation of OT in Isabelle/HOL (Kirchner 2017 [2021]) to confirm the consistency of separately adding (14), (20), and the generalization of (21), i.e., $\forall F \forall x \forall y(Fxy = [\lambda z\, Fxz]y)$, to OT.

- one can account for the intuition that there is *one* part of the world underlying these distinct states and, consequently,
- one can disarm the worry about a 'profusion' of states of affairs and completely clear the path for understanding the quantifiers of 2OL as quantifying over relations.

7 Distinct States, One Situation

What is driving MacBride's certainty that (a) $Fxy = Fyx$ when F is symmetric, (b) $Fxy = F^*yx$ when F is non-symmetric, and (c) $Fxy = [\lambda z Fxz]y = [\lambda z\, Fzy]x$ generally? The argument is most clearly stated for the case of non-symmetric relations, where he argues that if non-symmetric relations have distinct converses, then we end up with 'a profusion of states of affairs'. We laid out the argument in Section 4, in the quote from 2014 (p. 4), about there being one state of affairs (i.e., one cat-mat orientation) despite there being two kinds of undertakings (putting the cat on the mat and putting the mat under the cat). Since to undertake to do something is to attempt to bring about a state of affairs, one might then conclude that there are two distinct undertakings precisely because there are two distinct states of affairs to be brought about. But, as we saw earlier, MacBride and Fine both conclude that there is only one state and that to claim otherwise is counterintuitive. And we saw that the concern is that converse relations commit us to the principle that if F is non-symmetric, then the state of affairs Fxy is distinct from the state of affairs F^*yx. We have formally represented the principle that concerns them as follows:

$$\forall F\Box(\mathit{Non\text{-}symmetric}(F) \rightarrow \forall x\forall y(Fxy \neq F^*yx)) \tag{13}$$

But notice that the cases MacBride (and Fine) discuss involve necessarily non-symmetric relations, such as *on*, *on top of*, *above*), etc. So when we instantiate the above to a necessarily non-symmetric relation, say R, it would follow by the K axiom of modal logic that $\Box\forall x\forall y(Rxy \neq R^*yx)$. But of course, we can also infer, from the fact that (λC) is a universal, necessary truth, that $\Box\forall x\forall y(Rxy \equiv R^*yx)$.[36] So we can generalize to conclude that whenever we assert that R is a necessarily non-symmetric relation, (λC) and (13) combine to ensure that Rxy and R^*yx are necessarily equivalent but distinct states of affairs, for any values of the variables x and y.

The real problem is now laid bare: it is thought that the hyperintensionality of states of affairs doesn't account for the intuition that in these cases, there is only one piece of the world (e.g., one *cat-mat* orientation) that accounts for the truth of the relational claims 'Rxy' and 'R^*yx'. Note that this same problem arises for the other cases we're considering. I take it MacBride would similarly be concerned about the following principle regarding *symmetric* relations:

$$\forall F\Box(\mathit{Symmetric}(F) \rightarrow \forall x\forall y(Fxy \neq Fyx)) \tag{23}$$

And the concern extends generally to principles such as the following, which would govern every 2-place relation:

$$\forall F\Box\forall x\forall y(Fxy \neq [\lambda z\, Fzy]x) \tag{24}$$

$$\forall F\Box\forall x\forall y([\lambda z\, Fzy]x \neq [\lambda z\, Fxz]y) \tag{25}$$

In each case, a complaint about a profusion of states of affairs will arise, for it can be shown (a) that (λC) and (23) imply that for any necessarily symmetric relation R, Rxy and Ryx are necessarily equivalent but distinct,[37] and (b) that (λC), (24), and (25) imply that for any relation R, the states Rxy, $[\lambda z\, Rxz]y$, and $[\lambda z\, Rzy]x$ are all pairwise necessarily equivalent but all pairwise distinct.[38]

So if one accepts these principles, can we account for the intuition that there is only one state of affairs in play? To answer this question, we shall not invoke 'decompositions' and 'constituents', for the identity for states of affairs is given by (19). But we *can* address the intuition driving MacBride, Fine, and no doubt others, by appealing to the notion of a *situation* and defining the conditions under which a state of affairs p *obtains in* a situation s (i.e., the conditions under which s *makes* p *true*). Once these notions are defined, we can identify, for any state of affairs p,

[36] As an instance of (λC), we know $[\lambda xy\, Ryx]xy \equiv Ryx$. So by definition (7), $R^*xy \equiv Ryx$, which by the commutativity of the biconditional implies $Ryx \equiv R^*xy$. So by applying, in order, the Rule of Generalization (2×) and the Rule of Necessitation, we obtain $\Box\forall y\forall x(Ryx \equiv R^*xy)$. But $\Box\forall x\forall y(Rxy \equiv R^*yx)$ is an alphabetic variant.

[37] Suppose $\Box \mathit{Symmetric}(R)$. Then by the definition of a symmetric relation, both $\Box\forall x\forall y(Rxy \rightarrow Ryx)$ and $\Box\forall x\forall y(Ryx \rightarrow Rxy)$, where the latter follows by universal quantifier commutativity and substitution from $\Box\forall y\forall x(Ryx \rightarrow Rxy)$, which is an alphabetic variant of the former. So $\Box\forall x\forall y(Rxy \equiv Ryx)$. But by (23) and the K axiom, $\Box\forall x\forall y(Rxy \neq Ryx)$. So again we have that Rxy and Ryx are necessarily equivalent, but distinct.

[38] These three states of affairs are all necessarily equivalent, by (λC) and the Rule of Necessitation, but they are pairwise distinct by (24) and (25).

a canonical situation s in which obtain all and only the states of affairs necessarily equivalent to p. Then it will follow, for example, that the canonical situation in which obtain the states necessarily equivalent to Rab is identical to the canonical situation in which obtain the states necessarily equivalent to R^*ba. And a similar result follows for states arising from necessarily symmetric relations and for the states Rab, $[\lambda x\, Rxb]a$, and $[\lambda x\, Rax]b$. As I develop this response, I'll use R as an arbitrary 2-place relation, which is necessarily non-symmetric, symmetric, or unspecified, as the case may be.

In OT (Zalta 1993, 410), situations are defined as abstract objects that encode only properties constructed out of states of affairs, i.e., encode only properties F of the form $[\lambda z\, p]$, where p ranges over states of affairs:

$$Situation(x) =_{df} A!x \;\&\; \forall F(xF \rightarrow \exists p(F = [\lambda z\, p])) \tag{26}$$

A situation, thus defined, is not a mere mereological sum because encoding is a mode of predication and so a situation is *characterized* by the state-of-affairs properties of the form $[\lambda z\, p]$ that it encodes. In addition (1993, 411), a state of affairs p *obtains in* a situation s ('$s \models p$') just in case s encodes *being a z such that p*:

$$s \models p =_{df} s[\lambda z\, p] \tag{27}$$

In what follows, therefore, we sometimes extend the notion of encoding by saying that s encodes a state of affairs p, or that s *makes p true*, whenever p obtains in s. That is, when $s \models p$, then instead of always saying that s encodes $[\lambda z\, p]$, we may say instead that s encodes p or s makes p true.

Now consider some state of affairs, say Rab. Given the foregoing definitions, OT implies that there exists a situation s such that a state of affairs p obtains in s if and only if p is necessarily equivalent to Rab. To see this, note that the comprehension principle for abstract objects asserts that there is an abstract object that encodes exactly those properties F such that F is a property of the form $[\lambda z\, p]$ when p is some state of affairs necessarily equivalent to Rab:

$$\exists x(A!x \;\&\; \forall F(xF \equiv \exists p(\Box(Rab \equiv p) \;\&\; F = [\lambda z\, p]))) \tag{28}$$

Let s_1 be such an object, so that we know:

$$A!s_1 \;\&\; \forall F(s_1F \equiv \exists p(\Box(Rab \equiv p) \;\&\; F = [\lambda z\, p])) \tag{29}$$

Since s_1 is abstract and every property it encodes is a property of the form $[\lambda z\, p]$, it follows that s_1 is a situation, by definition (26). Moreover, the theory implies that s_1 is unique, i.e., that any abstract object that encodes all and only those states of affairs necessarily equivalent to Rab is identical to s_1. Since situations are abstract objects, they are identical whenever they encode the same properties.[39] And since situations encode only properties F such that $\exists p(F = [\lambda z p])$, by (26), they obey the principle: s and s' are identical just in case the same states of affairs obtain in s and s' (Zalta 1993, 412, Theorem 2). So there can't be two distinct abstract objects that encode all and only the states of affairs necessarily equivalent to Rab. Since (28) has a unique witness, we may treat s_1 as a name of this witness (introduced by definition) and treat (29) as a fact about s_1 implied by the definition.

Two modal facts about s_1 become immediately relevant:

- A state of affairs obtains in s_1 if and only if it is necessarily equivalent to Rab, i.e.,

$$\forall p(s_1 \models p \equiv \Box(Rab \equiv p)). \tag{30}$$

- s_1 is *modally closed* in the following sense: for any states of affairs p and q, if p obtains in s_1 and p necessarily implies q, then q obtains in s_1., i.e.,

$$\forall p \forall q((s_1 \models p) \;\&\; \Box(p \rightarrow q) \rightarrow (s_1 \models q)). \tag{31}$$

The proof of (30) is straightforward and, interestingly, relies on the object-theoretic definition for the identity for states of affairs (19).[40] Note that it immediately follows from (30) that Rab obtains in s_1, since $\Box(Rab \equiv Rab)$

[39]Strictly speaking, the definition of identity (16) implies that abstract objects x and y are identical if and only if *necessarily*, they encode the same properties. But since $xF \rightarrow \Box xF$ is an axiom of OT, it follows that if x and y encode the same properties, they necessarily encode the same properties, and so it is sufficient to show $\forall F(xF \equiv yF)$ to establish that $x = y$, for abstract x and y.

[40]Let q_1 be an arbitrary state of affairs. Then for the left-to-right direction, assume $s_1 \models q_1$, to show $\Box(Rab \equiv q_1)$. Then by definition of *obtains in* (27), $s_1[\lambda z\, q_1]$. So by facts about s_1 (29), it follows that $\exists p(\Box(Rab \equiv p) \;\&\; [\lambda z\, q_1] = [\lambda z\, p])$. Let q_2 be such a state of affairs, so that we know $\Box(Rab \equiv q_2) \;\&\; [\lambda z\, q_1] = [\lambda z\, q_2]$. By the definition of identity for states of affairs (19), the second conjunct implies $q_1 = q_2$. But then, substituting identicals into the first conjunct, we obtain $\Box(Rab \equiv q_1)$.

For the right-to-left direction, assume $\Box(Rab \equiv q_1)$. But by the reflexivity of identity, $[\lambda z\, q_1] = [\lambda z\, q_1]$. Hence $\Box(Rab \equiv q_1) \;\&\; [\lambda z\, q_1] = [\lambda z\, q_1]$. So $\exists p(\Box(Rab \equiv p) \;\&\; [\lambda z\, q_1] = [\lambda z\, p])$. Then by facts about s_1 (29), $s_1[\lambda z\, q_1]$, and by definition of *obtains in* (27), $s_1 \models q_1$.

is an instance of the modal principle $\forall p\Box(p \equiv p)$. The proof of (31) relies on both the definition of identity for states of affairs and the symmetry and transitivity of necessary equivalence, i.e., the facts that:

- $\forall p\forall q(\Box(p \equiv q) \rightarrow \Box(q \equiv p))$
- $\forall p\forall q\forall r(\Box(p \equiv q) \,\&\, \Box(q \equiv r) \rightarrow \Box(p \equiv r))$

The proof is left to a footnote.[41]

It is an immediate consequence of (30) that:

- if R is necessarily non-symmetric, then R^*ba obtains in s_1, for it is necessarily equivalent to Rab,
- if R is necessarily symmetric, then Rba obtains in s_1, for it is necessarily equivalent to Rab, and
- if R is any 2-place relation whatsoever, then $[\lambda x Rxb]a$ and $[\lambda x Rax]b$ both obtain in s_1, since these are necessarily equivalent to Rab.

Moreover, when R is necessarily non-symmetric, it follows that neither Rba nor R^*ab obtain in s_1, since neither is necessarily equivalent to Rab in that case.

It is interesting to observe that in each of the above scenarios, any one of the necessarily equivalent states of affairs in question can be used to define the unique situation in which they all obtain. The resulting situations become identified, since it is a theorem of modal logic that necessarily equivalent states of affairs are necessarily equivalent to the same states of affairs:

$$\forall p\forall q(\Box(p \equiv q) \rightarrow \forall r(\Box(p \equiv r) \equiv \Box(q \equiv r))) \tag{32}$$

To see why this fact helps us to show that the resulting situations are all identified, consider the case of necessarily non-symmetric R and consider the situation that can be introduced in a manner similar to s_1 but with R^*ba instead of Rab:

[41] Let p_1 and q_1 be arbitrary states of affairs, and assume $s_1 \models p_1$ and $\Box(p_1 \equiv q_1)$, to show $s_1 \models q$. The first implies $s_1[\lambda z\, p_1]$, by (27). So $\exists p(\Box(Rab \equiv p) \,\&\, [\lambda z\, p_1] = [\lambda z\, p])$, by the second conjunct of (29). Suppose r_1 is an arbitrary such state of affairs, so that we know $\Box(Rab \equiv r_1)\&[\lambda z p_1] = [\lambda z r_1]$. The second conjunct implies, by the identity of states of affairs (19), that $p_1 = r_1$. Hence $\Box(Rab \equiv p_1)$. But by the symmetry and transitivity of necessary equivalent, it now follows that $\Box(Rab \rightarrow q_1)$. Hence $\Box(Rab \rightarrow q_1) \,\&\, [\lambda z\, q_1] = [\lambda z\, q_1]$, by reflexivity of identity and conjunction introduction. So $\exists p(\Box(Rab \rightarrow p) \,\&\, [\lambda z\, q_1] = [\lambda z\, p])$. But by the second conjunct of (29), it follows that $s_1[\lambda z\, q_1]$. So $s_1 \models q_1$, by definition of *obtains in* (27).

$$\exists x(A!x \,\&\, \forall F(xF \equiv \exists p(\Box(R^*ba \equiv p) \,\&\, F = [\lambda z\, p])))$$

This is the (provably unique) situation that makes true all and only the states of affairs necessarily equivalent to R^*ba. Call this s_2. Clearly, facts analogous to (30) and (31) holds for s_2: a state of affairs p obtains in s_2 if and only if R^*ba is necessarily equivalent to p, and s_2 is modally closed.

But OT implies that $s_1 = s_2$.[42] Moreover, the reasoning in the proof applies to all the other canonical situations definable in terms of the necessarily equivalent states of affairs mentioned above: these canonical situations are pairwise identical. Thus, in each example, there is a single canonical situation in which all of the states of affairs mentioned in the example obtain.

Moreover, to account for the intuition that the situation in which the necessarily equivalent states obtain is *part of* the actual world, we turn to the principles (theorems and definitions) governing *part of*, *actual situations*, and *possible worlds*. Since x is a part of y is defined as $\forall F(xF \rightarrow yF)$, it follows that a situation s is *part of* a situation s' ($s \trianglelefteq s'$) just in case every state of affairs that obtains in s also obtains in s' (Zalta 1993, 412, Theorem 4). Moreover, an *actual situation* is a situation s such that every state of affairs that obtains in s obtains *simpliciter* (1993, 413). And a *possible world* is a situation s that might be such that it makes true all and only the truths (1993, 414). Formally:

$$s \trianglelefteq s' \equiv \forall p(s \models p \rightarrow s' \models p)$$

[42] *Proof*. To show $s_1 = s_2$, it suffices to show that they encode the same properties, for as we noted earlier in footnote 39, the object-theoretic principle $xF \rightarrow \Box xF$ implies that if s_1 and s_2 encode the same properties, then necessarily they encode the same properties. To show s_1 and s_2 encode the same properties, we show, for an arbitrarily chosen property, say P, that $s_1P \equiv s_2P$. Without loss of generality, we show only $s_1P \rightarrow s_2P$, since the proof of the converse is analogous. So, assume s_1P. Then, by definition of s_1,

$$\exists p(\Box(Rab \equiv p) \,\&\, P = [\lambda y\, p])$$

Let q_1 be such a state of affairs, so that we know $\Box(Rab \equiv q_1)$ and $P = [\lambda y q_1]$. Now earlier we saw that when R is necessarily non-symmetric, $\Box(Rxy \equiv R^*yx)$. Hence $\Box(R^*ba \equiv Rab)$. So by an appropriate instance of (32), it follows that $\forall r(\Box(R^*ba \equiv r) \equiv \Box(Rab \equiv r))$. Instantiating this last result to q_1, it follows that $\Box(R^*ba \equiv q_1) \equiv \Box(Rab \equiv q_1)$. But we already know $\Box(Rab \equiv q_1)$. Hence $\Box(R^*ba \equiv q_1)$. So we have established:

$$\Box(R^*ba \equiv q_1) \,\&\, P = [\lambda y\, q_1]$$

By existential generalization:

$$\exists p(\Box(R^*ba \equiv p) \,\&\, P = [\lambda y\, p])$$

But then, by definition of s_2, it follows that s_2P.

$Actual(s) =_{df} \forall p(s \models p \rightarrow p)$

$PossibleWorld(s) =_{df} \Diamond\forall p(s \models p \equiv p)$

OT then yields, as theorems (1993, Theorem 18 and 19):

There is a unique actual world, i.e.,
$\exists!s(PossibleWorld(s) \,\&\, Actual(s))$ ('w_α')

Every actual situation is a *part of* the actual world, i.e.,
$\forall s(Actual(s) \rightarrow s \trianglelefteq w_\alpha)$

The proof of the first theorem rests on the fact that there is a unique situation that encodes all and only the states of affairs that obtain, i.e., there is a unique situation s such that all and only the states that obtain in s are states that obtain *simpliciter*.[43]

So the canonical situations that exist in each of the examples validate the following claims:

- when R is necessarily non-symmetric and Rab obtains, there is a unique situation that (a) encodes all and only the states of affairs necessarily equivalent to Rab, (b) is actual, (c) is a part of the actual world, and (d) makes both Rab and R^*ba true.

- when R is necessarily symmetric and Rab obtains, there is a unique situation that (a) encodes all and only the states of affairs necessarily equivalent to Rab, (b) is actual, (c) is a part of the actual world, and (d) makes both Rab and Rba true.

- when R is any 2-place relation and Rab obtains, there is a unique situation that (a) encodes all and only the states of affairs necessarily equivalent to Rab, (b) is actual, (c) is a part of the world, and (d) makes Rab, $[\lambda x\, Rxb]a$, and $[\lambda x\, Rax]b$ true.

This addresses the intuition that served as the obstacle to treating states of affairs as hyperintensional entities. It lays to rest the claim that we don't understand the open formula 'Fab' and the claim that we can't interpret the quantifier in '$\exists F(Fab)$' as ranging over relations.

The foregoing analysis therefore preserves the conclusion that Russell develops concerning non-symmetric relations, when he says (1903, §219) regarding the terms *greater* and *less*:

> These two words have certainly each a meaning, even when no terms are mentioned as related by them. And they certainly have different meanings, and are certainly relations. Hence if we are to hold that "a is greater than b" and "b is less than a" are the same proposition, we shall have to maintain that both greater and less enter into each of these propositions, which seems obviously false.

One might reframe Russell's point by noting that if non-synonymous relational expressions signify or denote different relations, then the simple statements we can make using those expressions signify different states of affairs. That principle has been preserved, without sacrificing any contrary intuitions. MacBride says (in discussing this passage) "But Russell also felt the deep intuitive force of another answer: that we are saying just the same whether we employ 'a is greater than b' or 'b is less than a' to do so" (2012, 157). But I take it that the foregoing analysis accommodates the intuitive force of this other answer.[44]

8 Conclusion

OT is expressed in an extension of 2OL and offers a precise theory of relations and states of affairs that offers both existence and identity conditions for these entities. The primitives of OT are thus axiomatized, and the more one works through the theorems the better one has a grasp of

[43] The proof goes by way of an instance of comprehension that asserts:

$\exists x(A!x \,\&\, \forall F(xF \equiv \exists p(p \,\&\, F = [\lambda y\, p])))$,

One can then prove that any such object, call it a, is a possible world, is actual (i.e., every state of affairs that *obtains in a* obtains *simpliciter*), and that any other situation that is a possible world and actual is identical to a. So one can then legitimately introduce the name w_α in terms of the description: *the* actual world.

[44] This analysis of possible worlds, as abstract objects that encode states of affairs, avoids the objection that arises for those who (a) take possible worlds to be maximal and possible states of affairs and (b) say that q obtains at p if p necessarily implies q. On this view, one can't maintain that there is a unique actual world and that states are hyperintensional. For suppose that p_1 is a world, i.e., a state of affairs that is possible and such that for every q, either p_1 necessarily implies q or p_1 necessarily implies $\neg q$. And suppose p_1 is an actual world (i.e., p is a possible world that obtains). Then $p_1 \,\&\, (r \vee \neg r)$ will also be an actual world, since the same states are necessarily implied by p_1 and $p_1 \,\&\, (r \vee \neg r)$. But on a hyperintensional view of states, p_1 and $p_1 \,\&\, (r \vee \neg r)$ would be distinct actual worlds. See Zalta 1988 (73–74), Zalta 1993 (393–394), and also McNamara (1993), who makes a similar point. Such a consequence doesn't affect the present view.

what these primitives mean. Moreover, though sets and set membership aren't primitive in OT, once can, for the purposes of showing that OT is consistent, use set theory to formulate an intelligible formal semantics of OT. So OT is at least one natural formalism for quantifying over relations and states of affairs. It is simply a non-starter to suggest that OT doesn't quantify over relations. And since OT is an extension of 2OL, its theory of relations provides us with a means of precisely understanding the open and quantified formulas of 2OL. The suggestion that the quantifiers of 2OL *can't* be interpreted as ranging over relations fails to engage with at least one theory that shows that they do.

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