Twenty-Five Basic Theorems in Situation and World Theory*

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In [1981a] and [1981b], Barwise and Perry sketched what they called 'situation semantics' and argued that a full-blown theory based on their sketch would offer a better analysis of natural language than possible world semantics. Instead of taking objects, sets, and *total* ways the world might be (i.e., worlds) as basic and reconstructing properties, relations, and propositions as functions, Barwise and Perry took *partial* ways the world might be (i.e., situations) as basic along with certain 'uniformities' across them, such as objects and relations. They refused to take truth-values as the denotations of sentences, and rejected the idea that the significance of a sentence shifted in intensional contexts. Instead, using 'naked-infinitive reports' as their guide, they argued that sentences signified situations, and that these same situations were the objects of the propositional attitudes. Moreover, their new semantics was consistent with the idea that necessarily equivalent properties, relations, and propositions can be distinct. All of these ideas contrasted sharply with the standard assumptions of possible worlds semantics, as embodied by Montague [1974] and Cresswell [1973]. From its inception, then, situation semantics was thought to be incompatible with possible worlds semantics.

In their seminal work of [1983], Barwise and Perry left the impression that this incompatibility between semantic frameworks indicated an incompatibility between situations and worlds, considered as metaphysical entities. And in Barwise [1985], in which some of Aczel's ideas on nonwellfoundedness were incorporated into situation theory, we find the suggestion that 'reality, all that is, is not a situation' (p. 191). But in an interchange with Stalnaker, Perry offered a picture in which worlds could be viewed as just certain maximal situations (Perry [1986] and Stalnaker [1986a]). Eventually, however, in [1989], Barwise developed a set of 'branch points' in situation theory, through which alternative versions of the theory must travel. At one branch point, there are two alternatives: one which leaves room only for the actual world (conceived as a maximal situation), the other which leaves room for multiple possible worlds.

The question of whether situations and worlds can peaceably coexist in the foundations of metaphysics is complicated by the fact that world theorists disagree about what worlds are. Though many of the researchers working within the possible worlds framework are content to regard worlds as a useful theoretical tool, such an attitude does not satisfy a metaphysician. Our best theories quantify over worlds and so we become interested in them as metaphysical entities in their own right. In Lewis [1973] and [1986], Stalnaker [1976] and [1985], Adams [1974], Chisholm [1976], Plantinga [1974], Pollock [1984], and Fine [1977], we find various attempts to develop a theory of worlds, often by constructing them out of other basic metaphysical entities. Interestingly, Chisholm, Plantinga, and Pollock defined worlds in terms of the notion of a state of affairs, a notion which has turned out to be one of the building blocks of situation theory. In situation theory, states of affairs are basic constructions out of objects and relations, and they are the kind of thing that situations 'make factual'. This convergence of ideas, in which states of affairs are seen as basic to both world and situation theory, leads one to wonder whether there is a unified theory that can integrate all of these entities.

In this paper, I propose to assimilate states of affairs, situations, and worlds into a single theory that distinguishes, yet comprehends, all three kinds of entity. The theory is couched in some definitions and theorems, all of which are cast in a precise logical framework. However, *none* of the theorems are *stipulated* to be true; rather, they all result as consequences of a formal, axiomatic theory of objects and relations for which the logical framework was originally developed. An important feature of the subtheory of situations is the volume of important definitions and theo-

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rems of naive situation theory that it successfully captures. It articulates an interesting path through the many branch points of situation theory. However, the most important feature of this subtheory is that it resolves the apparent conflict between worlds and situations, for worlds are shown to be certain maximal situations. Neither situations nor worlds are taken to be primitive, nor does the theory make an essential appeal to possible world semantics. Rather, the notions of situation and world are both *defined* (worlds and situations are identified as objects of a special sort), and so we no longer have to decide which of these entities is more basic. The theory may justify much of the recent work both in situation theory and in world theory not just by showing that the two theories can be integrated successfully, but by showing that most of the basic principles of each theory can be derived.

I should like to emphasize that what follows constitutes a *theory* and *not* a model of situations and worlds. Situations and worlds will not be identified as elements of some mathematical structure. The background theory of objects and relations is not cast within the framework of a mathematical theory, nor do mathematical entities of any kind (not even sets) appear in the background ontology.[1] Though the theory *appears* to be highly technical, in fact, it is not. From a logical point of view, it requires only the sophistication of modal predicate logic, and so should be readable and accessible to anyone familiar with *S5* modal predicate logic. The various symbolizations that give the theory the appearance of being technical serve only to make everything precise (this allows logicians to inspect the logic and axioms for consistency and completeness) and to simplify the statement of the theorems and proofs.

To determine whether the theory proposed here is indeed a theory of *worlds* and *situations*, it will serve well to describe the principal, pretheoretic intuitions that govern our conceptions of these entities. Thus, the plan for the paper will be as follows. In §1, I examine the basic conception of a situation, as it has developed from Barwise and Perry [1980] through Barwise [1989]. In §2, I examine the two basic conceptions of possible worlds. In §3, I try to sketch the intuitions that connect situations and worlds with the foundational metaphysical theory of objects. Then, in §4, I develop the foundational theory in more detail, for those readers who may be unfamiliar with my previous work (readers already familiar with this work may skip this section).[2] §5 contains the main results of the paper, i.e., the statement of the basic definitions and theorems governing situations and worlds.[3]

[1]I take sets to be just a special kind of object, and membership to be just a special relation. As such, the study of sets (or any other kind of mathematical object or relation) is posterior to the study of metaphysics, which is the study of objects and relations in general. I have shown elsewhere how mathematical objects and mathematical relations can be identified among the general objects and relations postulated by the present theory; see [1983], pp. 147–53.

§1: The Conception of a Situation

The ideas underlying situation theory originated in Barwise and Perry [1980], [1981a] and [1981b]. In these early papers, the authors stressed their dissatisfaction with certain features that had become built into standard semantic theory, namely, (a) the denotation of a sentence is a truth value, (b) the denotation of a sentence shifts when the sentence appears in indirect, intensional contexts, and (c) properties and relations are reconstructed as functions from possible worlds to sets of (sequences of) individuals. Features (a) and (b) come to us from Frege [1892], whereas (c) is an application of possible world semantics. All three features were, in one way or another, either incorporated into Montague's intensional logic or presupposed in its application to natural language. Montague's intensional logic has been widely regarded as one of the two most formally elegant ways of capturing some of Frege's views about language.[4] Barwise and Perry, however, proposed to recover our pre-Fregean semantic innocence by rejecting (a) and (b). After undermining the argument frequently used to conclude that the denotation of a sentence had to be a truth value, they turned their attention to the development of a semantics in which *situations*, construed as complexes of objects and properties, played a more direct role in the interpretation of a sentence. To this end, Barwise and Perry rejected possible world semantics and along with it, the

[2]Readers who skip §4 should note that the notion of a proposition used in my earlier work is similar to the situation-theoretic notion of a state of affairs. So in order to square the language of my theory with the language of situation theory, I now call 0-place relations 'states of affairs' rather than 'propositions'. This should pose no problems when it comes to world theory, since there are various equivalent versions of world theory, some using states of affairs, others using propositions.

[3]Some of the twenty-five theorems that appear there (roughly a fifth) have been discussed in my previous work, but the others appear here for the first time, as part of a new application of the foundational theory and its underlying philosophy.

[4]The other is A. Church's logic of sense and denotation. See Church [1951] and especially Anderson [1984].

mathematical reconstruction of properties and relations. Instead, properties and relations were taken as primitive. This allowed them to regard situations as complexes of objects and properties and thus as pieces of reality. They proposed to employ situations 'at the level of reference' by having sentences stand for them. Naked infinitive reports such as 'John saw Mary run' played an important role in the development of the semantic theory, since these could be interpreted as expressing relationships between individuals and situations; the embedded, naked infinitive sentence 'Mary run' seemed to signify a limited and observable piece of reality.

These ideas were developed in much more detail in Barwise and Perry [1983]. Though they still thought of reality as a complex web of situations, the focus of this work was on semantic theory. The semantic theory, however, was developed in terms of a (set-theoretic) framework of *abstract* situations. Abstract situations were introduced principally to interpret false sentences. Without abstract situations, there is nothing for a false sentence to designate, since the conception of situations as parts of reality does not leave room for negative situations, or negations of situations. Abstract situations were therefore constructed as (set-theoretic) sequences of relations and objects, and thus constructed out of the kinds of entities one typically finds in real situations. The sequence itself has a kind of structure that was suppose to reflect the 'complex-of-objects-and-properties' structure of real situations. However, some of these sequences fail to correspond to, or classify, any real situations, since they relate objects and properties in ways not reflected by reality. These abstract situations were then incorporated into a relational theory of meaning. The meaning of a sentence was taken to be a relation between two abstract situations—an utterance situation u and a described situation s. Thus, a sentence ϕ, would express a relation between an utterance situation u and a described situation s, though in the case of false sentences, the related s fails to classify any real situations. Abstract situations also proved useful for handling false beliefs, for these could be analyzed as relations between persons and abstract situations to which no real situations correspond.

However, Barwise and Perry also introduced into their semantical theory lots of other abstract, set-theoretic constructions out of abstract situations: event types, roles, constraints, anchors, indeterminates, etc. This profusion of abstract objects led them to reconsider the extent to which their theory could be called 'realistic'. Reality supposedly consisted of

situations and a few uniformities across them (namely, objects and relations), yet all of the work in their theory was being done by abstract situations and constructs thereof. Reservations about this were expressed in Barwise and Perry [1984].[5] After realizing that they had been misled, Barwise and Perry changed the direction of their research.[6] Recognizing that their model was not a theory, they adopted a new goal, namely, to characterize situations directly, without the mediation of abstract situations, by elaborating the basic axioms of situation theory.

After some false starts, the rudiments of a theory began to emerge. Abstract situations reappeared in the more realistic guise of *states of affairs*. A state of affairs, or SOA (sometimes also referred to as an *infon*), is a basic piece of information, reflecting that some objects either do or don't stand in some relation. The expression used for designating basic states

[5] Consider the following passage:

> Then we allowed ourselves to introduce into our semantical theory any constructions from abstract situations and the other devices of set theory that we needed to make the semantics work. This seemed to allow us a lot of freedom at the level of our theory, while investing the world only with situations, objects, locations, relations, and the like.
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> Somewhere along the way, though, we realized that this was an illusion. After all, our theory was intended to be a theory about the world. To the extent that it is correct the sets we constructed did get at real uniformities in the world, so we are committed to all sorts of things. This dawned on us as we worked out the book... . After all, if we are going to get by without some Platonic realm like senses or possible worlds and find everything one needs in the reality we inhabit, we had better be prepared to recognize all the structure that is really there. (p. 9)

[6] The nature of this shift is expressed in the following passage, also in Barwise and Perry [1984]:

> **I:** *Why didn't you, instead, express those intuitions about types of things and roles directly? Why not develop a theory of situations, types, role and the like?*
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> ⋮
>
> **B** and **P:** Exactly. That is another way of saying what John said earlier about the illusion we were under when we thought we could get by with set theoretic objects to classify invariants, rather than admit the invariants as first class citizens of reality. Part II of our book was called "A Theory of Situations," but really all it is is a model of a theory of situations. It is a real theory of situations that we are working on, now. We have found it to be a very liberating idea. (p. 23)

of affairs is: $\langle\langle R, a_1, \ldots, a_n; \pm\rangle\rangle$. In this expression, the angled brackets are not to be construed set theoretically, but as primitive notation. The expression 'R' denotes a relation, while the expressions of the form 'a_i' denote objects. The expression '+' ('−') indicates that the objects do (don't) stand in the relation. In addition, there is a special relationship postulated between situations and SOAs, namely, the *supports* or *makes factual* relation. This relationship is designated as '$s \models \sigma$', where 's' designates a situation and 'σ' designates a state of affairs. The fact that some situations are partial is reflected by the fact that for some situations s, there are SOAs of the form $\langle\langle R, a_1, \ldots, a_n; \pm\rangle\rangle$, such that neither $s \models \langle\langle R, a_1, \ldots, a_n; +\rangle\rangle$ nor $s \models \langle\langle R, a_1, \ldots a_n; -\rangle\rangle$. In addition to the $\models$ relation, it is standardly assumed that the domain of situations is partially ordered by a *part-of* relation ($\trianglelefteq$), which is reflexive, anti-symmetric and transitive. Sometimes, when it is supposed that there is a maximal element of this partial ordering, such a maximal element is designated a 'world'.

However, there were still numerous conflicting intuitions about some of the basic properties of situations. This conflict of intuitions was canonized in Barwise [1989]. After positing a basic ordering of situations under the *part-of* relation, and assuming that there is at least one maximal element (i.e., world), Barwise presents a list of 19 questions, each of which constitutes a 'branch point' in situation theory. The various answers to these questions lead to different sets of first principles for situation theory. The more important questions that Barwise asked include: whether there is more than one world, whether every part of a situation is a situation, whether every world is a situation, whether there are nonactual situations, whether situations are well-founded, whether every state of affairs has a dual, and whether a richer algebra should be imposed on the domain states of affairs. These are obviously fundamental questions, and different answers to these questions lead to rather different theories of situations.

These, then, are the ideas that form the basic conception of a situation. As yet, no canonical version of situation theory has been defined. The various versions that have appeared so far have been forged by stipulating basic axioms at many of the choice points. By contrast, the theory we produce in §§4 and 5 *makes predictions* at 15 of the 19 branch points defined in Barwise [1989]. That is, the choices made by the theory are never just stipulations, but are rather consequences of both definitions and general principles which govern objects and relations.

§2: Conceptions of Possible Worlds

It is now customary to suppose that there are basically two conceptions of possible worlds: the possibilist (or concretist) conception and the actualist (or abstractionist) conception. Thus, in Stalnaker [1986b], we find:

> Philosophers who take possible worlds seriously are often divided into two camps. First there are the *possibilists* who hold that possible worlds and other possible objects may exist without *actually* existing. Other possible worlds, according to the possibilist, are concrete universes, spatially and temporally disconnected from our own, but just as real. The claim that such universes are not actual is, in effect, just the claim that we are not located in them. Second, there are the *actualists* who hold that nothing is real except what is actual—that is, except what exists as a part of the actual world. According to the actualist, the things that are (perhaps misleadingly) called 'possible worlds' are not really *worlds*, but are properties or states of the world, or states of affairs, or propositions or sets of propositions, or perhaps set theoretic constructions of some kind. There are many different versions of actualism; what they have in common is the thesis that possible worlds are things that can be instantiated or realized. A nonactual possible world is not a concrete object that exists in some nonactual place, but an abstract object that actually exists but is uninstantiated. (p. 121)

No one has gone further to develop the idea that there are two conceptions of possible worlds than van Inwagen in [1986]. He says:

> . . . Lewis did not content himself with saying that there were entities properly called 'ways things could have been'; nor did he content himself with implying that 'possible world' was a heuristically useful stylistic variant on 'way things could have been'. He went on to say that what most of us would call 'the universe', the mereological sum of all the furniture of earth and the choir of heaven, is one among others of these 'possible worlds' or 'ways things could have been', and that the others differ from it "not in kind but only in what goes on in them" (Lewis [1973], p. 85). And to suppose that the existence of a plurality of universes or cosmoi could be established by so casual an application of Quine's criterion of ontological commitment has been regarded by most of Lewis's readers as very exceptional indeed.

> Whether or not the existence of a plurality of universes can be so easily established, the thesis that possible worlds are universes is one of the two 'concepts of possible worlds' that I mean to discuss. ... The other concept I shall discuss is that employed by various philosophers who would probably regard themselves as constituting the Sensible Party: Saul Kripke, Robert Stalnaker, Robert Adams, R. M. Chisholm, John Pollock, and Alvin Plantinga.[3] These philosophers regard possible worlds as abstract objects of some sort: possible histories of *the* world, for example, or perhaps properties, propositions or states of affairs.
>
> I shall call these two groups of philosophers Concretists and Abstractionists, respectively. (pp. 185–6)

In his article, van Inwagen states a preference for the Abstractionist view of worlds, and discusses a criticism of this view that Lewis puts forward in [1986].

It will not be our concern here to adjudicate between these two conceptions of worlds, but rather to consider the extent to which the theory of worlds offered here captures the intuitions of the two camps. We shall address this question in the final section of the paper, but for now, we note that there is reason to be dissatified with both the possibilist and the actualist conceptions in their present state of development. The Lewisian possibilist conception seems to embrace claims that, for one reason or another, few philosophers find plausible. Take, for example, the analysis Lewis offers for the truth (of ordinary language) that there might have been a talking donkey. On his analytical scheme, this becomes: there exists both a possible world w and an object x such that x exemplifies being a talking donkey and x is a part of w. From this, it follows that there exists a talking donkey, though Lewis is careful to say that it is non-actual (i.e., that it is not one of our worldmates). Few philosophers seem to be able to accept that there exist talking donkeys, million-carat diamonds, and all the other possible but non-actual individuals, even with the proviso that these objects are actual only at other possible worlds. Such a result seems to conflict with the view that concrete things are precisely the things that are spatiotemporally related to us.

A second dissatisfaction with Lewis's possibilist scheme concerns the exact nature of the theory. Though Lewis offers a translation scheme between natural language and his own metaphysical views, this is not quite the same as offering a theory, based on first principles, that couches

his metaphysical views. The axioms of Lewis's combination of mereology, set theory, and world theory, have not yet been precisely formulated. And there is still the question of how to classify the informal claims of Lewis's theory. For example, at the beginning of Lewis [1986] (p. 2), we find:

> There are so many other worlds, in fact, that absolutely *every* way a world could possibly be is a way that some world is.

He also repeats this claim in several other places (for example, on pp. 71 and 86). Now what exactly is the status of this claim within Lewis's theory? Is it suppose to be an axiom? If so, then the theory must quantify over 'ways worlds could be' and postulate a correlation between these 'ways' and worlds. But 'ways the world could be' are not entities of the theory. If it is not an axiom, what exactly is the status of this claim? Questions of this sort still puzzle those interested in a more exact development of Lewis's views.

On the other hand, the actualist/abstractionist conception of worlds faces its own problems. The most important of these seems to me to be that the conception cannot simultaneously adopt a fine-grained view of states of affairs or propositions and yet preserve the intuition that there is a unique actual world. This latter intuition seems to be *incompatible* with any theory that treats worlds as states of affairs (or propositions) but which permits necessarily equivalent states of affairs to be distinct. Fine-grained theories of states of affairs allow us to distinguish certain necessarily equivalent states. For example, such a theory allows us to distinguish the state of affairs p from the necessarily equivalent state of affairs $p \;\&\; (q \vee \neg q)$, since the former is (let us suppose) simple, whereas the latter has a more complex structure. But on the abstractionist conception, worlds are defined, for example, to be any state of affairs p such that: $\Diamond(p \;\&\; \forall q(p \Rightarrow q \;\vee\; p \Rightarrow \neg q))$.[7] Now consider some particular state of affairs p_0 that satisfies the definition of a world. Note that for any arbitrary proposition q, the state of affairs $p_0 \;\&\; (q \vee \neg q)$ also satisfies the definition of a world. Since the latter is distinct from p_0, but equivalent, we have multiple, distinct copies of each world, contrary to intuition. And in particular, there will be multiple, distinct copies of the actual world. So it seems that either we have to give up our fine-grained conception of

[7] In this definition, '$p \Rightarrow q$' just abbreviates: $\Box(p \rightarrow q)$. Definitions roughly equivalent to this may be found in Chisholm [1976], Plantinga [1976], and Pollock [1984]. In Fine [1977], there is a similar definition, except using propositions.

states of affairs or the intuition that there is a unique actual world.

One way out of this dilemma is to suppose that worlds are *sets* of states of affairs (or sets of propositions). The suggestion is that a world is a set S such that (i) for every state of affairs p, either $p \in S$ or $\neg p \in S$, and (ii) the conjunction of all of the members of S is itself a possible state of affairs. On this view, the actual world is the set that contains all and only the states of affairs that obtain (there is a unique such set). The problem with this suggestion, however, is that it doesn't constitute a genuine metaphysical conception of a world. The suggestion might be useful as a *model* of worlds, but not as a *theory* of worlds. Whatever worlds are, they are not sets, nor any other kind of mathematical construction. Even if one is a Platonist about mathematics, it is a mistake to think that the fundamental properties of worlds and situations are exhibited by mathematical entities. Whatever else they are, worlds and situations are entities *in which some states of affairs obtain and others don't*, and it would seem that mathematical entities are the wrong kind of thing to play this role. A more plausible candidate needs to be found.

So at present, it is unclear whether abstractionists interested in developing a genuine theory of worlds can reconcile the uniqueness of the actual world with a fine-grained theory of propositions or states of affairs.

§3: The Intuitions Connecting Situations, Worlds, and Objects

In order to connect the central ideas of situation theory and world theory to those underlying our background metaphysics, we focus on a distinction that has directed the development of situation theory from its inception but which has never been made explicit at the level of theory. From the beginning, situation theorists have appealed to the distinction between the internal properties of a situation and its external properties. In [1981a], Barwise and Perry assert:

> Situations have properties of two sorts, internal and external. The cat's walking on the piano distressed Henry. Its doing so is what we call an external property of the event. The event consists of a certain cat performing a certain activity on a certain piano; these are its internal properties. (p. 388)

This distinction between the internal and external properties appears

throughout the course of publications on situation theory. In [1985], Barwise writes:

> If $s \models \sigma$, then the fact σ is called a fact of s, or more explicitly, a fact about the internal structure of s. There are also other kinds of facts about s, facts external to s, so the difference between being a fact that holds in s and a fact about s more generally must be borne in mind. (p. 185)

And in [1989], Barwise writes:

> The facts determined by a particular situation are, at least intuitively, intrinsic to that situation. By contrast, the information a situation carries depends not just on the facts determined by that situation but is relative to constraints linking those facts to other facts, facts that obtain in virtue of other situations. Thus, information carried is not usually (if ever) intrinsic to the situation.
>
> The objects which actual situations make factual thus play a key role in the theory. They serve to characterize the intrinsic nature of a situation. (pp. 263-4)

The main point here is that the conception of an object having intrinsic, internal properties as well as extrinsic, external properties is central to the idea of a situation. But, oddly enough, situation theory does not formally develop the distinction between internal and external properties.

However, this is just the idea that underlies the theory of objects developed in Zalta [1983] and [1988], for it is based on a language having two modes of predication, i.e., in which there are two basic ways to predicate properties of objects. On the one hand, a property F can be predicated *externally* of an object x, in which case it is said that x *exemplifies* F. On the other hand, F can be predicated *internally* of x, in which case it is said that x *encodes* F. It is axiomatic that ordinary objects such as electrons, tables, planets, people, etc., have only external properties. However, the theory asserts that there is a special subdomain of objects the members of which have an extraordinary nature—they have both internal and external properties. In other words, these objects both encode as well as exemplify properties. Formally, the distinction between encoding and exemplifying a property is captured by having two kinds of atomic sentences: we use the formula 'xF' to assert that an object x encodes F and use the formula 'Fx' to assert that x exemplifies F. Our special objects may both exemplify and encode the very same properties, or may encode properties that are distinct from the ones they exemplify. Though encoding is restricted to 1-place properties (which, nevertheless, may be quite complex), exemplification can be generalized to n-place relations in the usual way. Thus the theory allows us to (externally) predicate relations among objects (of any kind) in the usual way, using sentences of the form '$F^n x_1 \ldots x_n$'. Thus, exemplification is a notion that is familiar from standard predicate logic.

Ernst Mally first distinguished the notion of encoding in [1912], to solve the puzzles of Alexius Meinong's naive theory of intentional objects. Mally identified intentional objects as abstracta that are 'determined by' their associated properties without really 'satisfying' those properties. For example, Mally would say that 'the round square' was an abstract object 'determined by' roundness and squareness, but which did not 'satisfy' either of these two properties. Using our terminology, we would say that 'the round square' encodes just the two properties roundness and squareness, but does not exemplify either. Rather, given that it is abstract, it exemplifies the negations of these properties. Such an object is consistent with the non-logical law that whatever *exemplifies* being round fails to *exemplify* being square. Notice the following two things: (1) since 'the round square' encodes just two properties and no others, there is a sense in which it is a partial object (though it will be complete with respect to the properties it exemplifies); and (2) since encoding is a kind of predication (i.e., a way for an object *to be F*), there is a sense in which 'the round square' *is* round and square.[8] Using this example as a model, one can develop similar responses to the puzzles associated with such problematic objects as 'the existent golden mountain', 'the ghost John feared', and 'the non-square square'.[9]

The properties that an abstract object encodes are constitutive of its nature, and as such, are essential to its identity as an object. These encoded properties are even more essential to its identity than the properties it necessarily exemplifies. Some examples will demonstrate this.

[8]In my previously cited work, I have gathered evidence for thinking that there is a lexical and structural ambiguity underlying the the copula 'is', namely, the ambiguity between exemplifying and encoding a property.

[9]Recently, Mally's distinction in kinds of predication has resurfaced in the work of Castañeda [1974] and Rapaport [1978]. Castañeda distinguishes several modes of predication, whereas Rapaport distinguishes exemplification from *constituency*. See also E. Sosa's [1986], where a similar distinction is utilized in the analysis of fictional characters.

We noted that 'the round square' exemplifies the properties of being non-round and non-square. And given that it is abstract, we may also claim that it exemplifies the property of failing to have a shape. Indeed, the round square *necessarily* exemplifies all three of these properties, for since it is abstract, it is not the kind of thing that could exemplify a shape. But these negative properties are not as critical to the nature and identity of the round square as the properties of being round and being square. The theory will capture these facts by asserting both (1) that any properties encoded by an abstract object are necessarily encoded (so the properties that are constitutive of an object's nature will not vary from world to world), and (2) that two abstract objects are identical iff they necessarily encode the same properties.

Consider, as another example, mathematical objects such as the numbers of Peano number theory. The number 1, if treated as an object, contingently exemplifies having been thought about by Peano and being denoted by the numeral '1', whereas it necessarily exemplifies such properties as having no location, having no shape, having no texture, etc. On the other hand, the theoretical properties of the number 1, such as being greater than 0, being odd, being prime, etc., are even more crucial to its identity than any of the properties previously mentioned. The present theory treats these theoretical properties of the number 1 as the properties it encodes. These are the properties internal to its nature.

Consider, as a final example, an object of fiction, such as Sherlock Holmes. The present theory treats the properties attributed to Holmes in the Conan Doyle novels as the properties he encodes. These include: being a person, being a detective, living at 221B Baker Street in London, having a prodigious talent for solving crimes, etc.[10] These are regarded as more crucial to the identity of Holmes than such properties as being fictional, being the main character of the Conan Doyle novels, being an inspiration to modern criminologists, etc., which are properties that are externally exemplified by Holmes (note that many of these properties are relational, based on the external relations Holmes bears to ordinary objects). These exemplified properties are contingent ones; had circumstances been different, the internally constituted object we have identified as Holmes might not have had these properties. On the other hand, given that we have identified him as a certain theoretical object, we could say

[10]Many of these properties are not explicitly attributed to Holmes in the novels, but are reasonably inferred from a common sense based understanding of the novels.

that Holmes necessarily exemplifies such properties as not being a person, not being a detective, not living in London, not being a spoon, etc.[11]

Let us assume, as an intuitive principle comprehending the domain of special, abstract objects, that for every group of properties, there is an abstract object that encodes just the properties in the group and no others. As an identity principle, let us assume that two such objects are identical iff necessarily, they encode the same properties (whereas two ordinary objects are identical iff necessarily, they exemplify the same properties). So this populates the subdomain of abstract objects with a wide variety of objects. Many of these objects will be partial in the sense that there are properties F such that the object encodes neither F nor the negation of F ('$\bar{F}$'). But each special object is complete with respect to the properties it exemplifies. Indeed, for any object x whatsoever, ordinary or special, either Fx or $\bar{F}x$.

We can now begin to see how this conception of an object having internal and external properties can be applied to our intuitive understanding of situations. Situations are supposed to be internally characterized by states of affairs. Note that a state of affairs is not a property, and so, strictly speaking, doesn't characterize anything. States of affairs either obtain or they don't. However, there are properties that are intimately linked to states of affairs. These are properties that objects exemplify in virtue of a state of affairs obtaining. Consider, for example, the following two properties: being such that George loves Barbara, and being such that Barbara doesn't love George. These are properties that are constructed out of states of affairs—the former is constructed out of the state *George's loving Barbara*, whereas the latter is constructed out of the state *Barbara's not loving George*. If we let the formula 'Lgb' denote the first state of affairs and '$\neg Lbg$' the second, then we could use the λ-predicates '$[\lambda y\, Lgb]$' and '$[\lambda y\, \neg Lbg]$' to denote, respectively, the properties being such that George loves Barbara and being such that Barbara doesn't love George. In the complex λ-predicates, the variable 'y' is vacuously bound by the λ. Nevertheless, these are perfectly good predicates, and the properties they denote are perfectly well-behaved. Necessarily, an object x *exemplifies* being such that George loves Barbara iff George loves Barbara; or in formal terms: $\Box([\lambda y\, Lgb]x \leftrightarrow Lgb)$. And, necessarily, x exemplifies being such that Barbara doesn't love George iff Barbara doesn't love George; or in formal terms: $\Box([\lambda y\, \neg Lbg]x \leftrightarrow \neg Lbg)$. Since we are calling states of affairs 'SOAs' for short, we may call the properties constructed out of states of affairs 'SOA-properties'.

[11]The reader may consult Zalta [1983] and [1988] for further details the treatment of mathematical and fictional objects.

Now suppose that every state of affairs p has a corresponding SOA-property $[\lambda y\, p]$. Note that the corresponding SOA-property provides a means by which a state of affairs can characterize an object. A situation can be characterized by a SOA-property, for example, when we say that the situation *is such that p*. We might have said, for example, that the situation of unrequited love between George and Barbara is such that George loves Barbara, but such that Barbara doesn't love George. Unfortunately, as far as the logic of external predication goes, no object is distinguished by the SOA-properties that it exemplifies. If a state of affairs p obtains, then everything whatsoever exemplifies $[\lambda y\, p]$. If p fails to obtain, then nothing whatsoever exemplifies $[\lambda y\, p]$. But this standard feature of the logic of exemplification does not hold for the logic of encoding. Whether or not an object encodes $[\lambda y\, p]$ is independent of whether or not p obtains. In particular, if p obtains, it does not follow from the fact that every object exemplifies $[\lambda y\, p]$ that every abstract object encodes $[\lambda y\, p]$. Formally, whether or not a special object x encodes $[\lambda y\, p]$ will depend on the whether this property satisfies the defining condition of the relevant instance of the abstraction principle. Metaphysically, though, whether a special object x does or does not encode $[\lambda y\, p]$ is just a brute fact. Thus, as internally encoded properties, SOA-properties may serve to distinguish all sorts of special objects.

We now have a way to capture the intuition that situations are 'intrinsically characterized by states of affairs'. We just think of a situation as a special object that encodes (only) SOA-properties. This gives us a clear sense in which a state of affairs can be an internal property of a situation—the property constructed out of the state of affairs is encoded by the situation. Given the intuitive comprehension principle for special objects described above, it follows that for every group of SOA-properties, there is a situation that encodes just the SOA-properties in the group. This guarantees, for example, that there is an object that 'is' such that George loves Barbara and 'is' such that Barbara doesn't love George. This is the object that encodes $[\lambda y\, Lgb]$, $[\lambda y\, \neg Lbg]$, and no other properties. This constitutes a situation of unrequited love between George and Barbara. It will be a 'part' of any situation that encodes these properties and others as well. The principle of identity for special objects ensures that the identity of this situation is completely determined by its internal properties. Thus, the situation just described is 'partial' in nature, for its identity is linked just to the two states of affairs Lgb and $\neg Lbg$. Note that this conception of situations leaves us free to treat the external properties of a situation as ones that it exemplifies. Properties such as being distressing to Henry, being seen by Mary, carrying information (of a certain kind), etc., are all properties that situations exemplify. These happen to be examples of contingent properties that situations may exemplify, though some properties that situations exemplify will be necessary. Properties such as not being a number, not being a person, not being a building, being such that p-or-not-p, etc., are all properties that situations exemplify necessarily.

All of these remarks about situations apply equally well to worlds. A world may be thought of as having internal and external properties. Its internal properties are just the SOA-properties that characterize what goes on at that world, and which make it that world and not some other. These are even more important to the identity of the world than the properties the world has externally. We shall exploit this similarity between situations and worlds, for we shall think of the latter as objects that encode only SOA-properties but which are also maximal and possible in the appropriate senses.[12]

Before developing these ideas in a precise way, let us consider the viability of a proposal that some might think constitutes a natural alternative to this view of situations and worlds. One might suggest that the 'intrinsic' properties of situations and worlds that we have been discussing are just properties that situations (worlds) *necessarily exemplify*. And by contrast, the 'extrinsic' properties would be ones they contingently exemplify. This suggestion would bypass the entire distinction

[12]It is important to distinguish the encoding/exemplification distinction from a distinction employed by Adams [1981], Fine [1985], Deutsch [1990], and Menzel [1991]. These philosophers are trying to account for the problem of contingently existing propositions. In their actualist frameworks, if the constituent of a proposition doesn't exist at a world, then neither does the proposition. Yet these philosophers think such propositions can characterize worlds even if they don't exist at that world. So they distinguish a proposition's being *true in* a world w from its being *true at* w (Adams [1981], pp. 20-22), or its being true in an *inner* sense at w from its being true in an *outer* sense at w (Fine [1985], p. 163). Such a distinction is not necessary in the present framework, for unlike these systems, existence is not identified with quantification. If an object fails to exist at a world w, it lacks the property of being spatiotemporal at w (and every other property that implies that it is spatiotemporal). We may still quantify over it (there), as well as over all the propositions that have it as a constituent.

between exemplifying and encoding properties. Unfortunately, however, it faces a major obstacle. Consider just situations for the moment (what we say about situations applies to worlds as well). Suppose, according to the suggestion, that situations necessarily exemplified the SOA-properties intrinsic to their nature. So, for an arbitrary situation s and SOA-property $[\lambda y\ p]$ that characterizes the nature of s, we have: $\Box[\lambda y\ p]s$. But, in the context of modal logic, the λ-abstraction principle is a necessary truth, and yields: $\Box([\lambda y\ p]s \leftrightarrow p)$. It now follows that $\Box p$. Thus, any state of affairs p that characterizes the nature of situations has to be a necessary truth. Hence, situations would never be characterized by contingent states of affairs, and this clearly seems to be false.

One might try to repair the suggestion by saying that properties that characterize the intrinsic nature of situations are not SOA-properties such as $[\lambda y\ p]$, but rather existence-relative SOA-properties having the form $[\lambda y\ E!y \rightarrow p]$ (*being a thing y such that p if existing*). The amended suggestion is that situations have such properties necessarily. From this, it would not follow that the states of affairs involved are necessary. That is, from $\Box([\lambda y\ E!y \rightarrow p]s)$ and $\Box([\lambda y\ p]s \leftrightarrow p)$ it does not follow that $\Box p$. So the amended suggestion avoids the problem discussed in the previous paragraph. But it has its own problems, for it places properties of the form $[\lambda y\ E!y \rightarrow p]$ on a par with other properties that situations necessarily exemplify. For example, take the property of *not being a spoon* ($[\lambda y\ \neg Sy]$), or the property of *not being a spoon if existing* ($[\lambda y\ E!y \rightarrow \neg Sy]$), or any other property that situations exemplify necessarily (other than a relativized SOA-property) . It is an *a priori* fact that situations couldn't fail to have such properties (intuitively, a situation couldn't possibly be a spoon), just as they couldn't fail to exemplify certain properties of the form $[\lambda y\ E!y \rightarrow p]$. But now how do we capture the intuition that it is the latter and not the former properties that are intrinsic to the nature of situations? There doesn't seem to be a way to distinguish these (relativized) SOA-properties from the others as being *more* critical to their identity.

That is just what the distinction between encoding and exemplifying a property does. With this distinction, we can tie the identity of situations directly to, and only to, properties of the form $[\lambda y\ p]$ if we suppose that these are the properties they encode. These encoded properties become more critical to the identity of situations than the properties they necessarily exemplify (recall the proposed definition of identity for abstract objects). The property of *not being a spoon*, even though exemplified necessarily by all situations, is not one of the properties by which situations are identified. The same is true for most other properties necessarily exemplified by situations. By supposing that SOA-properties are encoded by situations, we have a better picture of why some properties are part of the nature of situations while others are not. Moreover, from the fact that a situation s encodes $[\lambda y\ p]$, and does so necessarily, it doesn't follow that p is necessary, since the relevant instance of λ-*Equivalence* governs only exemplified SOA-properties, not encoded ones. So puzzles like the ones raised in the previous two paragraphs have a simple solution on this conception. And it should be mentioned that all of these considerations apply to our understanding of worlds as well.

In fact, we've already outlined one other good reason for thinking that situations and worlds are best characterized in terms of encoded SOA-properties, namely, that no object whatsoever is distinguished by the SOA-properties it exemplifies. If p obtains, then everything whatsoever exemplifies being such that p; if p fails to obtain, then nothing exemplifies this property (everything would exemplify the property of *being such that* $\neg p$). But not every abstract object encodes the same SOA-properties. Recall that given our abstraction principle for abstract objects, we can expect that for every group of SOA-properties, there will be an abstract object that encodes just those properties and no others. This gives us a wide variety of situations, each one being what it is in virtue of the states of affairs that 'characterize' it.

§4: The Background Theory of Objects[13]

Our metaphysical foundations consist of a language, its logic, and a proper theory. The language is an almost trivial variant of quantified 'second order' modal logic. It has two primitive kinds of variables: object variables $x, y, z \ldots$, and n-place relation variables $F^n, G^n, H^n, \ldots$ $(n \geq 0)$. There is one distinguished predicate: $E!$. Recall that the distinction between exemplifying and encoding a property is captured by having two kinds of atomic formulas in the language. *Exemplification* formulas of the form '$F^n x_1 \ldots x_n$' assert that objects $x_1, \ldots, x_n$ exemplify (or stand in) relation F^n. *Encoding* formulas of the form 'xF^1' are to be read: x encodes F^1

[13]The following rough sketch should give one a good idea of what the system looks like, but it is not a substitute for the precise definitions found in Zalta [1983] and [1988]. Readers already familiar with these works may skip this section.

(no generalization to n-place relations, for $n > 1$, is necessary). Using these two atomic formulas as a basis, the language may be defined in the usual way, so as to contain the usual sorts of molecular, quantified, and modal formulas.

Here is an intuitive picture of the models of this language.[14] The variables $x, y, \ldots$ and $F^n, G^n, \ldots$ range over mutually exclusive domains of primitive entities, the objects and the n-place relations, respectively. The $\Box$ is a universal quantifier ranging over a fixed domain of primitive worlds (since we plan to use *S5*, no accessibility relation is needed). We emphasize that there are no world-relative domains of objects or relations, but rather one fixed domain for objects and one for relations. Each n-place relation receives an exemplification extension (a set of n-tuples) at each world, and moreover, each property (i.e., 1-place relation) receives, in addition, an encoding extension (this is just a set of objects that does not vary from world to world). In terms of this picture, we can sketch the truth conditions of our atomic formulas. The formula '$F^n x_1 \ldots x_n$' is true at a world $\mathbf{w}$ (relative to a model $\mathbf{M}$ and assignment $\mathbf{f}$ to the variables) just in case the n-tuple consisting of the objects denoted by the variables $x_1, \ldots, x_n$ (relative to $\mathbf{M}$ and $\mathbf{f}$) is a member of the exemplification extension at $\mathbf{w}$ of the relation denoted by F^n (relative to $\mathbf{M}$ and $\mathbf{f}$). The formula 'xF' is true at $\mathbf{w}$ (relative to $\mathbf{M}$ and $\mathbf{f}$) just in case the object denoted by x (relative to $\mathbf{M}$ and $\mathbf{f}$) is an element of the encoding extension of the property denoted by F (relative to $\mathbf{M}$ and $\mathbf{f}$). The truth conditions for the molecular, quantified, and modal formulas are the usual ones.

It should be reasonably clear from this picture that the classical axioms of propositional logic, quantification theory, and *S5* modal logic can be associated with our language.[15] Since the quantifiers and modal operators range over fixed domains, the Barcan formulas, both first and second

[14]The reader is cautioned not to take this picture too seriously. In particular, the fact that possible worlds appear as primitive entities in the semantics does not imply that the metaphysical theory expressed in the object language is committed to primitive possible worlds. It is not. It is committed only to two domains, objects and relations, and takes the modal operator as primitive. Worlds will be defined within the theory, and it is this definition, coupled with the object-theoretic theorems about worlds, that grounds and justifies our use of worlds as primitive in the semantics.

[15]By 'classical quantification theory', I am simply referring to the standard axioms and rules governing the introduction and elimination of quantifiers and terms. To prove the theorems in the present paper, the reader may choose his or her favorite natural deduction system or axiomatization of classical quantified modal logic. I have used the axiomatic method to sketch the proof of the modal theorems.

order, are valid.[16] Since the encoding extension of a property does not vary from world to world, if an encoding formula is true at some world, it is true at all worlds. So the modal logic of encoding is expressed by the following principle:

Logical Axiom: $\diamond xF \rightarrow \Box xF$

This helps to capture the intuition that the properties encoded by a special object are internal to its nature, and so do not change with the changing circumstances from world to world. From the usual rules and axioms of *S5*, it follows that:

Lemma: $xF \leftrightarrow \diamond xF \leftrightarrow \Box xF$

This *Lemma* plays a very important role in what follows.

Now the first thing that our proper theory asserts is that ordinary objects do not encode properties. Ordinary objects, like you, me, my desk, etc., and objects like us in other possible worlds, just exemplify the properties they have. To represent this formally, let the distinguished predicate '$E!$' denote the property of *existence*, where this is understood as the property of having a location in spacetime. Then we say what it is for an object x to be ordinary ('$O!x$'):

$$O!x =_{df} \diamond E!x$$

Things like you, me, my desk, etc., possibly have a location in space-time, and so we satisfy the definition. Any object x that is possibly a person or possibly a desk also satisfies the definition (given that being a person and being a desk are existence entailing properties) and is to be counted as ordinary. We may now capture the theoretical assertion that ordinary objects don't (and couldn't) encode properties by using our notation 'xF' for encoding as follows:

Proper Axiom: $\forall x(O!x \rightarrow \Box\neg\exists F\, xF)$

The most important principles of the theory will characterize not these ordinary objects but the extraordinary objects that encode properties. In previous work, I have called these objects 'abstract', employing the symbol '$A!$' defined as follows:

[16] They are quite harmless, however, given that we distinguish *quantifying* over an object x from asserting that x (physically) *exists*. So from the fact that we can conclude $\exists x\diamond\phi$ from $\diamond\exists x\phi$, it doesn't follow that the x in question physically exists.

$$A!x =_{df} \neg O!x$$

Thus, abstract objects are not the kind of thing that could have a location in spacetime.

An important principle of the theory guarantees that abstract objects are to be identified by the properties they encode: no two distinct abstract objects encode the same properties. Since the abstract objects and ordinary objects jointly exhaust the domain of objects, the following constitutes a completely general definition of '$x = y$':[17]

$$x{=}y =_{df}$$

$$[O!x \;\&\; O!y \;\&\; \Box\forall F(Fx \leftrightarrow Fy)] \vee [A!x \;\&\; A!y \;\&\; \Box\forall F(xF \leftrightarrow yF)]$$

This principle simply says that two objects x and y are identical iff either x and y are both ordinary objects and necessarily exemplify the same properties or they are both abstract objects and necessarily encode the same properties. The following principle shall govern our defined notion of identity:

Proper Axiom: $x = y \rightarrow [\phi(x, x) \leftrightarrow \phi(x, y)]$, provided that y is substitutable for x in ϕ

In this principle, $\phi(x, y)$ is the result of substituting y for x at some, but not necessarily all, free occurrences of x in $\phi(x, x)$, and the proviso that y be substitutable for x ensures that y is not 'captured' by any quantifier when substituted for x.

Now the main principle comprehending the domain of abstract objects asserts that for any expressible condition ϕ on properties F, there is an abstract object that encodes all and only the properties satisfying the condition:

Proper Axiom: $\exists x[A!x \;\&\; \forall F(xF \leftrightarrow \phi)]$, where ϕ has no free xs

Intuitively, this guarantees that for every set of properties determined by an expressible condition ϕ, there is an abstract object that encodes just the properties in the set. Since there is a wide variety of conditions

[17] The identity sign is not a primitive of the theory!

on properties, this axiom ensures that there is a wide variety of abstract objects.[18]

Note that, given our underlying modal logic, all of the principles of the theory described so far turn out to be necessary truths, for by the rule of necessitation, if ϕ is derivable, then so is $\Box\phi$. This means, in particular, that the comprehension principle for abstract objects is necessarily true.

Now the greater the variety of properties and relations that there is, the greater the variety of abstract objects. To guarantee variety in the former domain, the object theory is supplemented by a theory of relations, properties, and states of affairs. This theory, among other things, serves to make precise the idea that situations and worlds are objects that have internal properties of the form *being such that p*. In previous work, we have set things up so that the main comprehension axioms for relations and states of affairs are logical, rather than proper, axioms. The semantic theory is cast in such a way that these axioms are true in every model.

The main comprehension schema, which circumscribes the domain of relations, asserts that for any *exemplification* condition ϕ on objects having no quantifiers binding relation variables, there is a relation F^n which is such that necessarily, objects $x_1, \ldots, x_n$ exemplify F^n iff ϕ:

> *Relations*: $\exists F^n \Box \forall x_1 \ldots \forall x_n (F^n x_1 \ldots x_n \leftrightarrow \phi)$, where ϕ has no free Fs, no encoding subformulas, and no quantifiers binding relation variables.

Note that the theory doesn't guarantee that there are any new relations constructible in terms of encoding formulas. But all of the familiar, first-order definable, complex relations are constructible.[19] Here are some sample instances of this schema involving 1-place relations:

[18]Lots of examples of instances of this axiom schema may be found in [1988], pp. 22–27, and in [1983], pp. 13 and 35. The hypothesis that there are abstract objects that encode as well as exemplify properties has proven to be a useful one. In [1983], I tried to show that these objects provide an analysis of Platonic Forms, Leibnizian Monads, Possible Worlds, fictional characters, Fregean Senses, and mathematical objects. In [1988], I tried to show how they help us to analyze the problems of intensional logic.

[19]For a more complete discussion of this principle, see pp. 46–50 of [1988]. The restriction that ϕ have no encoding subformulas ensures that the domain of relations is not automatically altered by the introduction of encoding into the theory. This formulation of *Relations* reflects, in part, a certain choice about which theory it is we want to develop, namely, a theory in which we introduce encoding and abstract objects into the foundations of metaphysics *without changing our picture of the domain of relations*. In addition, the restriction serves to avoid a certain paradox—see Zalta [1983], pp. 158–160, and [1988], p. 27. The restriction that ϕ have no quantifiers binding relation variables is not as critical as the 'no encoding subformulas' restriction. See [1983], pp. 159–160.

$$\exists F \Box \forall x (Fx \leftrightarrow \neg Gx)$$
$$\exists F \Box \forall x (Fx \leftrightarrow Gx \,\&\, Hx)$$
$$\exists F \Box \forall x (Fx \leftrightarrow \Box(E!x \rightarrow Px))$$
$$\exists F \Box \forall x (Fx \leftrightarrow \exists z Rzx)$$

The first instance asserts that, given any property G, there is a property F which is such that, necessarily, an object x exemplifies F iff x fails to exemplify G. The second instance asserts that two properties G and H have a conjunction. And so forth.

The abstraction schema *Relations* is in fact a logical theorem schema. It is derivable from the logical axiom schema λ-*Equivalence*:

> λ-*Equivalence*: $\forall x_1 \ldots \forall x_n ([\lambda y_1 \ldots y_n \, \phi] x_1 \ldots x_n \leftrightarrow \phi^{x_1, \ldots, x_n}_{y_1, \ldots, y_n})$, where x_i is substitutable for y_i in ϕ $(1 \leq i \leq n)$.[20]

Here are some λ-expressions that correspond to the above instances of *Relations*:

$$[\lambda y \, \neg Gy]$$
$$[\lambda y \, Gy \,\&\, Hy]$$
$$[\lambda y \, \Box(E!y \rightarrow Py)]$$
$$[\lambda y \, \exists z Rzy]$$

Relations is derived from λ-*Equivalence* by first applying the rule of necessitation and then the rule of existential generalization. The schema ensures that the domain of relations is well-stocked with a *familiar* variety of complex relations.

[20]The notation $\phi^{x_1, \ldots, x_n}_{y_1, \ldots, y_n}$ stands for the result of replacing, respectively, x_i for y_i in ϕ, and the requirement that x_i be substitutable for y_i guarantees that x_i will not be 'captured' by a quantifier when the substitution is carried out.

The restrictions on the *Relations* schema are built right into the formation of λ-expressions. Consequently, the λ-expressions may not contain any formula ϕ having encoding subformulas or quantifiers binding relation variables. For the precise formation rules for λ-expressions, see [1983], p. 60, or [1988], p. 234.

The λ-expressions are interpreted by using an algebraic-style semantics, in which semantic counterparts of Quine's predicate functors in [1960] are used to generate complex relations out of simpler relations. The structure of the λ-expression is a guide to the structure of the relation that it denotes. See [1983], 20–27, 61–67; and [1988], 46–51. By requiring that the domain of relations be closed under these functions in every model of the language, λ-*Equivalence* becomes a logical truth.

To give a complete theory of relations, one must give both comprehension and identity conditions. Identity conditions for relations can be defined in terms of identity conditions for properties, which in turn are defined in terms of our notion of encoding. We stipulate that two properties are identical just in case, necessarily, they are encoded by the same objects:

$$F = G =_{df} \Box\forall x(xF \leftrightarrow xG)$$

In terms of this definition, we may say that two relations F^n and G^n are identical just in case no matter which order you plug $n-1$ objects into both F^n and G^n (plugging F^n and G^n up in the same order), the resulting properties are identical.[21] Note that our definition of property and relation identity is compatible with the idea that necessarily equivalent relations may be distinct. From the fact that two properties are necessarily exemplified by the same objects, it does not follow that they are necessarily encoded by the same objects. One may consistently assert that $F \neq G$ even though $\Box\forall x(Fx \leftrightarrow Gx)$. Nevertheless, our theory of properties is *extensional* in an important sense. That is because properties have encoding extensions in addition to having an exemplification extensions. The theory stipulates that two properties with the same encoding extensions (at every world) are identical, and this is the sense in which the theory of properties is extensional. It is an extensional theory of intensional entities.

Let us call a 0-place relation a 'state of affairs', and let the variables $p, q, r, \ldots$ (in lieu of $F^0, G^0, \ldots$) range over states of affairs. In earlier work, I called these entities 'propositions', but in situation theory, they seem to play the role of 'states of affairs'. Recall that we are adopting the convention of referring to a state of affairs as a 'SOA'. In what follows, the notion of a state of affairs *being factual* or *obtaining* is basic to the theory (i.e., not defined). To assert that a state of affairs p is factual (or obtains), one just *uses* the expression 'p'. To assert that p isn't factual, that both p and q are factual, and that p's factuality is necessary, one uses the expressions '$\neg p$', '$p \,\&\, q$', and '$\Box p$', respectively.

Now if we let $n = 0$, the following 'degenerate' case of the *Relations* principle asserts that for any complex exemplification statement ϕ, there is a state of affairs p such that, necessarily, p is factual (obtains) iff ϕ:

> *States of Affairs*: $\exists p\Box(p \leftrightarrow \phi)$, where ϕ has no free ps, no encoding subformulas, and no quantifiers binding relation variables.

Here are some relevant theorems, where the a_i are constants that (rigidly) denote particular objects, and R^n is a constant that (rigidly) denotes a particular relation:

$$\exists p\Box(p \leftrightarrow Ra_1 \ldots a_n)$$
$$\exists p\Box(p \leftrightarrow \neg Ra_1 \ldots a_n)$$
$$\forall q\exists p\Box(p \leftrightarrow \neg q)$$
$$\forall q\forall r\exists p\Box(p \leftrightarrow q \,\&\, r)$$
$$\forall q\exists p\Box(p \leftrightarrow \Box q)$$

The first example asserts that there is a SOA that is factual iff the objects $a_1,\ldots,a_n$ stand in the relation R. The second example asserts that there is a state of affairs that is factual iff the SOA $Ra_1 \ldots a_n$ fails to be factual. The third example asserts that every SOA q has a negation, the fourth that every two SOAs have a conjunction, and the fifth that every SOA has a necessitation.[22]

Since it constitutes the degenerate case of *Relations*, the schema *States of Affairs* is also derivable from λ-Equivalence, using λ-expressions with *no* variables bound by a 'λ'. To see why, let us read expressions of the form '$[\lambda\, \phi]$' as '*that*-ϕ.' Then the following constitutes a 0-place instance of λ-Equivalence:

$$[\lambda\, \phi] \leftrightarrow \phi$$

This simply asserts: *that*-ϕ obtains iff ϕ.[23] This means that each (complex) exemplification formula ϕ can be used to construct a term ('$[\lambda\,\phi]$')

[21] Formally, this can be stated as follows:

$$F^n = G^n \equiv_{df} \quad (\text{where } n > 1)$$
$$(\forall x_1)\ldots(\forall x_{n-1})([\lambda y\, F^n y x_1 \ldots x_{n-1}] = [\lambda y\, G^n y x_1 \ldots x_{n-1}] \;\& \\ [\lambda y\, F^n x_1 y x_2 \ldots x_{n-1}] = [\lambda y\, G^n x_1 y x_2 \ldots x_{n-1}] \;\& \ldots \& \\ [\lambda y\, F^n x_1 \ldots x_{n-1} y] = [\lambda y\, G^n x_1 \ldots x_{n-1} y])$$

[22] These results already resolve Choices 14 – 17 in Barwise [1989]. We can freely form states of affairs out of any objects and relations (Choice 14); not every SOA is basic (Alternative 15.2, Choice 15); there is a rich algebraic structure on the space of SOAs (Choice 16); and every SOA has a dual (Choice 17).

[23] To derive *States of Affairs* from this, apply necessitation and existential generalization.

that *denotes* a (complex) state of affairs.[24] To simplify the formalism, we shall henceforth use ϕ both as a formula and as a term (i.e., we shall abbreviate $[\lambda\ \phi]$ by ϕ). Note that in most works on situation theory, the expression $\langle\langle R, a_1, \ldots, a_n; 1\rangle\rangle$ is used to denote the SOA that is factual iff objects $a_1, \ldots, a_n$ stand in relation R; however, in the present theory, such a SOA is denoted by the formula $Ra_1 \ldots a_n$.

We are now in a position to consider some rather special instances of *Relations*. Here are just two examples of this special group:

$$\exists F\Box\forall x(Fx \leftrightarrow Ra_1 \ldots a_n)$$
$$\exists F\Box\forall x(Fx \leftrightarrow \neg Ra_1 \ldots a_n)$$

Note that in these two examples, the condition ϕ on objects x is vacuous. The first example asserts that there is a property F which is such that, necessarily, an object x exemplifies F iff objects $a_1, \ldots, a_n$ exemplify relation R^n. The second asserts that there is a property F which is such that, necessarily, an object x exemplifies F iff $a_1, \ldots, a_n$ fail to exemplify R^n. If $a_1, \ldots, a_n$ exemplify R^n, then every object whatsoever exemplifies the former of these two properties and no object exemplifies the latter. If $a_1, \ldots, a_n$ fail to exemplify R^n, then no object exemplifies the former and everything exemplifies the latter. Corresponding to these two examples are the following λ-expressions, respectively:

$$[\lambda y\ Ra_1 \ldots a_n]$$
$$[\lambda y\ \neg Ra_1 \ldots a_n]$$

Of course, these special kinds of properties can be generated for any SOA derived from *States of Affairs*. It is a simple consequence of *Relations* that for any state of affairs p, there is a property F which is such that, necessarily, an object x exemplifies F iff p (obtains):

$$\forall p\exists F\Box\forall x(Fx \leftrightarrow p)$$

So for every state of affairs p, there is a property of *being such that* p, i.e., $[\lambda y\ p]$, which either everything exemplifies or fails to exemplify, depending on whether or not p obtains. If $F = [\lambda y\ p]$, we say that F is *constructed out of* p and that F is a SOA-*property*.[25]

[24] Semantically, these complex states of affairs are constructed using the same logical functions that generate the complex relations. The resulting complexes are therefore highly structured. For more details, see Zalta [1988], pp. 57–61.

[25] In earlier work, I have called these 'propositional properties' or 'vacuous properties'.

To complete the theory of states of affairs, we need to define their identity conditions. Our recent discussion of SOA-properties serves to make the following definition of identity for states of affairs understandable: p and q are identical just in case the property of *being such that* p (i.e., $[\lambda y\ p]$) and the property of *being such that* q (i.e., $[\lambda y\ q]$) are identical.[26]

$$p = q =_{df} [\lambda y\ p] = [\lambda y\ q]$$

This definition, therefore, defines the identity of states of affairs in terms of the defined notion of property identity.[27] It also allows necessarily equivalent states of affairs to be distinct.

Finally, since we have now defined '$F^n = G^n$' for $n \geq 0$, we may assert that identical relations are substitutable for one another. Consequently, the following proper axiom shall govern our defined notions of relation identity ($n \geq 0$):

> *Proper Axiom*: $F^n = G^n \rightarrow [\phi(F^n, F^n) \leftrightarrow \phi(F^n, G^n)]$, provided G^n is substitutable for F^n in ϕ.[28]

This principle of substitution is entirely unrestricted. It completes the presentation of the theory of relations.

§5: The Theory of Situations and Worlds

Recall that in §3 we suggested that a situation is an abstract object (one that may both encode and exemplify properties) which is such that every property it encodes is a SOA-property. Formally, this suggestion can now be represented in terms of the following definition:

$$\mathit{Situation}(x) =_{df} A!x\ \&\ \forall F(xF \rightarrow \exists p(F = [\lambda y\ p]))$$

[26] A definition somewhat similar to this was proposed in Myhill [1963], p. 306.

[27] This definition yields certain theorems that decide Choice 13 in Barwise [1989] in favor of Alternative 13.2. Our semantic picture of basic SOAs treats them as structured complexes, in which the objects are 'plugged' into places of the relation. But there are so many abstract objects generated by the abstraction schema that the theory entails, for some abstract objects a and b, that $Pa = Pb$ even though $a \neq b$. If a and b are ordinary, however, then $Pa \neq Pb$ follows from $a \neq b$. See Zalta [1983], p. 75, footnote 8, and the discussion in Zalta [1988], pp. 31-2.

[28] The notation $\phi(F^n, G^n)$ stands for the result of substituting G^n for F^n at some, but not necessarily all, free occurrences of F^n in $\phi(F^n, F^n)$.

This definition sets the stage for the following series of definitions and theorems.[29] In what follows, we shall use the variable 's' to range over the situations.

The first and foremost theorem of situation theory is a comprehension theorem schema that falls directly out of the proper axiom schema *A-Objects*. The schema for situations asserts that for any condition on SOA-properties, there is a situation that encodes all and only the SOA-properties satisfying the condition. To represent this theorem formally, let us say that a formula $\phi(F)$ is a condition on SOA-properties iff every property F that satisfies ϕ is a SOA-property. The following is then a theorem schema that comprehends the domain of situations:

> *Theorems 1*: $\exists s \forall F(sF \leftrightarrow \phi)$, where ϕ is any condition on SOA-properties having no free ss.

Let us call this theorem scheme '*Situations*'. It forces the domain of situations to be rather rich, and evidence of this richness will be presented as we proceed through the theorems. For the present, let us look just at the instance that yields the situation discussed previously, namely, the situation which is only such that George loves Barbara and such that Barbara doesn't love George:

$$\exists s \forall F(sF \leftrightarrow F = [\lambda y\ Lgb] \lor F = [\lambda y\ \neg Lbg])$$

In this example, $\phi = \ulcorner F = [\lambda y\ Lgb] \lor F = [\lambda y\ \neg Lbg] \urcorner$. Any property F satisfying this ϕ is a SOA-property, and so the object encoding just such properties will be a situation.[30] Other instances of *Situations* will be found frequently in what follows.

Each instance of our theorem scheme asserts that there is a situation that encodes just the SOA-properties meeting a certain condition ϕ. In fact, for each instance, there is a unique situation that encodes just the SOA-properties satisfying ϕ. Where '$\exists! x\psi$' is defined in the usual way to assert that there is a unique x such that ψ, then the following is a lemma to *Situations*:

> *Lemma 1*: $\exists! s \forall F(sF \leftrightarrow \phi)$, for any condition ϕ on SOA-properties having no free ss

[29] The proofs of the theorems are all gathered in Appendix A.

[30] Some readers may find it useful to recall that the set $\{x \mid x = 1 \lor x = 2\}$ is a set that contains just two numbers, and that the set $\{F \mid F = [\lambda y\ Lgb] \lor F = [\lambda y\ \neg Lbg]\}$ contains just two properties. Therefore, the condition ϕ in the above instance of *Situations* yields an object that encodes just two SOA-properties.

To see why this is true, note that for any particular instance of *Situations*, there couldn't be two distinct situations encoding all and only the SOA-properties satisfying the condition ϕ in question, for by the definition of identity for abstract objects, distinct situations have to differ with respect to at least one of the properties they encode.

Now one of the principal notions of situation theory is that of a state of affairs p *being factual in* a situation s (sometimes it is said that s *makes* p *factual*). In most other developments of situation theory, the claim that p is factual in s is taken as basic (i.e., undefined) and it is formally represented as: $s \models p$. But we define this notion within the present theory. Given our basic theoretical understanding of situations, it should be apparent that the states of affairs encoded in a situation (via SOA-properties) are the ones *factual in* that situation. So we shall say: state of affairs p is *factual in* situation s (or, s *makes* p *factual*) iff s encodes the SOA-property of being such that p. So we may introduce the notation of situation theory in the following definition:

$$s \models p =_{df} s[\lambda y\ p]$$

It is very important to note that '$s \models p$' is defined in terms of the variable 'p' that ranges over states of affairs of any complexity. Thus, any formula ϕ that contains no encoding subformulas and no quantifiers binding relation variables may be substituted for the variable p, for these constitute terms that denote (complex) states of affairs. For such ϕ, the expression '$s \models \phi$' is well-defined as: $s[\lambda y \phi]$. Also, in order to disambiguate formulas containing '$\models$', we adopt the following convention: '$\models$' shall be dominated by all the other connectives in a formula. For example, a formula of the form '$s \models p \rightarrow p$' shall be short for '$(s \models p) \rightarrow p$.' We write '$s \models (p \rightarrow p)$' to assert that s makes the complex state of affairs $p \rightarrow p$ factual.

Given this definition, it now follows that two situations are identical just in case the same SOAs are factual in them.[31]

> *Theorem 2*: $s = s' \leftrightarrow \forall p(s \models p \leftrightarrow s' \models p)$

In other applications of the theory of abstract objects, it has proven useful to define the following notion of part-whole: x is a *part-of* y iff y encodes

[31] This determines another choice at one of the branch points of situation theory. In Barwise [1989], Choice 5 (p. 264) concerns the question of whether situations that support the same infons (or SOAS) are identical. The following theorem decides the issue in favor of Barwise's Alternative 5.1, namely, such SOAS are identical.

every property x encodes. To capture this definition formally, let us use the symbol '$\trianglelefteq$' to represent the notion *part-of*. We therefore have:

$$x \trianglelefteq y =_{df} \forall F(xF \rightarrow yF)$$

Consequently, it follows that every part of a situation is a situation:[32]

Theorem 3: $\forall x[x \trianglelefteq s \rightarrow \mathit{Situation}(x)]$

It is also an immediate consequence that a situation s is a part of situation s' iff every SOA factual in s is factual in s'.

Theorem 4: $s \trianglelefteq s' \leftrightarrow \forall p(s \models p \rightarrow s' \models p)$

This simple theorem is significant because it shows that the theory predicts a natural situation-theoretic analysis of the notion of *part-of*. The theory also makes two other simple predictions, namely, that two situations are identical iff each is part of the other, and that two situations are identical iff they have the same parts:

Theorem 5: $s = s' \leftrightarrow s \trianglelefteq s' \;\&\; s' \trianglelefteq s$

Theorem 6: $s = s' \leftrightarrow \forall s''(s'' \trianglelefteq s \leftrightarrow s'' \trianglelefteq s')$

In light of these results, we shall say that a situation s is a *proper* part of s' just in case s is a part of s' and $s \neq s'$.

In addition to these facts about parts and wholes, it turns out that the entire domain of situations is partially ordered by the notion of *part-of*:[33]

Theorem 7: *Part-of* ($\trianglelefteq$) *is reflexive, anti-symmetric, and transitive on the situations.*

Another important notion of situation theory is *persistency*. Following the situation theorists, we say that a state of affairs p is *persistent* iff whenever p is factual in a situation s, p is factual in every situation s' of which s is a part.[34]

[32]This rather simple theorem decides another choice point in situation theory, namely, Choice 2, where Barwise ([1989], p. 261) asks whether every part of a situation is a situation. Our theory asserts that it is.

[33]Note that whereas the partial ordering of situations is assumed in Barwise [1989] (p. 259), before the branch points of situation theory are even enumerated, this partial ordering turns out to be a consequence of our theory.

[34]This definition follows Barwise [1989], p. 265.

$$\mathit{Persistent}(p) =_{df} \forall s[s \models p \rightarrow \forall s'(s \trianglelefteq s' \rightarrow s' \models p)]$$

Clearly, persistency is built right into the theory, for given the above definition of *part-of*, it is an immediate consequence of the foregoing that all states of affairs are persistent:[35]

Theorem 8: $\forall p\, \mathit{Persistent}(p)$

In the next group of theorems, we consider what kinds of situations are to be found in our domain. One of the most important questions to ask about a situation is whether or not it is actual. In other versions of situation theory, philosophers have restricted themselves to the actual situations. For us, the actual situations constitute just a part of the domain of situations. Let us say that a situation s is *actual* iff every SOA factual in s is factual; i.e.,

$$\mathit{Actual}(s) =_{df} \forall p(s \models p \rightarrow p)$$

Given this definition, it follows that there are both actual and non-actual situations:[36]

Theorem 9: $\exists s\, \mathit{Actual}(s) \;\&\; \exists s \neg \mathit{Actual}(s)$

Moreover, it follows that no state of affairs and its negation both are factual in any actual situation,[37] and that some SOAs are not factual in any actual situations:

Theorem 10: $\forall s[\mathit{Actual}(s) \rightarrow \neg\exists p(s \models p \,\&\, s \models \neg p)]$

Theorem 11: $\exists p \forall s(\mathit{Actual}(s) \rightarrow s \not\models p)$.

Our comprehension principle also guarantees that for any two (actual) situations, there is an (actual) situation of which they are both a part.[38]

Theorem 12: $\forall s \forall s' \exists s''(s \trianglelefteq s'' \;\&\; s' \trianglelefteq s'')$

[35]Thus, the theory comes down in favor of Alternative 6.1 at Choice 6 (p. 265) in Barwise [1989]. It should also be clear that the theory resolves Choice 11 (p. 268) in favor of Alternative 11.1. No relations are perspectival; the argument places of a relation R involved in a situation s remain the same in any situation s' of which s is a part.

[36]The theory here decides Choice 4 (p. 262) of Barwise [1989] in favor of Alternative 4.2.

[37]Compare the Coherency Principle in Barwise [1989], p. 235.

[38]Compare the Compatibility Principle in Barwise [1989], p. 235.

On our theory of situations, there are two different notions of maximality, as well as two corresponding notions of partiality. Let us say that a situation s is *maximal*$_1$ iff every SOA or its negation is factual in s. A situation s is *partial*$_1$ iff some SOA and its negation are not factual in s. A situation s is *maximal*$_2$ iff every SOA is factual in s. A situation s is *partial*$_2$ iff some SOA is not factual in s. Formally:

$$Maximal_1(s) =_{df} \forall p(s \models p \vee s \models \neg p)$$
$$Partial_1(s) =_{df} \exists p(s \not\models p \,\&\, s \not\models \neg p)$$

$$Maximal_2(s) =_{df} \forall p(s \models p)$$
$$Partial_2(s) =_{df} \exists p(s \not\models p)$$

The reader should now be able to use *Situations* to demonstrate the following:

Theorem 13: *There are maximal*$_1$ *and partial*$_1$ *situations.*

Theorem 14: *There are maximal*$_2$ *and partial*$_2$ *situations.*

The discussion of maximality brings us naturally to the question of whether there are any maximal situations that could reasonably be called 'possible worlds'. Let us say that a situation s is a *world* iff it is possible that all and only factual SOAs are factual in s; i.e.,

$$World(s) =_{df} \Diamond\forall p(s \models p \leftrightarrow p)$$

In other words, those situations that *might* make factual all and only the facts are worlds.[39] Note that since the modal operator 'possibly' is defined in terms of the primitive modal operator 'necessarily', we are using a primitive notion of modality to define the notion of a world. Instead of taking possible worlds as primitive entities, as one does in 'possible world semantics', we are taking the first step in developing a *theory* of worlds.

This theory has lots of interesting consequences, but before we describe them, it is important to make a few observations about the modal behavior of situations. First, note that *Situations* is a necessary truth! This necessary truth, or $\Box$*Situations*, is derived from a comprehension axiom (the one for abstract objects) that is (provably) necessarily true. The derivation makes no appeal to contingent truths. So the rule of necessitation applies, yielding $\Box$*Situations*. Second, recall that the logical

[39]This decides Choice 3 in Barwise [1989] in favor of Alternative 3.1: worlds are situations.

axiom for encoding asserts that if an object possibly encodes a property, it necessarily does. This principle ensures that any SOA-property that a situation possibly encodes is one that it necessarily encodes, and *a fortiori*, one that it in fact encodes. In situation theoretic terms, this means that if it is possible that s makes p factual, it is necessary that s makes p factual. In fact, by the *Lemma* to the logical axiom for encoding (see §4), we have the following:

Lemma 2: $s \models p \leftrightarrow \Diamond s \models p \leftrightarrow \Box s \models p$

Lemma 2 and $\Box$*Situations* combine to produce the following effect: whenever we are describing a particular possibility, not only can we always appeal to $\Box$*Situations* to tell us what situations s there are relative to this possiblility (since $\Box$*Situations* is necessary), but furthermore, any truth of the form $s \models p$ *relative* to this possibility that we discover when appealing to $\Box$*Situations* turns out, by *Lemma 2*, to be true *simpliciter*, and a necessary truth at that. This effect proves to be crucial to the proofs of the theorems that follow. With this observation, we turn to the theorems of world theory.[40]

The first, but not foremost, theorem of world theory is that every world is maximal:[41]

Theorem 15: *All worlds are maximal*$_1$.

Let us say next that a situation s is *possible* iff it is possible that s is actual. Let us also say that a situation s is *consistent* iff s doesn't make incompatible states of affairs factual (i.e., iff no contradictory SOAs are factual in s). Formally, we have:

$$Possible(s) =_{df} \Diamond Actual(s)$$

[40]Some of the theorems that follow were proved in Chapter IV of Zalta [1983], and discussed further in Zalta [1987] and Zalta [1988]. They are recast here in the framework of situation theory.

[41]In many of the standard works on world theory, this principle often forms part of the very definition of a world. Specifically, the maximality of worlds forms part of the definition of 'world' given by Chisholm, Plantinga, and Fine. On their view, a world is a possible state of affairs p such that for every state of affairs q, either $p \Rightarrow q$ or $p \Rightarrow \neg q$. Since for these philosophers, a state of affairs q is *true at* a world w just in case $w \Rightarrow q$, it should be clear that maximality is built right into the definition of a world. On the other hand, in Lewis' world theory, and in possible world semantics, this principle is a consequence of defining a proposition p as a set of worlds and defining the negation of p as the complement set of worlds. See the works of these authors cited in the Bibliography.

$$Consistent(s) =_{df} \neg\exists p\exists q[\neg\Diamond(p \,\&\, q) \,\&\, s \models p \,\&\, s \models q].$$

It now follows both that:

Theorem 16: *All possible situations are consistent.*

Theorem 17: *All worlds are possible and consistent.*

Now that we have derived some basic truths about worlds in general, it seems reasonable to ask whether there are worlds that are actual, and if so, how many there are. On the present theory, the answers to these questions are 'yes' and 'one', for there is a unique actual world:

Theorem 18: $\exists! w Actual(w)$

In what follows, we use the symbol 'w_α' to (rigidly) designate the unique actual world.

The next basic intuition concerning situations and worlds predicted by the theory is that all and only actual situations are part of the actual world:

Theorem 19: $\forall s(Actual(s) \leftrightarrow s \trianglelefteq w_\alpha)$

Let us take a moment to reflect on these results before we turn to the next group of theorems. In some versions of situation theory, only actual situations are tolerated. It is assumed that there is an actual world, and that it is a maximal element under the relation of *part-of*.[42] Thus, the actual world is not a proper part of anything, but is a 'maximal element' in the sense that every (actual) situation is a part of it.

Contrast such versions of situation theory with the present one. *Theorem 19* tells us that w_α is a 'maximal element' in the sense that every actual situation is a part of it. But note that w_α is nevertheless part of lots of nonactual situations (though no nonactual situation will be part of it). For example, take a situation in which all the SOAs factual in w_α are factual, and in addition, the negation of one of those SOAs is factual as well. Such a situation is non-actual (for it makes a contradiction factual). It is maximal$_1$, but not maximal$_2$. It is neither a possible nor a consistent situation. But w_α is a part of it. And w_α is also a part of the 'universal situation', the situation in which every SOA is factual. The universal situation is maximal$_1$, maximal$_2$, and is a genuine maximal element under *part-of* (every situation is a part of it, but it is not a proper part of any situation). But it is neither actual, possible, nor consistent. Consequently, though the actual world is part of lots of nonactual situations, it is not a part of any actual situation other than itself. So our subdomain of actual situations looks almost exactly like the domain of situations posited by the philosophers who believe that there are only actual situations. Thus, when restricted to the subdomain of actual situations, the theory still yields theorems that capture most of the intuitions held by the 'actualist' situation theorists.

[42]For example, in Barwise [1989], on p. 259, it is *assumed* that there is an actual world and that it is a maximal element. On p. 261, the actual situations are *defined* to be the ones that are part of the actual world.

The next theorem gives us some basic information about the actual world. It follows from the definition of the actual world w_α that a state of affairs is factual (*simpliciter*) iff it is factual in w_α.

Theorem 20: $p \leftrightarrow w_\alpha \models p$

Recall that the notion of *being factual* is basic to the theory. But no new primitive notation was introduced to mark this notion. *Theorem 20* therefore shows that the theory offers an analysis of *being factual* (or *obtaining*) in terms of its other primitive notions.

Theorem 20 also points us toward some important results about the relationship between the internal and external properties of actual situations in general. We have been treating the internal properties of a situation s as encoded properties of the form $[\lambda y\ p]$. The external properties of a situation are the ones that it exemplifies. For example, all situations exemplify the property of *not being a spoon*. Some, but not others, exemplify the property of *being seen by Mary*. In general, however, situations, like all other objects, are *complete* with respect to the properties they exemplify, in the sense that: $\forall s \forall F(Fs \vee \bar{F}s)$, where $\bar{F} = [\lambda y\ \neg Fy]$. Consequently, situations will be complete with respect to the SOA-properties that they exemplify: $\forall s \forall p([\lambda y\ p]s \vee [\lambda y\ \neg p]s)$. If we now think about the relationship between the internal and external properties of an actual situation, it should be clear that actual situations exemplify (externally) every (internal) property they encode.

Theorem 21: $\forall s[Actual(s) \rightarrow \forall F(sF \rightarrow Fs)]$

We may express this theorem in situation theoretic terms as the following lemma:

Lemma 3: $\forall s[Actual(s) \rightarrow \forall p(s \models p \rightarrow [\lambda y\ p]s)]$

Of course, there are lots of properties that actual situations exemplify that they won't necessarily encode, such as not being a spoon, being seen by Mary, being depressing, etc. But if we restrict ourselves to the SOA-properties, then the actual world w_α turns out to be a rather special actual situation that has exactly the same internal and external SOA-properties. In situation theoretic terms, a state of affairs p is factual in w_α iff w_α exemplifies being such that p:

Theorem 22: $w_\alpha \models p \leftrightarrow [\lambda y\ p]w_\alpha$

Theorems 20 and *22* have the following interesting consequence about the actual world, namely that a state of affairs p is factual iff the state of affairs, w_α's being such that p, is factual in w_α:

Theorem 23: $p \leftrightarrow w_\alpha \models [\lambda y\ p]w_\alpha$

What is noteworthy about this theorem is its logical form: if we think of the formula '$[\lambda y\ p]s$' as a formula of the form $\phi(s)$, then *Theorem 23* shows that w_α is a situation s such that $s \models \phi(s)$. Intuitively (indeed, semantically) this suggests that w_α is a constituent of the facts that it makes factual.[43]

Let us turn to the final group of theorems—ones which verify our deepest intuitions about the relationship between modality, situations, and possible worlds. The foremost principle of world theory is that a state of affairs (proposition) is necessary iff it is factual (true) in all worlds. Of course, this principle led Kripke in [1959] and [1963] to conceive of his semantics of modal logic, and set the stage for thirty years of fruitful research in modal logic. In Kripke's work, this principle was the guiding force by which the primitive notions of modality were interpreted by the primitive semantic notions of world theory. In the present theory, however, the primitive notions of modality are couched in our object language and the notions of world theory are defined in terms of them. We *derive* the equivalence of necessity and factuality in all worlds as a theorem:

Theorem 24: $\Box p \leftrightarrow \forall w(w \models p)$

Of course, the dual of this claim is also a theorem, namely, that a state of affairs is possible iff it is factual in some world:[44]

Theorem 25: $\Diamond p \leftrightarrow \exists w(w \models p)$

This could be the most important ontological consequence of world theory. Anytime we add to the system a statement of the form $\neg q\ \&\ \Diamond q$ (i.e., that q doesn't obtain but might have), *Theorem 25* guarantees that there is a world that is distinct from the actual world and in which q is factual. For example, let q be the state of affairs: *George Bush lost the 1988 presidential election*. Then q is not factual but might have been. So, by *Theorem 25*, there is a world w that makes q factual. The world w is not the actual world w_α, since the former makes q factual while the latter makes $\neg q$ factual. So there is a possible world other than the actual world.

Thus, the principles of the theory support an argument for possible worlds other than the actual world. The great variety of intuitive truths of the form $\neg q\ \&\ \Diamond q$ *theoretically implies* that there is a great variety of possible worlds.[45]

Note that our worlds are individuals, not properties or states of affairs. They are not entities that can be instantiated (compare Stalnaker's

[43]In situation theory, statements of the form $s \models \phi(s)$ constitute the defining characteristic of 'nonwellfounded' situations. So the actual world w_α seems to be nonwellfounded in the sense that it makes factual states of affairs p of which it is a constituent. These theorems decide Choices 8, 9, and 10 in Barwise [1989]: situations can be constituents of facts; not every object is a situation; and at least some situations are non-well-founded.

There is other evidence for thinking that w_α, and actual situations in general, are nonwellfounded in some sense. And that has to do with what appears to be a 'natural' model of the theory, but which cannot be developed within the theory of wellfounded sets. To see this, note that the actual world, and other actual situations all have the following feature: they are objects that, for certain properties F, encode and exemplify the very same F. Now suppose you tried to model, within ZF, 'x encodes (internally) F' as '$F \in x$' (modeling x as a set of properties) and model 'x exemplifies (externally) F' as '$x \in F$' (modeling F as the set of individuals that exemplify it). This seems to be a natural way to use $\in$ to model encoding. But, then, this picture turns out to be in violation of the wellfoundedness of ZF sets, since for certain actual situations s and properties F, the fact that $sF\ \&\ Fs$ would require, in the model, that both $F \in s$ and $s \in F$. This result seems to square with the intuitions of the situation theorists who believe that nonwellfounded sets provide the best picture of situations.

[44]Thus we derive what D. Lewis must stipulate. Recall the quotation from p. 2 of his [1986], where he says, "There are so many other worlds, in fact, that absolutely *every* way a world could possibly be is a way that some world is." He repeats this claim in several other places (for example, pp. 71, 86). The present theorem captures this as the claim that for each possible state of affairs, there is a world in which that state of affairs is factual.

[45]So, given at least one claim of the form $\neg p\ \&\ \Diamond p$, the theory decides Choice 1 in Barwise [1989] in favor of Alternative 1.2: there is more than one world.

description). On the other hand, they are abstract rather than concrete or possibly concrete individuals, which distinguishes them from Lewisian worlds. It might be thought that our conception is incompatible with that of "*actualists* who hold that nothing is real except what is actual—that is, except what exists as part of the actual world" (Stalnaker's description). But, in fact, our conception could be made consistent with this actualistic view. We could simply use the predicate '$C!$' instead of '$E!$' as our primitive predicate for having a location in spacetime, reserving the notion of existence for reading the quantifier '$\exists$'. Then the abstraction schema for abstract objects would assert the *existence* of our abstract (i.e., not possibly concrete) objects. We could then agree with the actualists that "everything that exists (i.e., everything there is) is actual." Our possible worlds would all be 'actual' in the sense that they exist; they are all members of the single domain of objects, over which our all-embracing quantifier ranges. Thus, whereas Lewis's *other* possible worlds are existing concrete objects which are not actual (i.e., not spatiotemporally related to us), our worlds (on this reading of the quantifier) would be existing (actual) abstract objects, though 'nonactual' in the defined sense that they encode falsehoods.

Moreover, we should not be subject to the objection that actualists have raised against Lewis's view, namely, that it postulates the existence of nonactual *concrete* objects.[46] Lewis's worlds are a subset of his possibilia, but ours are not. Our possibilia are objects that are possibly concrete (i.e., they possibly have a location in spacetime), but which are not concrete. Unlike Lewis, we do not assert the existence of talking donkeys and million carat diamonds. We accept that it is *possible* that there are talking donkeys, and that it is *possible* that there are million carat diamonds. And given the Barcan formulas, it follows that there are (or if you prefer to read the quantifier as existentially loaded, that there exist) things that are *possibly* talking donkeys, and that are *possibly* million carat diamonds. But these possibly concrete objects are not concrete, and so they do not exemplify the properties of being a talking donkey or being a million carat diamond, respectively. Rather, they exemplify the negations of both of these properties. We assert that there are (or again, with existentially loaded quantifiers, that there exist) no objects that exemplify the property of being a talking donkey or that exemplify

[46]See van Inwagen [1986], and Lycan [1988].

the property of being a million carat diamond.[47]

Nor do we face the dilemma faced by abstractionists who want to treat worlds as states of affairs or properties. Our *Theorem 18* establishes that there is a unique actual world. Within the present theory, this theorem is quite compatible with the idea that necessarily equivalent states of affairs may be distinct. Compare this with the predicament, described at the end of §2, facing philosophers who accept only individuals and fine-grained properties, relations, and propositions (or states) in their ontology. The attempt to identify worlds as maximal and possible propositions (or states, or properties) runs up against the problem that, on such a conception, there seem to be multiple, distinct copies of each of the possible worlds, and in particular, multiple copies of the actual world. We have made no *ad hoc* adjustments to reconcile the uniqueness of worlds with the fine-grainedness of states of affairs.

The distinction between exemplifying and encoding a property seems to capture what it is for a situation and world to have a *nature* that is defined by the states of affairs that they make factual. It is part of the very nature of situations and worlds that they make states of affairs factual. This demonstrates that our worlds are not *ersatz* worlds. Our worlds do not "represent the entire concrete world in all its detail, as it is or might have been." Unlike the other abstractionist 'worlds', our worlds are *characterized* by the states of affairs they make factual, for encoding is a mode of predication. If a world (or situation) makes p factual, then that world (situation) *is* such that p, in an important new sense of the copula. This fact, I believe, will serve to undermine the attempt to apply Lewis's objections regarding ersatz worlds to the present theory.[48]

Conclusion

The foregoing set of theorems forms an effective foundation for the theory of situations nd worlds. All twenty-five theorems seem to be basic,

[47]Notice that if one were to read the quantifier as existentially loaded, all objects would exist necessarily (since $\Box\exists y\, y = \tau$, for any term τ, is a theorem of our simple quantified modal logic), but this is not to say that any object would be necessarily concrete. On this reading of the quantifier, the notion of a 'contingent being' is properly analyzed as the notion of 'contingently concrete being'. Our ordinary objects, we may assert, are contingently concrete, and this is how our theory allows for contingent beings.

[48]See Lewis [1986], pp. 136–42, and 174–91.

reasonable principles that structure the domains of properties, relations, states ofd affairs, situations, and worlds in true and philosophically interesting ways. They resolve 15 of the 19 choice points defined by Barwise [1989] (see footnotes 22, 27, 31, 32, 35, 36, 39, 43, and 45). Moreover, important axioms and principles stipulated by situation theorists are derived (see footnotes 33, 37, and 38). This is convincing evidence that the foregoing constitutes a theory of situations.[49] Note that worlds are just a special kind of situation, and that the basic theorems of world theory, which were derived in previous work, can still be derived in the situation-theoretic setting. So there seems to be no fundamental incompatibility between situations and worlds—they may peacably coexist in the foundations of metaphysics. The theory may therefore reconcile two research programs that appeared to be heading off in different directions. And we must remind the reader that the general metaphysical principles underlying our theory were not designed with the application to situation theory in mind. This suggests that the general theory and the underlying distinction have explanatory power, for they seem to relate and systematize apparently unrelated phenomena.

[49]For a more extensive discussion of the way in which the theory captures the standard conception of a situation, see Zalta [1991], §4.

Appendix A: Proofs of the Theorems

Proof of T1: The comprehension principle for abstract objects is: $\exists x(A!x \;\&\; \forall F(xF \leftrightarrow \phi))$, where ϕ has no free xs. But the conditions ϕ on SOA-properties constitute a subset of the conditions that may be used in this comprehension schema for abstract objects. Moreover, any object 'generated' by such a condition on SOA-properties encodes only SOA-properties, and will therefore be a situation.

Proof of T2: ($\leftarrow$) Our hypothesis is that the same SOAs are factual in both s and s', and we want to show that these two situations are identical. Since both s and s' are situations, and hence abstract objects, to show that they are identical, we must show that necessarily, they encode the same properties. We reason by showing, for an arbitrary property Q, that $\Box(sQ \leftrightarrow s'Q)$, for then by universal generalization and the Barcan formulas we are done.

The first step is to show $sQ \leftrightarrow s'Q$, and then we'll show that this is necessary. ($\rightarrow$) Assume sQ. Then since s is a situation, Q must be a SOA-property, say $[\lambda y\, q]$ (for some state of affairs q). So s encodes $[\lambda y\, q]$, and by the definition of 'factual in', q is factual in s ($s \models q$). But our initial hypothesis is that the same SOAs are factual in s and s', and so $s' \models q$, i.e., $s'[\lambda y\, q]$. So s' encodes the property Q. ($\leftarrow$) Reverse reasoning. Thus we have $sQ \leftrightarrow s'Q$.

Now, for *reductio*, suppose that this biconditional is *not* necessary. Then, it must be possible that sQ and $s'Q$ differ in truth value. So, without loss of generality, let us say $\Diamond(sQ \;\&\; \neg s'Q)$. But if so, then (a) $\Diamond sQ$ and (b) $\Diamond\neg s'Q$. Now in virtue of the *Logical Axiom* $\Diamond xF \rightarrow \Box xF$, it follows from (a) that $\Box sQ$, and so in fact sQ. Now (b) is equivalent to $\neg\Box s'Q$, and so it also follows from the *Logical Axiom* (this time by *Modus Tollens*) that $\neg\Diamond s'Q$, i.e., $\Box\neg s'Q$. So in fact, $\neg s'Q$. But we have now proved both sQ and $\neg s'Q$, and this contradicts our first result that sQ and $s'Q$ in fact have the same truth value.

Thus, $\Box(sQ \leftrightarrow s'Q)$. And so by universal generalization and the Barcan formulas,$\Box\forall F(sF \leftrightarrow s'F)$. Thus, s and s' are identical. ⋈

Proof of T3: Suppose x is a part of situation s and that x encodes G (to show G is a SOA-property). Then, since s encodes every property x encodes, s encodes G. But since s is a situation, every property it encodes is a SOA-property. So G is a SOA-property.

Proof of T4: ($\rightarrow$) Assume s is a part of s' and that $s \models q$ (to show $s' \models q$). By definition of $s \models q$, we have $s[\lambda y\ q]$. Since every property encoded by s is encoded by s', $s'[\lambda y\ q]$, i.e., $s' \models q$. ($\leftarrow$) Assume $\forall p(s \models p \rightarrow s' \models p)$ and that s encodes G (to show s' encodes G). Since s is a situation, $G = [\lambda y\ q]$, for some SOA q, and so $s \models q$. By hypothesis, then, $s' \models q$, and so s' encodes G.

Proof of T5: ($\leftarrow$) If s and s' are both parts of each other, then they encode exactly the same properties. So by the definition of identity for abstract objects, they are identical.

Proof of T6: ($\leftarrow$) Suppose s and s' have the same parts. To show that s and s' are identical, we must show that they encode the same properties. For *reductio*, suppose they don't. Then (without loss of generality) s encodes a property G that s' fails to encode. Now by the lemma to *Situations*, there is a unique situation, call it 's_0', that encodes just the property G. Clearly, s_0 is a part of s, but since s_0 encodes a property s' doesn't encode, s_0 is not a part of s'. Thus, s and s' don't have the same parts, contrary to hypothesis.

Proof of T7: Reflexivity is straightforward. To see that *part-of* is anti-symmetric, assume $s \trianglelefteq s'$ and $s \neq s'$. Then, there is a property s' that is not encoded in s. So, $\neg(s' \trianglelefteq s)$. To see that *part-of* is transitive, assume $s \trianglelefteq s'$ and $s' \trianglelefteq s''$ and that s encodes property G (to show that s'' encodes G). Since s is a part of s', s' encodes G. Since s' is a part of s'', s'' encodes G.

Proof of T8: Assume $s \models p$ and that $s \trianglelefteq s'$. Then by *Theorem 4*, $s' \models p$.

Proof of T9: Consider the following two instances of *Situations*:

$$\exists s \forall F(sF \leftrightarrow F = [\lambda y\ q])$$
$$\exists s \forall F(sF \leftrightarrow F = [\lambda y\ \neg q])$$

Now if q obtains, then $\neg q$ doesn't. So the first instance gives us an actual situation (in which q and no other SOA is factual), while the second instance gives us a non-actual situation (in which $\neg q$ and no other SOA is factual). However, if $\neg q$ obtains, then q doesn't obtain. Then, the first instance gives us a non-actual situation whereas the second gives us an actual one. But either q or $\neg q$ obtains.

Proof of T10: Assume s is actual. Then $\forall p(s \models p \rightarrow p)$. For *reductio*, assume that there is a SOA q such that both $s \models q$ and $s \models \neg q$. Then, since s is actual, both q and $\neg q$ obtain, which is impossible.

Proof of T11: By *States of Affairs*, for an arbitrary SOA q, there is a complex SOA $q \,\&\, \neg q$. Assume for an arbitrary situation s that s is actual and that $s \models (q \,\&\, \neg q)$. Then by the actuality of s, $q \,\&\, \neg q$, which is impossible. So, for any actual situation s, if s is actual, $s \not\models (q \,\&\, \neg q)$. So there is a SOA p that is not made factual by any situation.

Proof of T12: By *Situations*, there is a situation that encodes all and only the SOA-properties F constructed out of SOAs factual in either s or s'; i.e.,

$$\exists s'' \forall F[s''F \leftrightarrow \exists p((s \models p \vee s' \models p) \,\&\, F = [\lambda y\ p])]$$

Note that if s and s' are both actual, so is s''.

Proof of T13: Consider the following two instances of *Situations*:

$$\exists s \forall F[sF \leftrightarrow \exists p(F = [\lambda y\ p])]$$
$$\exists s \forall F(sF \leftrightarrow F = [\lambda y\ q])$$

The first instance yields a situation ('s_1') that makes every state of affairs factual. *A fortiori*, s_1 is maximal$_1$. The second instance yields a situation ('s_2') that makes just q factual. Then, for any SOA r such that $q \neq r$ and $q \neq \neg r$, s_2 makes neither r nor $\neg r$ factual. So s_2 is partial$_1$.

Proof of T14: Consider the same two instances of *Situations* utilized in the previous proof. Situation s_1 is maximal$_2$, and s_2 is partial$_2$.

Proof of T15: Suppose s is a world. Then $\Diamond\forall p(s \models p \leftrightarrow p)$. We first try to establish, for an arbitrary SOA q, that $\Diamond(s \models q \vee s \models \neg q)$, for then it will follow by *Lemma 2* that $s \models q \vee s \models \neg q$, and hence that *Maximal*$_1(s)$. Now if we momentarily assume $\forall p(s \models p \leftrightarrow p)$, we can use the fact that $\Box(q \vee \neg q)$ to establish that $s \models q \vee s \models \neg q$. So by conditional proof: $\forall p(s \models p \leftrightarrow p) \rightarrow (s \models q \vee s \models \neg q)$. Since this conditional was proved without appealing to any contingencies, the rule of necessitation applies and we get: $\Box[\forall p(s \models p \leftrightarrow p) \rightarrow (s \models q \vee s \models \neg q)]$. From this fact, and the original fact that $\Diamond\forall p(s \models p \leftrightarrow p)$, we may apply the following well known theorem of modal logic: $\Box(\phi \rightarrow \psi) \rightarrow (\Diamond\phi \rightarrow \Diamond\psi)$. Applying this theorem yields: $\Diamond(s \models q \vee s \models \neg q)$, which is our first objective.

From this fact, it follows that $\Diamond s \models q \vee \Diamond s \models \neg q$. But by *Lemma 2*, each disjunct gives us a nonmodal truth about s, and so it follows that $s \models q \vee s \models \neg q$. Since q was arbitrary, we have shown: *Maximal*$_1(s)$.

Proof of T16: Assume s is possible. Then, $\Diamond\forall p(s \models p \rightarrow p)$. For *reductio*, assume s is not consistent. Then, there are states of affairs q and r such that $\neg\Diamond(q \,\&\, r)$ and for which both $s \models q$ and $s \models r$. Note that by *Lemma 2*, these last two facts are necessary. Moreover, they are all that is needed to establish: $\forall p(s \models p \rightarrow p) \rightarrow (q \,\&\, r)$. Since this conditional is provable using only necessary truths, the rule of necessitation applies and yields: $\Box[\forall p(s \models p \rightarrow p) \rightarrow (q \,\&\, r)]$. But, by hypothesis, $\Diamond\forall p(s \models p \rightarrow p)$. So it follows by a previously mentioned principle of modal logic that $\Diamond(q \,\&\, r)$, which contradicts the fact (derived from our *reductio* hypothesis) that $\neg\Diamond(q \,\&\, r)$.

Proof of T17: Suppose s is a world. Then it follows immediately that s is possible. So by *Theorem 16*, it follows that s is consistent.

Proof of T18: Consider the situation that encodes all and only those properties F constructed out of SOAs that are factual; i.e,

$$\exists s \forall F[sF \leftrightarrow \exists p(p \,\&\, F = [\lambda y\, p])]$$

Call such a situation 's_0.' It is straightforward to show that s_0 has the following feature, for an arbitrary SOA q: $s_0 \models q \leftrightarrow q$. So, *a fortiori*, s_0 is both a world and actual. Now to see that there couldn't be two distinct actual worlds, suppose for *reductio* that s' is a distinct actual world. Since s' and s_0 are distinct, there must be a SOA q factual in one but not in the other (by *Theorem 2*). Suppose, without loss of generality, that $s_0 \models q$ and $s' \not\models q$. Then since s' is a world, it is maximal$_1$. So $s' \models \neg q$. But since both s_0 and s' are actual, both q and $\neg q$ must obtain, which is a contradiction.

Proof of T19: ($\rightarrow$) Suppose s is actual and that q is a state of affairs factual in s. Then q must be factual. But since all and only the factual SOAs are factual in w_α (by definition of w_α), q is factual in w_α. So by *Theorem 4*, $s \trianglelefteq w_\alpha$. ($\leftarrow$) By reverse reasoning.

Proof of T20: By definition of w_α.

Proof of T21: Assume s is actual and encodes G (to show s exemplifies G). Then, for some p, $G = [\lambda y\, p]$. So s makes p factual, and since s is actual, p obtains. But, by λ-abstraction, necessarily, an object x exemplifies $[\lambda y\, p]$ iff p obtains (i.e., $[\lambda y\, p]x \leftrightarrow p$). So, in particular, s exemplifies $[\lambda y\, p]$, i.e., s exemplifies G.

Proof of T22: ($\rightarrow$) By *Lemma 3*. ($\leftarrow$) Suppose $[\lambda y\, p]w_\alpha$. Then, by λ-abstraction, p is factual. So, by *Theorem 20*, $w_\alpha \models p$.

Proof of T23: ($\rightarrow$) Suppose p. Then by *Theorems 20* and *22*, $[\lambda y\, p]w_\alpha$. But let $q = [\lambda y\, p]w_\alpha$. Then, by *Theorem 20*, $w_\alpha \models q$, i.e., $w_\alpha \models [\lambda y\, p]w_\alpha$. ($\leftarrow$) By reverse reasoning.

Proof of T24: ($\rightarrow$) Assume $\Box q$. We want to show, for an arbitrarily chosen world w, that $w \models q$. Since w is a world, $\Diamond\forall p(w \models p \leftrightarrow p)$.

Moreover, by appealing to $\Box q$, it is easy to establish: $\Box[\forall p(w \models p \leftrightarrow p) \rightarrow w \models q]$. Since we know $\Diamond\forall p(w \models p \leftrightarrow p)$, it follows by now familiar reasoning that $\Diamond w \models q$, and by *Lemma 2*, $w \models q$. ($\leftarrow$) Assume that $\forall w(w \models q)$. By *Lemma 2*, we know that if $w \models q$ then $\Box w \models q$. So $\forall w \Box(w \models q)$, and by the Barcan formulas that $\Box\forall w(w \models q)$. Now if we can show $\Box[\forall w(w \models q) \rightarrow q]$, then by a familiar theorem of $S5$, namely, $\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$, we are done. But recall that, by hypothesis, $\forall w(w \models q)$. So, in particular, $w_\alpha \models q$. So q. By conditional proof, $\forall w(w \models q) \rightarrow q$. Since no contingent information was used in the proof, $\Box[\forall w(w \models q) \rightarrow q]$.

Proof of T25: By contraposition and modal negation of *Theorem 24*.

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