In Defense of the Simplest Quantified Modal Logic*

Bernard Linsky
Department of Philosophy
University of Alberta

and

Edward N. Zalta
Department of Philosophy
Stanford University

By combining the laws of classical quantification theory with the modal propositional logic K in the most direct manner, one produces the simplest Quantified Modal Logic. The models of this simple QML relativize predication to possible worlds and interpret the quantifiers as ranging over a single, fixed domain. But simple QML has many controversial features, not the least of which are that it validates the Barcan formula and appears to require quantification over possibilia. Whereas possibilists employ distinctions that render these features of simple QML unobjectionable,[1] actualists find the distinctions and the controversial features

*This paper was published in *Philosophical Perspectives 8: Logic and Language*, J. Tomberlin (ed.), Atascadero: Ridgeview Press, 1994, pp. 431–458. The authors would like to acknowledge support from the Social Sciences and Humanities Research Council of Canada (SSHRC Grant #410-91-1817), and generous support from the Center for the Study of Language and Information (CSLI) at Stanford University. We would also like to thank Harry Deutsch, Greg Fitch, Chris Menzel, and Nathan Tawil for giving us insightful comments on an earlier draft of this paper.

[1]We are describing here those possibilists who accept that predication is relativized to worlds. David Lewis [1986], however, is a possibilist who would reject simple QML because he can not make sense of world-relativized predication. Our remarks in what follow therefore concern those philosophers who find world-relativized predication un-

difficult to accept. Many thought that Kripke-models, with their varying domains and restricted quantifiers, promised to rid QML of the Barcan formula, quantification over possibilia, and other objectionable features we haven't yet described. Unfortunately, Kripke-models themselves have features at which actualists balk, and so these philosophers have had to modify (our understanding of) Kripke-models to find an acceptable QML.

Though we are possibilists, we understand the suspicion with which actualists regard simple QML and Kripke-models. But in their attempt to modify Kripke-models to produce an acceptable QML, actualists employ sophisticated maneuvers and machinery. To us, this seems analogous to introducing epicycles into an already flawed geocentric astronomy. For we have reexamined the simplest QML and have discovered that it has the following, unheralded virtue: in addition to having an interpretation compatible with possibilism, it has an interpretation compatible with actualism! In this paper, we concentrate our energies on motivating and describing the interpretation of simple QML that is compatible with actualism, since the possibilist interpretations of simple QML are well known. We argue that actualists need not use nor reconceive the varying domains of Kripke-models to avoid the objectionable features of simple QML. Our actualist interpretation does not require a commitment to nonexistent or nonactual objects, nor does it render the Barcan formula objectionable. Moreover, our interpretation is compatible with 'serious' actualism because it does not require that objects have properties at worlds where they don't exist. The existence of both possibilist and actualist interpretations shows that simple QML, to a larger extent than heretofore suspected, is independent of certain metaphysical views.

Our new interpretation has been overlooked because of a pervasive, yet mistaken view of the distinction between abstract and concrete objects. The abstract/concrete distinction is mistakenly seen as an absolute difference in the nature of objects. Thus, abstract objects are thought to be essentially abstract, and concreteness is thought to be part of the nature of concrete objects, something they couldn't fail to have (whenever they exist). We question these ideas by motivating and introducing what might be called 'contingently nonconcrete objects'. Contingently nonconcrete objects exist and are actual, and they shall replace 'possibilia'. That is why our new actualist interpretation is not committed to nonexisting

derstandable. We have discussed Lewis' views at length in our paper Linsky and Zalta [1991].

or nonactual objects. Once the abstract/concrete distinction is properly conceived to make room for such objects, the other controversial theorems and features of QML should be quite acceptable to actualists. For example, the Barcan formula will have a reading that doesn't violate the principles of actualism. We argue that no technical complications are required to make the simplest QML philosophically satisfactory to an actualist. Furthermore, most actualists should find contingently nonconcrete entities acceptable, since in various ways, they already invoke objects that straddle the alleged categorial divide between the abstract and concrete.

Our conception of the distinction between the abstract and concrete is developed in §4, where we provide the details of our new actualistic interpretation of simple QML. Before we get to this, however, we proceed first to define simple QML in a precise manner. We do this in §1. Then, in §2, we describe the features of simple QML that have been so controversial. We conclude this section by noting how the varying domains of Kripke-models appear to eliminate the objectionable features of simple QML. In §3, we describe the features of Kripke models that still bother actualists and serious actualists. We examine how such philosophers as Salmon, Deutsch, Plantinga, Fine, and Menzel modify (our understanding of) Kripke-models to produce an acceptable QML. However, in each case, we conclude that the modifications themselves either (a) still have puzzling features, or (b) are incompatible with actualistic principles, or (c) needlessly complicate our conception of modality and modal logic, at least when compared with the (interpretation of) simple QML we offer in §4.

§1: A Sketch of the Simplest QML

The simplest QML is easy to describe. It involves a standard language having individual constants $a, b, \ldots$, individual variables $x, y, \ldots$, and predicates $P^n, Q^n, \ldots$, along with the usual atomic formulas and the usual molecular, quantified and modal formulas constructed out of $\neg$, $\rightarrow$, $\forall$, and $\Box$. Once the formulas are defined inductively in the standard way, $\exists$ and $\Diamond$ are given their usual definitions. The axiomatic basis of simple QML is formed by combining the axioms and rules of propositional logic, the modal axiom and rule that characterize the simplest modal logic K, and

the axioms and rules of classical quantification theory.[2] We'll use the following axioms and rules of propositional logic:

Axioms: Tautologies of propositional logic

Rule of Modus Ponens (MP): if $\vdash \varphi \rightarrow \psi$ and $\vdash \varphi$, then $\vdash \psi$

To form the propositional modal system K, we include the instances of the K axiom and the rule of necessitation:

K Axiom: $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$

Rule of Necessitation (RN): if $\vdash \varphi$, then $\vdash \Box\varphi$

To this basis, we add the following axioms and rules of Classical Quantification Theory (CQT):[3]

Axiom: $\forall x\varphi \rightarrow \varphi^{\tau}_{x}$, where τ is any term substitutable for x

Axiom: $\forall x(\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \forall x\psi)$, where x is not free in φ

Rule of Generalization (Gen): if $\vdash \varphi$, then $\vdash \forall x\varphi$

When identity is added to the system, we employ the following axioms:

$x = x$

$x = y \rightarrow (\varphi(x, x) \rightarrow \varphi(x, y))$, where $\varphi(x, y)$ is the result of substituting y for some, but not necessarily all, occurrences of x in $\varphi(x, x)$, provided that y is substitutable for x at those occurrences.

We complete our definition of the simplest QML with the addition of the Barcan Formula (Marcus [1946]):

(BF) $\forall x\Box\varphi \rightarrow \Box\forall x\varphi$.

[2]For philosophical applications of QML, one may prefer to use *S4* or *S5*. But K is simpler and is the simplest QML for which the problems about possibilia, the Barcan formula, etc., arise.

[3]We formulate classical quantification theory in the manner of Mendelson [1964], which uses the quantifier rule Gen (unlike Enderton [1972], which instead of a quantifier rule, adds extra axiom schemata to the basis and takes the universal closures of axiom-schemata as axioms). Note that we do not follow Mendelson's explicit identification of (three) basic axioms for propositional logic. Given the effective truth table method of determining whether a formula is a tautology, we prefer to follow the more recent practice of simply citing the tautologies and using the rule Modus Ponens as our propositional logic.

The reason for adding BF is that the simplest models of the above language validate this formula, yet it is not derivable from the basis we have so far. The simplest model $\mathbf{M}$ is $\langle \mathbf{W}, \mathbf{w}_\alpha, \mathbf{D}, \mathbf{R}, \mathbf{V} \rangle$, where $\mathbf{W}$ is a non-empty domain of possible worlds, $\mathbf{w}_\alpha$ is a distinguished member of $\mathbf{W}$ (the actual world), $\mathbf{D}$ is a non-empty domain of individuals, $\mathbf{R}$ is the standard accessibility relation on worlds, and $\mathbf{V}$ is a valuation function that: (a) assigns each constant to a member of the domain of objects, and (b) assigns each n-place predicate an 'intension' (i.e., a function from possible worlds to sets of n-tuples drawn from $\mathbf{D}$).[4] In the usual way, an assignment function $\mathbf{f}$ maps each variable to some member of the domain $\mathbf{D}$, and in terms of a given assignment, we define the denotation of term τ with respect to model $\mathbf{M}$ and assignment $\mathbf{f}$ (notated: $\mathbf{d}_{\mathbf{M},\mathbf{f}}$) as follows: if τ is a constant, say a, then $\mathbf{d}_{\mathbf{M},\mathbf{f}}(a)$ is $\mathbf{V}(a)$ and if τ is a variable, say x, then $\mathbf{d}_{\mathbf{M},\mathbf{f}}(x)$ is $\mathbf{f}(x)$. The model-relative definitions of satisfaction with respect to a world, truth at a world, and truth can be given in the usual way. These are as follows:

Satisfaction ($\mathbf{f}$ *satisfies*$_\mathbf{M}$ φ *with respect to* $\mathbf{w}$):

1. $\mathbf{f}$ satisfies$_\mathbf{M}$ $P^n\tau_1 \ldots \tau_n$ wrt $\mathbf{w}$ iff

   $$\langle \mathbf{d}_{\mathbf{M},\mathbf{f}}(\tau_1), \ldots, \mathbf{d}_{\mathbf{M},\mathbf{f}}(\tau_n) \rangle \in [\mathbf{V}(P^n)](\mathbf{w})$$

2. $\mathbf{f}$ satisfies$_\mathbf{M}$ $\neg\psi$ wrt $\mathbf{w}$ iff $\mathbf{f}$ fails to satisfy$_\mathbf{M}$ ψ wrt $\mathbf{w}$

3. $\mathbf{f}$ satisfies$_\mathbf{M}$ $\psi \rightarrow \chi$ wrt $\mathbf{w}$ iff

   either $\mathbf{f}$ fails to satisfy$_\mathbf{M}$ ψ wrt $\mathbf{w}$ or $\mathbf{f}$ satisfies$_\mathbf{M}$ χ wrt $\mathbf{w}$

4. $\mathbf{f}$ satisfies$_\mathbf{M}$ $\forall x\psi$ wrt $\mathbf{w}$ iff

   for every $\mathbf{f}'$, if $\mathbf{f}' \overset{x}{=} \mathbf{f}$, then $\mathbf{f}'$ satisfies$_\mathbf{M}$ ψ wrt $\mathbf{w}$

5. $\mathbf{f}$ satisfies$_\mathbf{M}$ $\Box\psi$ wrt $\mathbf{w}$ iff

   for every $\mathbf{w}'$, if $\mathbf{R}\mathbf{w}\mathbf{w}'$, then $\mathbf{f}$ satisfies$_\mathbf{M}$ ψ wrt $\mathbf{w}'$.

For identity, we add:

$$\mathbf{f} \text{ satisfies}_\mathbf{M}\ \tau = \tau' \text{ wrt } \mathbf{w} \text{ iff } \mathbf{d}_{\mathbf{M},\mathbf{f}}(\tau) = \mathbf{d}_{\mathbf{M},\mathbf{f}}(\tau')$$

[4] Throughout this paper, we use boldface to refer to items employed in the semantics of a quantified modal logic.

Truth at a World in a Model: φ is *true*$_\mathbf{M}$ *at* world $\mathbf{w}$ iff every assignment $\mathbf{f}$ satisfies$_\mathbf{M}$ φ with respect to $\mathbf{w}$.

Truth in a Model: φ is *true*$_\mathbf{M}$ iff φ is true$_\mathbf{M}$ at $\mathbf{w}_\alpha$.

By inspecting the quantifier and modal clauses for satisfaction, it should be clear that one may validly commute the universal quantifier and the box. So the Barcan formula is valid and must be added as an axiom.[5]

That is all there is to the simplest QML. Henceforth, we designate this system '*QML*'. An easy-to-remember formula that captures this system is: $QML = K + CQT + \text{BF}$. *QML* is complete with respect to the semantics outlined above. The completeness proof is not only straightforward but the simplest of all the quantified modal systems.[6] Note that *QML* is practically identical with Hughes and Cresswell's first formulation of quantified modal logic in [1968].[7] The only difference (other than the fact that we allow for identity) is that we base the logic on the K axiom rather than on the T axiom. It is also identical with Garsons' [1984] system *Q1*, which he takes as the starting point in his elaborate taxonomy of modal systems. *QML* has both *de re* and *de dicto* modal contexts—*de re* formulas involving quantification into modal contexts have perfectly well-defined truth conditions. The system suffers no modal collapse of the sort that worried Quine, nor do any interesting essentialist sentences appear as theorems.[8]

In addition to BF, there are two important theorems of *QML* that play a central role in what follows, namely:

(NE) $\forall x \Box \exists y\, y = x$

(CBF) $\Box \forall x \varphi \rightarrow \forall x \Box \varphi$

To see that NE is a theorem, note that $\exists y\, y = x$ is a theorem of CQT with identity. The latter is a theorem because in CQT, the domain $\mathbf{D}$ of objects is nonempty, and every assignment function to the variables gives

[5] Of course, if we add the power of the modal systems *B* or *S5* (by adding either the T and B axioms or the T and 5 axioms), then we can derive BF and so no longer need it as an axiom. See Hughes and Cresswell [1968], p. 145, and Prior [1956].

[6] Strictly speaking, the completeness proofs for *S5* under models without an accessibility relation are simpler. But this is just a special case, for models without an accessibility relation are not generalizable as models for the other modal systems.

[7] See pp. 141–169. This is what they call LPC + T + BF, which stands for Lower Predicate Calculus plus the T axiom plus the Barcan formula.

[8] See Parsons [1967] and [1969].

each variable an assignment into **D**. In *QML*, these remain as facts, and moreover, the denotation and assignment functions are *not* relativized to worlds.[9] RN therefore validly lets us infer $\Box\exists y\, y = x$ from this theorem of *CQT*. And NE is then derivable by a simple application of Gen to the free variable x. Moreover, an even stronger results falls out by another application of RN, namely:

($\Box$NE) $\Box\forall x\Box\exists y\, y = x$

Similarly, CBF ('Converse Barcan Formula') is derivable in a few simple steps.[10] This is just another instance of the semantic independence, and hence commutativity, of the universal quantifier and necessity operator.[11]

Possibilists can interpret BF, NE, and CBF in a way they see as unobjectionable. For example, some possibilists read the quantifier $\exists$ as 'there is', without an implication of *existence*. In other words, they interpret the quantifier $\exists$ as '*existentially unloaded*'.[12] To assert existence, they use an existence predicate ('$E!$') that is not defined in terms of $\exists$. Like other predicates, the extension of the existence predicate may vary from world to world. Consider the effect of distinguishing the quantifier and the existence predicate on the equivalent form of BF, which asserts: $\Diamond\exists x\varphi \rightarrow \exists x\Diamond\varphi$. If we take it as a perfectly acceptable modal intuition

[9]All terms are therefore (strongly) rigid.

[10]Following Kripke [1963], consider:

1) $\forall x\varphi \rightarrow \varphi$, by *CQT* axiom
2) $\Box(\forall x\varphi \rightarrow \varphi)$, by RN on (1)
3) $\Box\forall x\varphi \rightarrow \Box\varphi$, by MP on (2) and an instance of the K axiom
4) $\forall x[\Box\forall x\varphi \rightarrow \Box\varphi]$, by Gen on (3)
5) $\Box\forall x\varphi \rightarrow \forall x\Box\varphi$, by MP on (4) and an instance of a *CQT* axiom

[11]Given the derivability of CBF, there is an alternative way to derive NE and $\Box$NE (suggested in Deutsch [1990]):

1) $\forall x\exists y\, y = x$, by *CQT*
2) $\Box\forall x\exists y\, y = x$, by RN on (1)
3) $\Box\forall x\exists y\, y = x \rightarrow \forall x\Box\exists y\, y = x$ (instance of CBF)
4) $\forall x\Box\exists y\, y = x$, by MP on (2) and (3)
5) $\Box\forall x\Box\exists y\, y = x$, by RN on (4)

While our original proof of NE and $\Box$NE uses RN on open formulas, this proof seems to use it only on sentences. The antecedent of the special instance of CBF in this proof is just $\Box\forall x\exists y y = x$, which only requires one step of necessitation on a theorem of *CQT*, in other words, a sort of 'propositional' inference, involving no operations inside quantifiers or on open formulas. However, this proof appeals to the K axiom, since K is used to prove CBF.

[12]We have borrowed the phrase 'existentially loaded' from Lewis [1990].

that sisterless person b might have had a sister, BF yields only that *there is* an object that possibly is b's sister, not that such a possible sister *exists*. Similarly, NE asserts that, for any object x, necessarily there is such a thing as x, not that x necessarily exists. $\Box$NE is harmless for the same reason. So the NE theorems allow for 'contingent' objects. Finally, CBF implies, for a property P that necessarily everything has, that everything has P necessarily. This doesn't require that everything *exist* necessarily, for possibilists allow that objects can have properties in worlds where they don't exist (though they wouldn't allow that an object x has a property in a world where there is no such thing as x). Possibilists find these results unobjectionable because they believe they understand the difference between using an existentially unloaded quantifier and using an existence predicate, if only by interpreting the existence predicate as a primitive property.

Of course, there are other sorts of possibilists. For example, some possibilists accept that the quantifier $\exists$ is existentially loaded, but distinguish, among the objects that exist, the *actual* ones from the *possible* ones. With this distinction, these possibilists reason that from the fact that b might have had a sister, BF requires only that there exists a possible but not actual sister of b. Though NE asserts that everything exists necessarily, it doesn't assert that everything is actual necessarily. And whereas CBF permits objects to have properties in worlds where they aren't actual, it doesn't permit them to have properties in worlds where they don't exist. Possibilists taking this line find such results acceptable because they believe they can make sense of the distinction between existing actual objects and existing possible objects.[13]

§2: Actualist Objections to *QML*

For actualists and serious actualists, however, *QML* is not acceptable. Actualism is a program in logic and modal metaphysics that is based on the following thesis:[14]

[13]We again note that David Lewis [1986] doesn't advocate one of the above kinds of possibilism, for he would reject *QML* altogether, in part on the grounds that it involves world-relativized predication and doesn't have domains. For a discussion of world-relativized predication and its connection with possibilism, see Linsky [1991].

[14]This thesis has its source in the following works: Adams [1974], p. 202; Plantinga [1976], p. 257; Kaplan [1975], p.220; and Loux [1979], p. 48 (all of these page references are to the reprinted versions in Loux [1979]). We have followed the most recent

> Thesis of Actualism: Everything which exists (i.e., everything there is) is actual.

Note that there are really two parts to the actualist thesis: (1) treating the quantifier $\exists$ as existentially loaded, and (2) rejecting the hypothesis that there exist possible but nonactual objects. These two parts of the actualist thesis rule out, respectively, the two kinds of possibilism we just described. Actualists treat the quantifier as existentially loaded because they can make no sense of the distinction between 'there is' and 'there exists'.[15] Moreover, the very idea of a possible but nonactual object is metaphysically suspicious, and so they reject the hypothesis that there exist possible but nonactual objects to rid quantified modal logic of offensive ontological commitments. These two aspects of the actualist program are part of our legacy from Russell [1905] and Quine [1948], who have complained, respectively, about nonexistent objects and unactualized possibles. Actualists see themselves as realists in the anti-Meinongian tradition of denying that there are any special intentional objects. Thus actualists reject the distinctions or hypotheses that might otherwise render BF, NE, and CBF unobjectionable.

It is instructive and revealing to consider, however, just why these theorems violate the actualist thesis. From the fact that it is possible that b has a sister, BF requires that *there exists* something that is possibly b's sister. Since b has no sisters, which existing object is it that is possibly b's sister? Some actualists, notably Ruth Marcus [1986], might defend BF by pointing to an existing woman (possibly one closely related to b) and suggesting that she is the thing which both exists and which is possibly b's sister. But the great majority of actualists don't accept this idea, for they subscribe to certain essentialist views about the nature of objects. For example, they believe that women who aren't b's sister could not have been (in a metaphysical sense) b's sister. This is a fact about their very nature, one concerning their origins.[16] Since there seems to

formulation of the actualist thesis in Menzel [1990], p. 355.

[15] Indeed, they would just define the existence predicate in terms of the quantifier: $E!x =_{df} \exists y\, y = x$.

[16] These actualists would accept certain ideas in Kripke [1972] (pp. 110–15, 140–2). They do not deny that there may have been a situation in which b has for a sister someone who looks very much like this woman, or that there may have been a situation in which this woman has for a sibling someone who looks very much like b, for neither would have been a situation in which *this very woman* and *this very person* b were siblings. Given that b and this woman have a certain genetic makeup, they couldn't

be no *actually existing* thing which is possibly b's sister, they conclude BF is false. We think the essentialist intuitions leading to this conclusion are not unreasonable, and so understand why these actualists take BF to be false. Indeed, it seems that BF, in general, is incompatible with the intuition that there might have been something distinct from every actual thing. It is hard to see how that intuition could be compatible with a principle which seems to require that every possibility be grounded in something that exists. This is further evidence actualists have against the acceptability of BF. But since they still want to make sense of modal discourse in terms of possible world semantics, they reject the Barcan formula as having unacceptable consequences, and search for a modal semantics on which it is not valid.

Consider next NE ($\forall x \Box \exists y\, y = x$). For actualists, this explicitly says that for any object x, necessarily something exists that is identical with x. In other words, everything necessarily exists (hence our abbreviation 'NE').[17] This applies even to those objects not named by a constant of the language. But since *CQT* allows us to instantiate universal quantifiers to constants, NE semantically implies that the constants of our modal language couldn't be used to name contingently existing objects. And $\Box$NE seems to suggest that there couldn't have been contingent objects. All of these consequences run counter to our ordinary (modal) intuitions. Actualists see this as an additional and independent reason to abandon *QML*.

Finally, there is CBF. The main problem with CBF is that in *QML* it implies NE, not only directly (see footnote 11), but also in conjunction with *serious* actualism. Serious actualism is the thesis that it is not possible for an object to have a property without existing, i.e., the thesis that exemplification entails existence.[18] In semantic terms, this amounts to the constraint that an object in the extension of a property at a world must fall under the range of the quantifier at that world. Serious Actualism is often expressed by the following schema of the object language:

(SA) $\Box[\varphi(x) \rightarrow \exists y\, y = x]$, where φ is atomic and contains x free.

As such, SA is a simple thesis of *QML*. But from SA and CBF, one

possibly be siblings. At least, that is the essentialist intuition we are now describing.

[17] Prior in [1957] was especially concerned by this, pointing out that classical quantified modal logic was "haunted by the myth that whatever exists exists necessarily."

[18] See Plantinga [1983], [1985], Menzel [1991], Pollock [1985], and Deutsch [1994] for various discussions of serious actualism.

can rederive NE from any (logically) necessary property in the system (henceforth, we assume there are such). To see how, let property R be such that $\Box\forall x Rx$. R might be the property of *being self-identical*, for example. From this, CBF yields $\forall x \Box Rx$. But instantiate this to an arbitrary object, say a, to get $\Box Ra$. Then instantiate SA to a to get: $\Box(Ra \rightarrow \exists y\, y = a)$. Thus, by the K axiom, it follows that $\Box\exists y\, y = a$. And since a was arbitrary, it follows by CQT that NE (apply Gen).[19] Thus, even if there were a way to block the direct derivation of NE, the alternative derivations of NE from CBF (using SA as we just did, or without SA, as in footnote 11) show that serious actualists could not accept *QML* unless CBF is somehow invalidated.

Traditionally, actualists have had one other reservation about *QML* in addition to reservations about BF, NE, and CBF. It concerns the nature of the 'possible worlds' that are part of every model of *QML*. The same intuitions that give rise to actualist concerns about unactualized possibilia seem to apply to the very worlds of the models of *QML*. The proponents of any semantic theory are committed to the existence of whatever entities are appealed to in its account of truth. Since the semantics of *QML* appeals to possible but nonactual worlds, the proponents of such a semantics would appear to be committed to large-scale examples of problemmatic possible objects.

However, many actualists believe there is a way around this problem, by treating possible worlds as existing abstract objects. They accept worlds, but not as possibilia. Some take them to be maximal, consistent states of affairs, others as maximal, consistent properties or propositions, while still others treat them as maximal consistent sets of some sort.[20] Given some such treatment, the truth definition of *QML* commits one only to a domain of worlds conceived as abstract entities rather than as possibilia. For the remainder of this paper, we shall assume that some actualistic theory of worlds is viable, and as much as possible, concentrate our energies solely on the problems that arise in connection with BF, NE, and CBF.

To summarize, then, theorems of *QML* conflict with various actualist principles and intuitions. BF suggests that possibilia actually exist. NE suggests that there are no contingent objects. CBF forces one to confront the issue of whether a thing can have properties in worlds where it doesn't exist. Given serious actualism, CBF implies that there are no contingent beings.

[19] Even in a language without identity, in which *existence* is expressed by a predicate '$E!$' and serious actualism is expressed by the formula $\Box[\varphi(x) \rightarrow E!x]$, we would still get the result that $\forall x \Box E!x$.

[20] See Plantinga [1974] and [1976], Chisholm [1976], Fine [1977], Adams, [1974], van Inwagen [1986], or Zalta [1983] and [1988a].

It is no wonder, then, why Kripke-models appeal to actualists and serious actualists. Kripke's system in [1963] invalidates BF, NE, $\Box$NE, and CBF. We shall assume that the reader has some familiarity with how Kripke managed this. But we note that the basic insight underlying Kripke-models was to replace the single domain $\mathbf{D}$ in the models of *QML* with a function ψ that assigns a domain of objects to each world.[21] When quantifiers are evaluated at a world, they range only over the objects that exist in the domain of that world. Predicates are, notwithstanding this restriction, assigned extensions at each world from the set that includes all of the possible objects that exist in the domain of some world (so the logic is two-valued). Beyond these formal accomplishments, there was an insight in Kripke's 'variable domains' approach that had natural appeal to many philosophers. Given the Quinean notion that the objects over which a quantifier ranges, its domain, are those that exist, and given the very different intuition that existence is like a property and varies in extension from world to world, it is natural to think that contingent existence is to be represented by variable domains. At the same time, existence in the actual world is distinguished—that is the only genuine existence. Everything that exists, that is, everything in the domain of the actual world, is actual. But there might have been things that are distinct from everything in the actual world.

§3: The Problems with Kripke-models

Kripke-models, however, don't completely satisfy the typical actualist or serious actualist. Though BF, NE, and CBF are no longer valid, the techniques used to invalidate them introduce three problems:

[21] That is, this is the basic insight underlying Kripke's *quantified* modal logic. The basic insight underlying his *propositional* modal logic was the definition of necessary truth in terms of an accessibility relation. With the addition of the accessibility relation $\mathbf{Rww'}$, one could define: $\Box\varphi$ is true at $\mathbf{w}$ iff for every $\mathbf{w'}$, if $\mathbf{Rww'}$, then φ is true at $\mathbf{w'}$. This defines necessary truth at $\mathbf{w}$ in terms of a condition on $\mathbf{w}$. Previous definitions failed in this regard, and always validated the S4 and S5 axioms. So whereas the accessibility relation made it possible to investigate different sets of propositional modal axioms, the variable domains were the moving force behind Kripke's quantified modal logic.

(1) *The thesis of actualism fails for Kripke's metalanguage.* If a semantic theory is committed to whatever entities are required by its account of truth, then Kripke-models are committed to possibilia. The quantifiers of Kripke's metalanguage still range over possible objects. There are perfectly good Kripke-models in which there are objects in the domains of other worlds that are not in the domain of the actual world. From the point of view of the object language, such objects are 'mere possibilia', since the quantifiers of the object language, when evaluated at the actual world, don't range over them.[22] But from the point of view of the semantic metalanguage, these objects seem to be existing-but-unactualized possible objects. Since the distinction between actual and possible objects is required to make sense of the semantics, the thesis of actualism fails for Kripke's metalanguage.

(2) *One must either eliminate terms or abandon classical quantification theory.* Kripke invalidates NE and CBF by banishing constants from the language and free variables from assertable sentences, allowing only closed formulas to be axioms. He gives open formulas the 'generality interpretation', following Quine's [1940] development of mathematical logic. The absence of free variables blocks the direct proofs of NE and CBF.[23] But the constants have to go as well, for were Kripke to introduce them without adopting a free logic, there would be alternative derivations of NE and CBF.[24] So the problem of developing a modal logic which includes terms and which doesn't reintroduce the offending theorems still remains.

(3) *The thesis of serious actualism fails.* Not only does the semantic version of the thesis of serious actualism fail in Kripke's metalanguage, but the object language expression of the thesis, SA, is invalid. The definitions in Kripke [1963] allow objects to have properties at worlds where they don't exist. The function assigning predicates an extension at a world doesn't require that the objects in a predicate's extension at a world be a member of the domain of that world. Restricting this function so that only members of the domain at a world are found in the extension of properties at that world introduces all sorts of logical complications.

There is one other problem for Kripke-models which has affected the study of quantified modal logic, and that concerns the relationships between the objects in the various domains. It would seem that the same objects that appear in one domain may appear in others, but some philosophers think that there are severe metaphysical problems in having the same object in the domain of more than one world, while others think there are epistemological problems in re-identifying an object at various worlds. This has led some philosophers to think that the domains are all disjoint, and that objects in the various domains are at best counterparts of each other. But actualists typically take the 'haecceitist' approach on which it makes sense to suppose that the same object can appear in more than one world, if only as a matter of stipulation. For the purposes of this paper, we shall suppose that it does make sense for an object to appear in more than one world, and we will not pursue this problem for the variable domains of Kripke-models further.

Some actualist philosophers don't believe that there is a problem with Kripke-models. Salmon and Deutsch, for example, are concerned primarily with finding the best way to introduce constants and free variables into Kripke's language.[25] They solve problem (2) in different ways. Salmon [1987], possibly following Fine [1978], adopts a free logic. By adopting free logic, Salmon invalidates CBF and NE (since the instantiation and generalization rules require additional premises in order to be applied). But, the particular free logic that Salmon uses allows that sentences containing terms naming merely possible objects can be true at a world even though the object doesn't exist at that world. Deutsch [1990], on the other hand, avoids the use of free logic. He relativizes the interpretation of constants and variables to contexts of origin and worlds, using a special 3-place denotation function and an exemplification relation that is rela-

[22] Thus, Kripke's object language does conform to the thesis of actualism. The quantifier is existentially loaded and no distinction between possible and actual is made.

[23] And by blocking CBF, the less direct proofs of NE described in the text and in footnote 11 are also blocked.

[24] Consider the proof of CBF presented in footnote 10. If we could instantiate line (1) to a constant, and then generalize on that constant in the next to last step, the proof goes through. In the proof of NE discussed in footnote 11, we would get a similar result.

[25] Salmon and Deutsch may subscribe to the view that ontological commitment is determined solely by the range of the object language quantifiers. This might explain why they don't see (1) as a problem. We say more about their view on problem (3) below.

tivized to a pair of worlds rather than to a single world. For Deutsch, an atomic formula 'Fx' is true at a pair of worlds $\langle \mathbf{w}, \mathbf{w}' \rangle$ iff the assignment to x exists at the context of origin $\mathbf{w}$ and is in the extension of F at the world of evaluation $\mathbf{w}'$. Although x must exist in the context of origin $\mathbf{w}$, it need not be in the domain of $\mathbf{w}'$. The resulting logic invalidates the rule of necessitation RN. So, for example, $\exists y\, y = x$ is valid (and hence a theorem), but $\Box\exists y\, y = x$ is not. The loss of RN blocks the derivation of CBF and NE.

But there are still features of both systems that are problemmatic for actualists. The metalanguages of both logics quantify over possibilia. Like Kripke's logic, the possibilia constitute a kind of *Aussersein*, since they are there in the semantics, even though the object-language quantifiers can't reach them. The doctrine of '*Aussersein*' is often attributed to Meinong on the basis of his [1904] remark "there are objects of which it is true that there are no such objects." Meinong seemed to be committed to entities he would not quantify over, and this commitment reappears in both Salmon and Deutsch. To an actualist, Salmon may be worse off in this regard than Deutsch, since he allows us to *name* these objects.[26] Moreover, Salmon describes a metaphysics that is objectionable to serious actualists, since he allows objects to exemplify properties in worlds where they don't exist. Not only does Salmon think we can name possibilia, he thinks we can predicate properties of them as well. He explicitly rejects SA ([1987], p. 91). Deutsch also rejects SA but suggests that serious actualism can do without it. Since in his system objects have properties with respect to a *pair* of worlds, he argues that serious actualism requires only that objects exist in the first member of the pair (the context of origin) and not necessarily in the second. At least the schema $\varphi(x) \rightarrow \exists y\, y = x$ (for atomic φ) is valid in Deutsch's system, though given his weakened rule of necessitation, one cannot infer SA from this. Finally, whereas both Salmon and Deutsch can block the derivation of CBF and NE even with the presence of free variables and constants, they do so at significant cost. Salmon is forced to adopt the complications of free logic, while

[26]Merely possible objects don't exist, Salmon argues, they just might have. The fact that the semantics involves quantifiers ranging over 'possibilia' doesn't concern him, for he thinks a logic is committed only to those things which the object language quantifiers range over. Indeed, he uses 'Noman' as the name for his possible twin brother. But, he concludes, that doesn't mean that Noman exists. Thus, Noman doesn't exist but we can name 'him'.

Deutsch must abandon the rule of necessitation.[27] The resulting systems are complicated, respectively requiring the logic of nondenoting terms (Salmon) and the logic of contexts and doubly-indexed denotation and predication relations (Deutsch), all of which might be kept distinct, one would hope, from pure issues of modality.

Whereas Salmon and Deutsch are concerned more by the problem of introducing constants and free variables into Kripke's system, other actualists are more concerned by problems (1) and (3), namely, how to rectify the semantics so that it accords with the theses of actualism and serious actualism. Plantinga would suggest that Kripke-models be redefined and modified. He would: (a) substitute properties for possibilia, ridding the semantics of the latter, and (b) adjust the definitions to satisfy the constraints of serious actualism. Fine's approach is, by contrast, less direct. He would leave Kripke's definitions intact and instead translate Kripke's metalanguage into a more basic language in which modality was primitive, eliminating the violations of the actualist and serious actualist theses in the process. Menzel has elements of both Plantinga's approach and Fine's approach in his work, yet solves the problems of Kripke-models in a new and interesting way. We discuss these views in turn.

Plantinga [1974] would replace the objects in Kripke-models with their corresponding individual *essences*, regarding 'possibilia' simply as unexemplified essences.[28] Essences are properties that can be exemplified by only one particular individual, such as *being Reagan*, *being Socrates*, and *being that man*.[29] Since essences are properties, they are abstract and

[27]We are not suggesting here that there aren't other reasons that might lead one to adopt free logic or restrict RN. For example, one might want to adopt free logic to handle definite descriptions that fail to denote. And one might want to restrict RN to accomodate operators and terms (such as an actuality operator or rigid definite descriptions) that look back to the actual world for their evaluation (Zalta [1988b]). But though limited and judicious use of free logic and restrictions on RN may be justified in certain special cases, it does not follow that these techniques are justifiable in the present context. We are considering a language without (rigid) definite descriptions or actuality operators, and neither technique (free logic, restrictions on RN) solves all the actualist and serious actualist problems of Kripke-models. Our view is that these techniques are not justified in the present case, and even if they should prove to be useful for solving other problems when expanding the system, they should not be used to handle the problems we are presently discussing.

[28]Plantinga's suggestions are, to a large extent, captured formally in Jager [1982].

[29]An essence is a property E such that: (a) E is possibly exemplified, (b) for any x, if x exemplifies E, x necessarily exemplifies E, and (c) for any x, if x exemplifies E, then it is not possible that there be a y other than x such that y exemplifies E. Note

are thought to exist necessarily. Plantinga would redefine Kripke-models so that: (i) the domain **D** consists only of individual essences, (ii) the domain function ψ assigns to each world the set of essences that are exemplified there (so the contingent existence of objects is represented by the fact that essences are exemplified at some worlds and not at others), and (iii) the extension of a predicate 'P' at world **w** is the set of those essences which are in $\psi(\mathbf{w})$ and which are *coexemplified* with the property 'P' expresses.[30] Since these models appeal not to possibilia but to abstract, existing essences, the metalanguage requires no distinction between the possible and the actual. So the metalanguage of the redefined models conforms to the thesis of actualism. The redefined models also satisfy the constraints of serious actualism, since an essence must be 'in the domain' of **w** (i.e., be exemplified at **w**) to be coexemplified with a property at **w**.[31]

that the examples *being Reagan* and *being Socrates* satisfy clauses (b) and (c), unlike individual concepts such as *being the 40th U. S. President* and *being the most famous snub-nosed philosopher*.

[30]To see the effect of this redefinition, compare the truth conditions of the following three examples with those offered by standard Kripke semantics. For these examples, let **P** be the property expressed by 'P', and for simplicity, ignore the accessibility relation:

> '$\Diamond Pa$' is true iff there is some possible world **w** such that the individual essence denoted by 'a' is in the domain of **w** and is coexemplified at **w** with **P**.
>
> '$\Diamond \exists x Px$' is true iff there is both a world **w** and an essence **E** such that **E** is in the domain of **w** and **E** is coexemplified at **w** with **P**.
>
> '$\exists x \Diamond Px$' is true iff there is an essence in the domain of the actual world $\mathbf{w}_\alpha$ which at some world **w** is coexemplified with **P**.

Note that the truth conditions of $\Diamond\exists x Px$ do not entail the truth conditions of $\exists x \Diamond Px$. Thus, BF is invalid.

[31]Note a certain tension that arises concerning the fact that essences are abstract objects which necessarily exist and which therefore constitute a fixed domain that doesn't vary from world to world. If the quantifiers range over a fixed domain of essences, why do we need the variable domains and restricted quantifiers of Kripke-models? Why bother with a domain function that defines another sense for an essence to 'exist at' a world, especially given that the domain function ψ defines 'exists at' as 'is exemplified at' a world. This tension suggests that Plantinga's idea for rectifying Kripke-models by substituting essences for objects undermines the actualist criticisms of *QML*. For the same technique of substituting essences for objects would seem to offer an actualistic interpretation of *QML*. Here is how.

Just treat the domain **D** of *QML* as a domain of essences, and treat world-relativized predication as the world-relativized coexemplification of relations and ordered sets of essences. The resulting interpretation would render BF, NE, and CBF uncontroversial and undercut the need for Kripke models. Thus BF ($\Diamond\exists x Px \rightarrow \exists x \Diamond Px$) would simply assert (where **P** is the property expressed by 'P' and ignoring accessibility): if at some world **w**, there exists an essence **E** that is coexemplified at **w** with **P**, then there exists an essence **E** at the actual world $\mathbf{w}_\alpha$ which at some possible world **w** is coexemplified with **P**. This is uncontroversial if essences exist necessarily. NE would assert only that every essence exists necessarily. CBF ($\Box\forall x Px \rightarrow \forall x \Box Px$) would assert only that if at every world every essence **E** is coexemplified with **P**, then every essence **E** is coexemplified with **P** at every world.

However, it will be seen that this actualistic interpretation of *QML* is not a good one. It suffers from many of the problems that are described in the next paragraph.

Unfortunately, however, Plantinga's attempt to rectify Kripke-models by substituting essences for objects faces difficulties. The first is that it is unlikely that there are any unexemplified essences, yet there must be such to play the role of possibilia in Kripke-models. Adams [1981], McMichael [1983], and Menzel [1990] have produced effective reasons for thinking that an essence such as *being Reagan* ontologically depends on Reagan himself. If so, then if Reagan hadn't existed, the essence *being Reagan* wouldn't have existed, and so essences couldn't exist unexemplified.[32] A second problem is that Plantinga's modal semantics abandons our ordinary ways of thinking in nonmodal cases. Ordinarily, '$\exists x Px$' expresses the fact that some object exemplifies property P. However, for Plantinga, it expresses the fact that some essence is coexemplified with P, and we are left without a way to express the fact that an *individual* x exemplifies a property.[33]

[32]The problem is this: an essence such as *being Reagan* could exist at a world where Reagan doesn't exist only if it is purely qualitative (i.e., doesn't involve Reagan as a constituent). But a purely qualitative property could be exemplified by different objects at different worlds, violating clause (c) of the definition of an essence (see a previous footnote). So essences seem to require a nonqualitative component. But if so, then if the nonqualitative component is all that there is to an essence, the essence can no longer be seen as a property, for such non-qualitative, non-repeatable entities are not distinguishable from possible objects. If the nonqualitative component is just a part of the essence, then what else could such a component be but the contingent object itself? But then essences would ontologically depend on contingent objects. Thus, they could not exist unexemplified.

[33]We can't even express the intuition that the coexemplification of two properties P and Q is to be cashed out as: some individual x exemplifies both P and Q. This raises the question: just what language is Plantinga using to state the definition of an essence? In footnotes 30 and 31, we used a boldface '**E**' to refer to essences when giving the semantic readings of modal formulas, since essences apprear in the semantics. But individuals don't appear in Plantinga's revised semantics for quantified modal logic. Yet the definition of an essence (footnote 29) presupposes a language in which there are variables that range over individuals. This is confusing, to say the least.

Jager [1982] suggests that '**P** and **Q** are coexemplified at **w**' should be understood as follows: the state of affairs, *something has both* **P** *and* **Q**, would have been true if

In fact, Plantinga's entire logic of coexemplification must be disconnected from the traditional logic of exemplification that captures our ordinary ways of thinking, for if coexemplifications were 'witnessed' by facts of the form x *exemplifies* P (having individual x as a constituent), the sentence '$\Diamond\exists x Px$' would imply the existence of individual witnesses in the domains of other worlds, thus reintroducing possibilia. A final problem concerns the constraints of serious actualism Plantinga places upon Kripke-models. By requiring an essence to be in the domain of a world **w** if coexemplified with a property at **w**, a logical problem with negation arises, namely, what to do when the negations of properties are added to the logic as properties. The formula '$[\lambda y\, \neg Py]x$' would be false at a world **w** not containing the essence denoted by 'x' in its domain. But '$\neg Px$' would be true at **w**. This would force the rejection of the classical conversion principle: $\Box([\lambda y \neg Py]x \leftrightarrow \neg Px)$, and thus force the rejection of predicate excluded middle: $\Box(Px \vee [\lambda y \neg Py]x)$. These problems become magnified when one considers adding a fully general theory of complex properties.[34]

Fine [1977] solves problems (1) and (3) of Kripke-models not by direct redefinition but indirectly by translating Kripke's metalanguage into a more basic language so as to eliminate the quantifier over possibilia. He doesn't substitute quantification over abstract objects for quantification over possibilia, but rather analyses away the apparent quantification altogether. To carry out the analysis, Fine constructs 'the proper language for a modal actualist', which involves quantification over contingent objects and contingent propositions, and a primitive modal operator. Fine is therefore a *modalist*, for he thinks there is nothing more basic than primitive modal notions. Using his modalist language, Fine defines a 'world-proposition' as a proposition that *might* be such that it necessarily implies ('entails') everything true, and defines 'q is true at world-proposition p' as: p entails q. Using these definitions, Fine is able to preserve the quantification over possible worlds in Kripke-models as quantification over world-propositions.[35] So though '$\Diamond q$' is a primitive form of expression in Fine's preferred language, the truth conditions it receives in Kripke's semantics are in turn reanalyzed in Fine's language as: there is a world-proposition p which entails q. However, the standard Kripkean semantic analysis of 'b might have had a sister', in terms of possible worlds and possible objects, would receive the following retranslation: there is a world-proposition (say p_1) which might have been true and which entails the proposition *some x is b's sister*. This analysis doesn't require the distinction between possible and actual objects, and so Fine squares Kripke's metalanguage with the thesis of actualism. Moreover, Fine would modify Kripke-models to accomodate serious actualism by regarding propositions as contingent. If an object doesn't exist at a world, no propositions about that object exist at that world either, and so an object may not have properties at worlds where it doesn't exist. To manage this, he employs a free logic of terms both for individuals and propositions.

However, Fine's therapy for Kripke-models is even more problematic than Plantinga's. He, too, must sever ordinary quantified claims from facts involving witnesses—the world-proposition p_1 that makes 'b might have had a sister' true will entail the proposition *some x is b's sister* without entailing, for some x, any witness of the form *x is b's sister*. The force of this problem is compounded by McMichael's [1983] examples of such 'iterated modalities' as $\Diamond\exists x(Px \& \Diamond Qx)$. Under Fine's translation scheme, this is true iff there is a world-proposition (say p_2) which entails *some x is P and is possibly Q*. But Fine cannot then analyze the occurrence of the modality '*is possibly Q*' as: some world-proposition (say p_3) entails *x is Q* (for an x satisfying *some x is P*), for that x would be a witness. Fine

w had been actual. But it is unclear what state of affairs Jager is referring to, since the quantifier 'something' clearly seems to be a quantifier ranging over individuals, of which there are none postulated in his semantics. Moreover, we seem once again committed to possibilia, for what else could Jager mean other than: if **w** had been actual, there would have been a individual that was both **P** and **Q**? Finally, Jager's suggestion also threatens circularity. Coexemplification is suppose to be a semantical notion used to interpret a quantified modal logic having a primitive modal operator. But Jager here seems to be offering a modal analysis of coexemplification in terms of a primitive modality.

It seems that both Jager and Plantinga ultimately retreat to a language in which there are separate variables and quantifiers ranging over individuals, essences, and states of affairs. Modality is primitive in this language. But as yet, they have not explicitly formulated their language, and this leaves one wondering not only about their most basic metaphysical views but also why they get rid of all individuals in the semantics of quantified modal logic if ultimately they plan to quantify over them.

[34] Jager [1982] attempts to solve this problem, but at the cost of introducing two kinds of necessity. The cost is not readily apparent, for he uses the terms 'de re' and 'de dicto' to label the two kinds of necessity. See p. 338.

[35] Similarly, Plantinga would substitute possible, maximal states of affairs for possible worlds in his redefinition of Kripke-models. However, we could ignore Plantinga's reconstruction of worlds because his conception of the possibilia in the variable domains didn't depend on that reconstruction but rather on essences. But we can't ignore Fine's account of worlds as propositions because it crucial to his technique for ridding Kripke-models of possibilia and solving problems (1) and (3).

denies the existence of p_3, but argues that p_3 *might* have existed.[36] He can do so because in capturing the idea that propositions are contingent, he rejects BF for the quantifiers ranging over propositions in his 'proper language for modal actualism'. In this proper language, $\exists q \Diamond (\ldots q \ldots)$ doesn't follow from $\Diamond \exists q (\ldots q \ldots)$.[37] But the failure of BF for the proposition quantifiers means that there must be varying domains of *possible* propositions, only one of which contains the actual propositions. That is, Fine's proper language of modal actualism would require, for its interpretation, Kripke-models that appeal to *existing-but-non-actual propositions*. So Fine's therapy for eliminating possibilia from Kripke-models requires possibilia of another kind.

Menzel [1990] solves problems (1) and (3) of Kripke-models by a new and innovative strategy which retains some ideas from Plantinga and Fine. Unlike these other actualists, Menzel makes no attempt to identify worlds as acceptable abstract entities of some sort, but rather abandons the notion of a possible world altogether. Like Plantinga, however, Menzel redefines Kripke-models, in his case by replacing possible worlds with the Tarski-models of CQT. These Tarski-models are to consist simply of existing objects (say, pure sets). Some of these existing objects, it doesn't really matter which, constitute a domain of individuals, others a domain of properties, and, of course, one is a function (i.e., a set) that maps the 'properties' into their extensions in the domain of 'individuals'. Then, like Fine, Menzel relies ultimately upon a language in which modality is primitive, offering the following truth conditions for modal formulas: '$\Diamond\varphi$' is true under Kripke-model $\mathbf{M}$ iff there is some Tarski-model in $\mathbf{M}$ that would have been a model of (i.e., would have faithfully mapped) the actual world (i.e, the way things would have been) had φ been true. And '$\Diamond \exists x Px$' is true in $\mathbf{M}$ iff there is a Tarski-model $\mathbf{T}$ in $\mathbf{M}$ containing an object in the extension of the property denoted by 'P' such that $\mathbf{T}$ would have faithfully mapped the actual world had something been P. Since the Tarski-models that substitute for worlds in Kripke-models are constructed entirely out of existing objects, Menzel's metalanguage for modal logic requires no distinction between what is actual and what is possible. It therefore conforms with the thesis of actualism. And it is seriously actualistic, since any individual having a property in a Tarski-model must be in the domain of that model. With this understanding of Kripke-models, Menzel in [1991] goes on to defend a particular modal system which, it should be noted, requires restrictions on the rule of necessitation.

But the more serious problem with Menzel's defense of Kripke-models is that it jettisons our traditional ideas about truth and modality. First and foremost, by both arguing that possible worlds semantics carries no commitment to possible worlds and eschewing the notion of possible world altogether, Menzel must abandon the seminal insight that necessary truth is truth in all (accessible) possible worlds. This not only undermines the rather nice, extensional characterization (of the truth conditions) of modal claims, but also disallows the (actualist) idea that there exist alternative *ways the world might have been*, not necessarily as possible objects but as actual abstract objects of some sort. Menzel also rejects the idea that there is an 'intended' Kripke-model, which represents pieces of the world itself as configured in a way that correctly reflects modal reality. He gets by instead with a notion of *intended* models*, that is, those Kripke-models (constructed out of pure sets) that, roughly, would have been structurally isomorphic to the intended model had there been one. These suffice, he argues, since there is nothing more to modal truth than the structure that they capture.[38] But surely there is something more to modal truth than this; surely *necessity* and *possibility* are about something besides the structure of intended* models, something which *grounds* modal truth and which is modeled by an intended model. Menzel suggests that modal se-

[36] As an example, McMichael [1983] uses: John F. Kennedy might have had a (second) son who might have become an astronaut. The problem is that the first modal operator implies the existence of a possibly true world-proposition p_2 entailing *something is Kennedy's second son*. But the embedded (iterated) modal operator implies the existence of a world-proposition p_3 which entails that some particular individual might have been an astronaut. That individual would seem to be a possible object. Fine, however, denies that there is such world-proposition as p_3. Since there is no possible second son (i.e., no witness to *something is Kennedy's second son*), there is no proposition involving 'him'. However, Fine would assert that p_3 might have existed.

[37] If BF fails for quantification over propositions, it fails for quantification over world-propositions as well. As a consequence, embedded modal operators must be understood as quantifiers that range over worlds (i.e., world-propositions) that don't exist but might have. As McMichael [1983] points out, this gives up the interpretation of the modal operators in possible worlds semantics as ranging over one single domain of possible worlds. This is an important feature of the possible worlds analysis of the modal operators that isn't captured by Fine's translation scheme.

[38] Note that, in a certain sense, none of these *intended** models are in fact genuine *models* of any*thing*. At best, they have the property of being actual objects that *possibly* model *the structure* of modal reality. But a model of the pure structure of modal reality is not the same as a genuine model of modal reality.

mantics need not try to say what this something is. But not only does this attitude sever the traditional connection between semantics and ontology, it seems a bit arbitrary given that Menzel accepts, with respect to nonmodal language, that there is something more to truth than the structure captured in pure Tarski-models. With Menzel's defense of Kripke-models, we cannot say that modal language is in part *about* the objects over which the quantifiers range, at least not in the same way that we can say that nonmodal language is about these objects.

§4: An Actualistic Interpretation of *QML*

We've now seen a variety of actualist attempts to solve the problems of Kripke-models and none have seemed completely successful. All involve some unsatisfying complication or reintroduce objects that look suspiciously like possibilia. But we think there is no reason to defend Kripke-models with their variable domains anyway, for there is an interpretation of (the fixed domain) *QML* that should satisfy both actualists and serious actualists. This interpretation is based on a more subtle understanding of the difference between the abstract and the concrete. Once the distinction between abstract and concrete objects is properly reconceived and incorporated into our view of the models of *QML*, the objectionable features of *QML* disappear. We shall introduce our new understanding of the abstract/concrete distinction by retracing the contemporary conceptions of possibilia.

One conception of a possible object is inherited from Quine [1948], who takes it that a possible F is an F but of a certain shadowy sort. A possible fat man in the doorway is fat, a man, and in the doorway, but has some level of being short of existence ('mere possibility'). Why else would Quine wonder how many such men are *in* the doorway? They can only be *in* the doorway if they are spatial objects, and since he thinks they are fat men of some sort, they fit the bill and so the question can be meaningfully asked. It is no wonder that actualists find it easy to reject this conception of possibilia. The thesis of actualism admits no special, shadowy category of being, since all that there is is actual. So only an existing, actual fat man would be in the doorway, and inspection shows there is none.[39]

[39]Quine's conception has now been absorbed into the standard view of the 'possibilist quantifier', the quantifier that ranges over more than just actual objects. See for

However, many philosophers now accept that possible Fs *don't* in fact have to be F. The data don't require it, and in particular, BF doesn't require it. As Marcus [1986] herself points out using the same example, BF requires only that there be something that *could have been* a fat man in the doorway (given that there might have been a fat man in the doorway), and requires only that there be something that *could have been* b's sister (given that b might have had a sister). BF does not require that there be anything which is in fact a fat man in the doorway or in fact b's sister. Formally, BF ($\Diamond\exists x Px \to \exists x \Diamond Px$) requires only something at the actual world which *at some other world* is P. For example, it requires only something at the actual world which *at some other world* is a fat man in the doorway, and only something at the actual world which *at some other world* is b's sister. These things need not be fat men or b's sister at all in the actual world. So strictly speaking, BF doesn't violate the reasonable intuition that the actual world contains nothing which is in fact, a fat man in the doorway, b's sister, a million carat diamond, or the like.

But, then, if they are not in fact fat men or b's sister, what are these things like at the actual world, these things required by BF which *could have been* fat men in doorways, b's sister, million carat diamonds, etc.? Actualists think there is no satisfactory answer to this question. As we pointed out in §2, given certain essentialist intuitions, they can't be concrete objects. Though some actual man might well have been fat and in the doorway, no actual woman could have been b's sister. Nor could any other concrete object have been b's sister. The only alternative is that they are abstract objects. But, then, actualists rule them out on the basis of the further intuition that what is abstract is essentially abstract and couldn't have been concrete. No abstract object *could have been* a fat man or b's sister or a million carat diamond, etc.

However, we believe that this bit of reasoning involves a distinctive mistake that many actualists have made in thinking about possibilia. Actualists mistakenly think the distinction between abstract and concrete is one of category, that whatever is abstract or concrete is essentially

example, Salmon [1987] (p. 57), Fine [1977] (p. 130), Forbes [1985] (pp. 243–4), and Pollock [1985] (p. 130). Moreover, this Quinean conception has appeared, though in somewhat altered form, in David Lewis' possibilist views, on which any object that has a property at a world has that property simpliciter. Lewis believes that a possible million carat diamond is a million carat diamond albeit not in this world, and that a possible fat man in a doorway is a man in a doorway, but not a doorway in this world.

so. On our view, this is unjustified. We see no reason not to recognize entities that are contingently nonconcrete, that is, objects that are in fact nonconcrete but which at other worlds are concrete. We suppose that there are nonconcrete objects which, at other worlds, are (variously) fat men, b's sister, and million carat diamonds. They are not, of course, fat men, b's sister, etc., at our world, but they exist and are actual. We can assert this because the actualist's existentially loaded quantifier doesn't carry any spatiotemporal connotations, for otherwise they would not be able to use it to assert the existence of abstract objects. BF, we claim, requires only the existence of contingently nonconcrete objects.

So, in answer to the above question, what are the objects required by the BF like in this world, we respond with the usual intuitions philosophers have concerning nonconcrete objects: the properties they have at this world are the same ones that numbers, sets, and other abstract, nonconcrete objects have. They are nonphysical, nonspatiotemporal, lacking in shape, size, texture, etc. We just appeal to the same intuitions actualists are prepared to use when describing ordinary (essentially) abstract objects.[40] However, we should note that contingently nonconcrete objects have different modal properties than essentially abstract objects.

So, in what follows, we agree with the actualists that the actual world contains no sister of b, nor million carat diamonds, etc., and agree that there is nothing beyond existing entities. But we differ about the nature of the realm of the nonconcrete. We see it as containing not only the familiar abstract particulars, which are necessarily nonconcrete, but also the contingently nonconcrete. This will make perfect sense if one supposes that 'abstract' and 'concrete' designate properties that are the negations of each other. By letting 'concrete' mean 'spatiotemporal' and defining 'abstract' as 'not concrete', the domain of abstract objects becomes the domain of the nonspatiotemporal objects. The familiar abstract particulars, such as numbers and sets, have the property of being abstract in

[40] It is always preferable if such intuitions are supported by a theory. Actualists rarely offer a general theory of the abstract. Fortunately, for our purposes, the theory of (essentially) abstract objects developed by one of the present authors will do just fine (Zalta [1983] and [1988a]). Even though on Zalta's view, essentially abstract objects both 'encode' as well as exemplify properties, the properties such abstracta exemplify are typically the negations of ordinary properties, as well as intentional and ontological properties. These same properties, we claim, are *exemplified* by the contingently nonconcrete at the actual world (the contingently nonconcrete do not *encode* properties because they are not essentially abstract).

every possible world.[41]

Once it is seen that BF requires only contingently nonconcrete objects and not possibilia, it is natural to reconceive the nature of concrete objects. We suppose that ordinary concrete objects are not necessarily concrete, that is, they are not concrete at every possible world. This in fact is why they are contingent objects. At worlds where they are not concrete, actualists want to say they don't exist or have any kind of being, whereas we just rest with their nonconcreteness. So we find it natural to reject the view that concrete objects are essentially concrete (i.e., concrete in every world in which they exist). This is the one essentialist intuition that we abandon. But the loss is not a grievous one, for we find no appreciable difference between a world where an object, say Reagan, exists but is not concrete and a world where Reagan doesn't exist. If there is none, then why not just express the actualist intuition that objects are essentially concrete by saying that concrete objects are not necessarily concrete?

Thus, while our contingently *non*concrete objects have in this world the properties that the familiar abstracta have, they are more similar *in kind* to ordinary concrete objects. Ordinary concrete objects are concrete at our world and are nonconcrete at others. The contingently nonconcrete are nonconcrete at our world and yet concrete at others. Together, they form the class of 'possibly concrete' objects, i.e., objects that are concrete at some world or other. We can therefore preserve the essentialist intuitions that actualists are fond of by defining a notion of essential property that applies to possibly concrete objects x: F is essential to x iff necessarily, if x is concrete, then x is F. Then, to say that x is essentially human is to say that in every world where x is concrete, x is human. But more importantly, consider the intuition that no object *could have been* b's sister. This was the intuition actualists use to reject BF. On our view, actualists were justified in rejecting BF *given* that they looked only among the concrete objects and the necessarily abstract objects. No concrete object could have been b's sister, and moreover, each concrete object

[41] Thus, we reject the idea that the familiar abstract objects can exist in some worlds and fail to exist in others. Such an idea, for example, seems to be needed for Hartry Field's [1989] views on mathematical objects. Mathematical objects, for him, are 'abstract' objects which contingently happen not to exist (and that is why he thinks mathematical sentences are literally false). But we see no reason why such objects would fail to exist in our world, yet exist in others. So we feel no need to be able to express the kinds of claims he wants to make.

x is essentially *not* b's sister, for in each world in which x is concrete, x is not b's sister.

Moreover, no necessarily abstract object could have been b's sister. Necessarily abstract objects are necessarily nonconcrete, but to be b's sister at some world, such an abstract object would have to be concrete at that world.[42] Moreover, we formally separate the notion of *essential* property as it applies to possibly concrete objects from the notion that applies to necessarily abstract objects. If x is necessarily abstract, then F is essential to x iff necessarily x exemplifies F. For example, to say that a number is essentially not a building is to say that the number necessarily fails to be a building. By distinguishing two notions of *essential property*, one for the necessarily abstract and one for the possibly concrete, we eliminate an otiose aspect of the *traditional* notion of an essential property. The traditional notion is that F is essential to x iff necessarily, if x exists, x has F. But the antecedent of the definiens is unnecessary in the case of abstract objects, since on the usual understanding, abstract objects exist necessarily. So the traditional notion of essential property doesn't efficiently characterize properties essential to abstracta. We are exploiting this fact in distinguishing two notions of essential property.

The upshot of the above ideas is that by recognizing the existence of contingently nonconcrete objects and by reconceiving both the contingency of concrete objects and the notion of an essential property in what seems to be harmless ways, we can interpret *QML* so that it is consistent with actualism and serious actualism. Just read the quantifier $\exists$ of the language of *QML* as 'there exists' or 'there is'. By actualist lights, these mean the same. Moreover, let us suppose that everything that exists is actual. This squares the object language with the thesis of actualism. Since the quantifer ranges over everything in domain $\mathbf{D}$ in the models of *QML*, everything in $\mathbf{D}$ therefore both exists and is actual. $\mathbf{D}$ includes concrete objects, contingently nonconcrete objects, and necessarily abstract objects, all of which, we claim, exist and are actual. So our metalanguage conforms to the thesis of actualism as well. There are no objects of any shadowy sort,[43] not even separated from us by a gap that separates worlds, nor is there some realm of *ausserseiende* entities beyond the

[42]Indeed, in our view, the property of being a sister, among many others, is 'concreteness-entailing': F is *concreteness-entailing* iff necessarily, for all x, if x exemplifies F, x is concrete.

[43]Being nonspatiotemporal, we shall presume, doesn't imply being shadowy, otherwise Quine is committed to shadowy entities in virtue of being committed to sets.

scope of the quantifiers. Note that modal claims of the form $\exists x \Diamond Px$ and $\Diamond \exists x Px$ receive standard truth conditions for the quantifiers—existential sentences are true because there exist objects exemplifying properties at the actual world as witnesses, and sentences having existential quantifiers embedded within modal operators are true because there exist objects exemplifying properties at other possible worlds. With this interpretation, we can reconsider the various actualist objections to *QML*.

First and foremost, the problems associated with BF disappear. It should now be clear that from the fact that b might have had a sister, BF asserts only that there exists an actual object, namely, a contingently nonconcrete object, that could have been b's sister.[44] Not only is this result consistent with the actualist thesis and the various actualist (essentialist) intuitions described above, we can now see why BF is compatible with the intuition that there might have been something distinct from every actual thing. This is in fact true precisely because the contingently nonconcrete objects are not concrete. Our analysis is: There might have been a concrete thing distinct from every actual concrete thing. This will entail: there is something (non-concrete) which could have been concrete (and so is clearly different from every actual concrete thing). The intuition that there might have been something distinct from every actual thing is grounded on the idea that when we look around, we notice with respect to the concrete objects that there might have been other kinds of concrete objects.[45] This idea is captured by our interpretation.

[44]Obviously, this doesn't require that any object in fact *is* b's sister, and so we avoid the Quinean conception of possibilia. We can answer the question, how many possible fat men are there in the doorway, by saying 'None, there is nothing in the doorway, though most likely an infinite number of contingently nonconcrete objects could have been fat men in the doorway (each in a slightly different way)'.

[45]Someone might object that since we now locate the contingently nonconcrete objects among the 'actual' things, we have still not captured the intuition 'there might have been something distinct from every actual thing'. Indeed, how could we, since we have a single fixed domain of objects all of which are actual. To this we respond that such a suggestion prepackages the datum with a philosophical interpretation, and in particular, treats 'actual' in terms of the theoretical notion that actualists connect with it. We see no reason to think that the datum in question, if given its ordinary, pretheoretical sense, is based on any technical notion of 'actual' on which it applies to either abstract or contingently nonconcrete objects. Indeed, if given such a wider meaning, there is no reason to think the intuition is true. For example, the intuition doesn't apply to the abstract objects—there is no reason to think that there might have been other abstract objects than the ones there in fact are. Instead, the claim seems defensible only with respect to the concrete objects, grounded in the ideas we

NE is also acceptable. Though NE asserts that everything necessarily exists, there is no conflict with intuition given that the actualist quantifier has no spatiotemporal connotations. The important thing is that neither NE nor $\Box$NE assert that everything (or indeed anything) is necessarily concrete. The intuition that a particular concrete object x might not have existed is captured in our logic by the idea that x is not necessarily concrete, i.e., that at some world **w**, x is nowhere to be found in spacetime at **w**. What more could be meant by saying that *it*, *qua* concrete object, doesn't exist there? The serious actualist might say that 'it' has no properties at all there. We agree that it has none of the physical properties of concrete objects there, but why not suppose that with respect to that world it is just like the contingently nonconcrete of our world? So it will have the nonphysical properties of other abstract entities there. In fact, we claim that our interpretation is seriously actualistic because we accept the idea that exemplification entails existence. SA is a thesis of *QML*, and semantically, no object has a property at any world without falling under the range of *QML*'s ubiquitous quantifier. This interpretation doesn't require that contingent concrete objects disappear from the logical scene just because they disappear from the physical scene at other worlds.

With these results, the validity of CBF is no longer a problem. Though it does follow from $\Box\forall x\varphi$ that $\forall x\Box\varphi$, the quantifier involved ranges over the same domain of objects in each case. The fact that there are various ways to derive NE from CBF in *QML* is no longer objectionable, since NE is an acceptable theorem. In particular, the derivation of NE from CBF and SA using necessary properties is no longer a problem for serious actualism. Note that these results do not imply that if x is concrete then x is necessarily concrete. It may be true that necessarily, everything x is such that if x is concrete, x has property P. But it doesn't follow from this that everything x is such that if x is concrete, then necessarily x has P. For example, it is true that necessarily, every object is such that if it is concrete, it is physical. But it doesn't follow that every concrete object is such that it is necessarily physical, though it will follow that every object is such that necessarily, if it is concrete, then it is physical.

Note that iterated modality is handled in a straightforward way. The sentence $\Diamond\exists x(Px \& \Diamond Qx)$ has straightforward truth conditions. Possibilia are not required, for the thing x which might have been both P and possibly Q is a contingently nonconcrete object that at some other world is P and at yet another world is Q. No essences or contingently existing propositions are needed.

described in the text.

We note that none of the problems plaguing the attempts to rectify Kripke-models arise. Free logic is unnecessary—a truth of the form Pa logically implies $\exists xPx$ without an additional premise $\exists x\, x = a$. Every suitable candidate for a name is already in the domain of the ordinary existential quantifier. Note that our contingently nonconcrete objects are not forced to have a name; one might argue, given a certain theory of naming, that they can't be singled out or 'baptized' in the usual way and so can't be named.[46] Moreover, there are no puzzles about the truth conditions of formulas containing constants that don't refer at certain worlds. No constant lacks a denotation at any world, and no formula containing a constant lacks a truth value at any world. This is not to suggest that our denotation function is a binary function. It is unary, taking terms as arguments and members of the single domain as values. This latter feature stands in contrast to Deutsch's system, who avoids the use of free logic by employing a ternary denotation function, and who solves the puzzle of how a constant can denote an object at a world where the object doesn't exist by an elaborate mechanism involving contexts of origins. In contrast to Deutsch system, our interpreted *QML* has no restrictions on the Rule of Necessitation.

Our interpretation also simplifies the logic of serious actualism. Contra Plantinga, there is an objectual interpretation of quantified modal logic which allows us to express the fact that individuals exemplify properties and which leaves property negation simple. In contrast to Fine, a retranslation of the semantic truth conditions for sentences of *QML* into the language of modal actualism requires no contingent or merely possible propositions, and a free logic for proposition terms is unnecessary. In contrast to Menzel, we preserve the idea that necessary truth is truth in all possible worlds. Our intended model has in its domain all of the objects that actually exist and distributes extensions to properties at worlds in just the way that is required by the modal facts. Modal language, as such, is directly about an independent reality free of possibilia, and the relationship between the formal language and the intended model exactly

[46] Our view is therefore compatible with the 'new theory of direct reference' (Kripke [1972]), assuming it is suitably modified to handle reference to necessarily abstract objects. Such modifications would not require the introduction of names for the contingently nonconcrete, but only that such objects be in the range of the bound variable.

mirrors the relationship between ordinary modal language and the reality that grounds modal truth.

Finally, it will be seen that even some of the other complications that arise in connection with quantified modal logic have natural solutions. The issue of cross-world identity, and its solution either with counterparts or haecceitism, doesn't arise. No distinction between two kinds of necessity, such as between weak necessity (not false at any possible world) and strong necessity (true at every possible world) is needed.

Conclusion

We believe that actualists should accept *QML* under the interpretation we have offered, not simply because it is consistent with their principles, but also because it eliminates a certain puzzle they repeatedly run up against in their own work. On the one hand, they accept that there are abstract objects of some sort, either sets, numbers, propositions, states of affairs, properties, or essences, etc. But on the other hand, these abstracta often have contingent objects as constituents in some way. The modal properties of these abstract complexes with contingent constituents become puzzling at worlds where the constituents don't exist. This happens especially for Plantinga's essences and Fine's contingent propositions, both of which ontologically depend on their constituents. So Plantinga, Fine, and other actualist philosophers already accept things that straddle the apparent categorial divide between the abstract and the concrete. But that is just what contingently nonconcrete objects do, except they cross over this divide without exhibiting any modally puzzling behavior. At our world, they are like abstracta in that they exemplify many of the properties abstract objects typically exemplify. But at other worlds, they are exactly like the objects that are concrete at our world. As such, they are no more strange than are familiar ordinary concrete objects *at* worlds where they are not among the inhabitants of spacetime. Indeed, we suggest that contingently nonconcrete objects are less strange than abstract entities that go out of existence at worlds where one of their parts physically fails to exist. It almost seems as if actualists are making the mistake of thinking of such complex abstracta as *physical objects*, since like physical objects, they cannot exist if one of their parts physically fails to exist. So actualists should find, in the contingently nonconcrete, a kind of entity that eliminates the need for complexes or other abstracta that disappear

from logical space whenever their components disappear from physical space.

We conclude that Kripke-models for quantified modal logic introduce many more problems than they solve. Variable domains and restricted quantifiers are not required for actualism. Recall why it was that philosophers found the variable domains of Kripke-models natural—the models validate the intuitions that existence goes with the quantifier and that existence varies from world to world. We accept the first (even when quantifiers appear inside modal operators) and strictly speaking, abandon the second, though we recapture the underlying intuition by allowing a real property, namely concreteness, to vary from world to world. To us, Kripke-models make the mistake of conceiving the quantifier in the same way one conceives of a predicate, namely, as having an extension that varies from world to world. Moreover, the quantifier under Kripke-models, when evaluated at the actual world, is too geocentric. By restricting the range of the quantifier so that it covers a smaller ontological realm, all sorts of logical and metaphysical problems arise. And even with those problems, the belief persisted that larger ontologies would create even graver logical and metaphysical problems. But we have shown that that is not the case. By postulating more existing objects, the logical and metaphysical problems connected with quantified modal logic have rather simple solutions. The infamous technical complexity of quantified modal logic is unnecessary; we need not follow a path down Garsons' [1984] tree in search of an acceptable logic, for the system at the top of the tree, namely *QML*, suffices.

Indeed, it suffices no matter what one's own view is about the possibilism/actualism debate on which we have focused. If we are right, then *QML* should be acceptable to both the possibilists described at the end of §2 and to the actualists and serious actualists described in §3. Of course these two opposing groups of philosophers will interpret the formalism in different ways to suit their own purposes. But it is always a virtue in a logic if it proves to be metaphysically neutral. And *QML*, as a formal system, has this virtue to a larger extent than was thought heretofore.

Bibliography

Adams, R. M., 1974, "Theories of Actuality," *Nous* **8**: 211–31; reprinted in Loux [1979], pp. 190–209

Adams, R. M., 1981, "Actualism and Thisness," *Synthese* **49**: 3–41
Chisholm, R., 1976, *Persons and Objects*, La Salle: Open Court
Deutsch, H., 1990, "Contingency and Modal Logic," *Philosophical Studies* **60**: 89–102
Deutsch, H., 1994, "Logic for Contingent Beings," *Journal of Philosophical Research*, 19: 273–329.
Enderton, H., 1972, *A Mathematical Introduction to Logic*, San Diego: Academic Press
Field, H., 1989, *Realism, Mathematics and Modality*, Oxford: Blackwell
Fine, K., 1977, "Postscript," in Prior [1977]
Fine, K., 1978, "Model Theory for Modal Logics: I," *Journal of Philosophical Logic* **7**: 125–56
Forbes, G., 1985, *The Metaphysics of Modality*, Oxford: Clarendon
Garson, J., 1984, "Quantification in Modal Logic," in *Handbook of Philosophical Logic: Volume II*, D. Gabbay and F. Guenthner (eds.), Dordrecht: D. Reidel
Hughes, G. and Cresswell, M., 1968, *An Introduction to Modal Logic*, London: Methuen
van Inwagen, P., 1986, "Two Concepts of Possible Worlds," in *Midwest Studies in Philosophy, XI*, P. French, T. Uehling, and H. Wettstein (eds.), Minneapolis: University of Minnesota Press, 185–213
Jager, T., 1982, "An Actualist Semantics for Quantified Modal Logic," *Notre Dame Journal of Formal Logic* **23**/3 (July): 335–49
Kaplan, D., 1975, "How to Russell a Frege-Church," *Journal of Philosophy* **72**: 716–29; reprinted in Loux [1979], pp. 210–24
Kripke, S., 1963, "Semantical Considerations on Modal Logic," *Acta Philosophica Fennica* **16**: 83-94.
Kripke, S., 1972, "Naming and Necessity," Cambridge: Harvard, 1980
Lewis, D., 1986, *On The Plurality of Worlds*, Oxford: Blackwell
Lewis, D., 1990, "Noneism or Allism," *Mind* **99**: 23–31
Linsky, B., 1991, "Truth at a World is Modality," *Philosophia* **20**/4: 387–94
Linsky, B., and Zalta, E., 1991, "Is Lewis a Meinongian?," *Australasian Journal of Philosophy* **69**/4 (December): 438–53
Loux, M., (ed.), 1979, *The Possible and the Actual*, Ithaca: Cornell
Marcus, R. Barcan, 1986, "Possibilia and Possible Worlds," *Grazer Philosophische Studien*, R. Haller (ed.), **25/26** (1985/1986): 107–33
Marcus, R. Barcan, 1946, "A Functional Calculus of First Order Based on Strict Implication," *Journal of Symbolic Logic* **11**: 1–16

McMichael, A., 1983, "A Problem for Actualism about Possible Worlds," *Philosophical Review* **92**, 49–66
Meinong, A., 1904, "On Object Theory," in *Realism and the Background of Phenomenology*, R. Chisholm (ed.), Glencoe: The Free Press, 1960; translation of "Über Gegenstandstheorie," in *Untersuchungen zur Gegenstandstheorie und Psychologie*, A. Meinong (ed.), Leipzig: Barth, 1904
Mendelson, E., 1964, *Introduction to Mathematical Logic*, New York: D. Van Nostrand
Menzel, C., 1990, "Actualism, Ontological Commitment, and Possible Worlds Semantics," *Synthese* **85**: 355–89
Menzel, C., 1991, "The True Modal Logic," *Journal of Philosophical Logic* **20**: 331–374
Parsons, T., 1967, "Grades of Essentialism in Quantified Modal Logic," *Nous* **1**/No. 2 (May): 181–91
Parsons, T., 1969, "Essentialism and Quantified Modal Logic," *The Philosophical Review* **78**/1 (January): 35–52
Plantinga, A., 1976, "Actualism and Possible Worlds," *Theoria* **42**: 139–60; reprinted in Loux [1979], pp. 253-73
Plantinga, A., 1974, *The Nature of Necessity*, Oxford: Oxford University Press
Plantinga, A., 1983, "On Existentialism," *Philosophical Studies* **44**: 1–20
Plantinga, A., 1985, "Replies," in *Alvin Plantinga*, J. Tomberlin and P. van Inwagen (eds.), Dordrecht: D. Reidel, pp. 313–96
Pollock, J., 1985, "Plantinga on Possible Worlds," in *Alvin Plantinga*, J. Tomberlin and P. van Inwagen (eds.), Dordrecht: D. Reidel, pp. 121-44
Prior, A., 1977, *Worlds, Times, and Selves*, Amherst: University of Massachusetts Press
Prior, A., 1957, *Time and Modality*, Oxford: Clarendon
Prior, A., 1956, "Modality and Quantification in S5," *Journal of Symbolic Logic* **21**: 60–2
Quine, W. V. O., 1940, *Mathematical Logic*, Cambridge, MA: Harvard University Press; 2nd edition, revised, 1951
Quine, W. V. O., 1948, "On What There Is," in *From a Logical Point of View*, New York: Harper, 1953, 1–19
Russell, B., 1905, "On Denoting," *Mind* **14** (October): 479–493

Salmon, N., 1987, "Existence," in *Philosophical Perspectives 1*, J. Tomberlin (ed.), Atascadero: Ridgeview Press

Zalta, E., 1988a, *Intensional Logic and the Metaphysics of Intentionality*, Cambridge: MIT Press/Bradford Books

———, 1988b, "Logical and Analytic Truths That Are Not Necessary," *Journal of Philosophy* **85**/2 (February): 59–89

———, 1983, *Abstract Objects: An Introduction to Axiomatic Metaphysics*, Dordrecht: D. Reidel