Possible Worlds, The Lewis Principle, and the Myth of a Large Ontology

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Lewis's Conception of Worlds

- Worlds are mereological sums of spatiotemporal objects.
- Worlds are *maximal* in the following sense: if x is a world, then any object that bears any (positive) spatiotemporal relation to x is part of x.
- The actual world is the world of which we are a part.
- There are worlds other than the actual world.
- Recombining duplicates of parts of different worlds yields another possible world, size and shape permitting.
- Aliens (i.e., an individual no part of which is a duplicate of any part of this world) exist.
- The principle of recombination applies to aliens: we can recombine parts of aliens to yield a possible world.

The Abstract (Wittgensteinian) Conception of Worlds

- A world is in some sense all that is the case
- Propositions are *true at* worlds (where this can be defined).
- Worlds are abstract objects of some sort, since their essential components are propositions and not concrete parts.
- Worlds are *maximal* in the following (or some similar) sense: if w is a world, then for every proposition p, either p is true at w or the negation of p is true at w.
- Worlds are *possible*; for any world w, all of the propositions true at w could have been jointly true.
- There is a unique actual world.
- There are worlds other than the actual world.

Fundamental Existence Principle

- The Lewis Principle: Every way that a world could possibly be is a way that some world is. (Lewis 1986, pp. 2, 71, 86)
- This can be expressed in a way that even those holding a more abstract view of worlds can accept.
- We'll see that this principle need not be taken as axiomatic, but can be derived from more general principles.
- We use automatic reasoning tools to confirm the derivation and to look for the smallest models
- We then build a model of the more general principles and show that the principles are true in very small models.
- We draw some epistemological conclusions (about justifying belief in possible worlds) with regard to this metaphysical foundation for modality.

Representing the Lewis Principle Formally

- The Lewis Principle: $\Diamond p \rightarrow \exists w(w \models p)$
- The Lewis Principle is derivable from the Leibniz/Kripke principle that necessary truth is truth in all possible worlds (given maximality):
  1. $\Box p \equiv \forall w(w \models p)$ — Assumption
  2. $\Box \neg q_0 \equiv \forall w(w \models \neg q_0)$ — from 1, by UE
  3. $\neg\Box\neg q_0 \equiv \neg\forall w(w \models \neg q_0)$ — from 2, by contraposition
  4. $\Diamond q_0 \equiv \exists w \neg(w \models \neg q_0)$ — from 3, by definition
  5. $\Diamond q_0 \equiv \exists w(w \models q_0)$ — from 4, by maximality
  6. $\Diamond p \equiv \exists w(w \models p)$ — from 5, by UI
- *A fortiori*, $\Diamond p \rightarrow \exists w(w \models p)$.
- Plantinga, Chisholm, Adams, Pollock, Zalta, etc., (sometimes only implicitly) endorse this.
- So are we done?
- No. We haven't derived it from more general principles and the definition of a world.

Principles Needed to Prove the Lewis Principle

Start with a '2nd-order' modal predicate calculus with encoding formulas and (relational) λ-expressions (no encoding formulas), interpreted in fixed domains with 1st-order 2nd-order BFs:

(S5) The modal propositional logic S5, including the Rule of Necessitation (RN).

(L2) Monadic second-order quantification theory (i.e., with both 0- and 1-place predicate variables),

($\mathbf{Id}_1$) $F = G \ =_{df} \ \Box\forall x(xF \equiv xG)$

($\mathbf{Id}_0$) $p = q \ =_{df} \ [\lambda y\, p] = [\lambda y\, q]$

(OC) $\exists x(A!x \ \& \ \forall F(xF \equiv \varphi))$, x not free in φ (Object Comprehension)

(RE) $\Diamond xF \rightarrow \Box xF$ (Rigidity of Encoding)

(Sit) $Situation(x) \ =_{df} \ \forall F(xF \rightarrow \exists p(F = [\lambda z\, p]))$

(Tr) p is *true in* x ('$x \models p$') $=_{df}$ $x[\lambda z\, p]$

(PW) $PossibleWorld(x) \ =_{df} \ Situation(x) \ \& \ \Diamond\forall p(x \models p \ \equiv \ p)$

Proof Strategy for the Lewis Principle

- Show: $\Diamond p \to \exists w(w \models p)$.
  - First: Show $\Diamond p \to \Diamond\exists w(w \models p)$
  - Second: Show $\Diamond\exists w(w \models p) \to \exists w(w \models p)$
- First: Assume $\Diamond p$ (to show: $\Diamond\exists w(w \models p)$)
  - Assume p and show $\exists w(w \models p)$) [on subsequent slides]
  - By CP and RN, $\Box(p \to \exists w(w \models p))$
  - We have $\Diamond p$ (our global assumption)
  - Apply: $\Box(\varphi \to \psi) \to (\Diamond\varphi \to \Diamond\psi)$
  - Conclude: $\Diamond\exists w(w \models p)$
- Second: Assume $\Diamond\exists w(w \models p)$ (to show: $\exists w(w \models p)$)

  $\Diamond\exists w(w \models p)$ implies $\exists w\Diamond(w \models p)$, by BF. Pick an arbitrary such world w_1, so that we know $\Diamond w_1 \models p$. By df, $\Diamond w_1[\lambda y\, p]$. By RE, $\Box w_1[\lambda y\, p]$. By T schema, $w_1[\lambda y\, p]$, i.e., $w_1 \models p$. So $\exists w(w \models p)$.

Show $p \to \exists w(w \models p)$: Begin

Assume p.

By (OC),

$$\exists x(A!x \,\&\, \forall F(xF \equiv \exists q(q \,\&\, F = [\lambda y\, q])))$$

Let x_0 be such an object:

$$A!x_0 \,\&\, \forall F(x_0F \equiv \exists q(q \,\&\, F = [\lambda y\, q])) \qquad (\vartheta)$$

To show $\exists w(w \models p)$, we have to show:

1. $Situation(x_0)$
2. $\Diamond \forall q(x_0 \models q \;\equiv\; q)$
3. $x_0 \models p$

1. *A fortiori*, from the right conjunct of ϑ:

$$\forall F(x_0F \to \exists q(F = [\lambda y\, q])).$$

So, $Situation(x_0)$.

Show $p \to \exists w(w \models p)$: End

2. To establish $\Diamond\forall q(x_0 \models q \equiv q)$, we establish $\forall q(x_0 \models q \equiv q)$ and apply the dual of the T schema ($\varphi \to \Diamond\varphi$).

   To establish $\forall q(x_0 \models q \equiv q)$, pick r and show $x_0 \models r \equiv r$.

   ($\to$) Assume $x_0 \models r$, i.e., $x_0[\lambda y\, r]$.

   Then by df (x_0), $\exists q(q \,\&\, [\lambda y\, r] = [\lambda y\, q])$. (‘$s$’).

   So s and $[\lambda y\, r] = [\lambda y\, s]$.

   So by (Id_0), $r = s$.

   But since s is true, so is r

   ($\leftarrow$) Assume r (show: $x_0 \models r$, i.e., $x_0[\lambda y\, r]$).

   By =I, $r \,\&\, [\lambda y\, r] = [\lambda y\, r]$.

   So $\exists q(q \,\&\, [\lambda y\, r] = [\lambda y\, q])$.

   By df (x_0), $x_0[\lambda y\, r]$.

3. $x_0 \models p$ follows from our global assumption p and $\forall q(x_0 \models q \equiv q)$ (which we proved as part of (2)). Thus, since (1) and (2) yield $PossibleWorld(x_0)$, we have thus established: $\exists w(w \models p)$.

Implementation in Prover9: I

(Fitelson and Zalta 2007)

- One must translate modal claims into statements quantifying over 'propositions' and 'points'. This allows 'worlds' to be defined. $\Box p$ becomes:

```
all d (Point(d) -> True(p,d)).
all p all d (True(p,d) -> (Proposition(p) & Point(d))).
```

- Predication requires sorts and is relativized to points:

```
all F all x all d (Ex1At(F,x,d) ->
  Property(F) & Object(x) & Point(d)).
```

- Rigidity of encoding:

```
all x all F ((Object(x) & Property(F)) ->
  ( (exists d (Point(d) & EncAt(x,F,d))) ->
    (all d (Point(d) -> EncAt(x,F,d))) )).
```

Implementation in Prover9: II

- Prover9 then clausifies the premises and conclusions.
- Example: The definition of a world:
  - $PossibleWorld(x) \;=_{df}\; Situation(x) \,\&\, \Diamond\forall p(x \models p \;\equiv\; p)$
- This gets input into Prover9 as:
  - ```
    all x all d (Object(x) & Point(d) -> (WorldAt(x,d) <->
     SituationAt(x,d) & (exists d2 (Point(d2) &
     (all p (Proposition(p) -> (TrueInAt(p,x,d2) <-> True(p,d2))))))))
    ```
- Prover9 clausifies this: open clauses-df-world.html
- Example: The Lewis Principle:
  - $\Diamond p \rightarrow \exists w(w \models p)$
- This gets input into Prover9 as:
  - `Point(W).`
  - `all p (PossiblyTrue(p) <-> (exists d (Point(d) & True(p,d)))).`
  - `all x (World(x) <-> WorldAt(x,W)).`
  - `all x all p (TrueIn(p,x) <-> TrueInAt(p,x,W)).`
  - ```
    all p (Proposition(p) -> (PossiblyTrue(p) ->
     (exists x (World(x) & TrueIn(p,x))))).
    ```

Proof of Lewis Principle

Input file: theorem25a.in. What it used:

- `all x all d (WorldAt(x,d) -> (Object(x) & Point(d))).`
- `WorldAt(x,d)` is rigid
- `TrueInAt(p,x,d)` is rigid
- `WorldAt(x,d) & ActualAt(x,d) ->`
  `all p (TrueInAt(p,x,d) iff True(p,d)).`
- `all d (Point(d) ->`
  `(exists x (WorldAt(x,d) & ActualAt(x,d)))).`
- `Point(W).`
- `all p (PossiblyTrue(p) <-> exists d (Point(d) & True(p,d))).`
- `all x (World(x) <-> WorldAt(x,W)).`
- `all p all x (TrueIn(p,x) <-> TrueInAt(p,x,W)).`

Smallest Computational Models of the Lewis Principle

- Input file without the conclusion and use Mace (demo theorem25a.in without goal in mace4)
- Analyze the model: two elements in the domain: element 0 is an object, a point, and a world, element 1 is arbitrary (mace4 assumes smallest models have 2 elements); no propositions. (Prover9 is a modal realist and a nominalist.)
- Of course, this is not the smallest non-trivial model. Let's examine model with additional assumptions: add additional.in to demo)
- Analysis: 4 elements in the domain: 0 and 2 are propositions, 1 and 3 are both distinct objects, distinct worlds, and distinct points.
- This give us hints about how to find smallest models of object theory.

Object Theory with Property/Proposition Comprehension

(S5) S5 modal logic, including RN

(L2) Monadic second-order quantification theory

($\mathbf{Id}_1$) $F = G \;=_{df}\; \Box\forall x(xF \equiv xG)$

($\mathbf{Id}_0$) $p = q \;=_{df}\; [\lambda y\, p] = [\lambda y\, q]$

(OC) $\exists x(A!x \,\&\, \forall F(xF \equiv \varphi))$, x not free in φ (Object Comprehension)

(RE) $\Diamond xF \rightarrow \Box xF$ (Rigidity of Encoding)

(Sit) $\mathit{Situation}(x) \;=_{df}\; \forall F(xF \rightarrow \exists p(F = [\lambda z\, p]))$

(Tr) p is *true in* x (‘$x \models p$’) $=_{df}$ $x[\lambda z\, p]$

(PW) $\mathit{PossibleWorld}(x) \;=_{df}\; \mathit{Situation}(x) \,\&\, \Diamond\forall p(x \models p \;\equiv\; p)$

(Λ_1) $[\lambda \nu\, \varphi]\tau \equiv \varphi^{\nu}_{\tau}$, where τ is free for ν in φ

(Λ_0) $[\lambda\, \varphi] \equiv \varphi$

Interpretations

- An intensional interpretation $\mathcal{I}$ is an 8-tuple: $\langle \mathbf{W}, \mathbf{w}^*, \mathbf{P}, \mathbf{D}, \mathbf{Op}, \mathbf{ex}, \mathbf{en}, \mathbf{V} \rangle$
  - $\mathbf{W}$, $\mathbf{D}$, and $\mathbf{P}$ are nonempty sets;
  - $\mathbf{w}^*$ is a distinguished element of $\mathbf{W}$;
  - $\mathbf{P}$ is the union of two mutually disjoint, nonempty sets $\mathbf{P}_0$ and $\mathbf{P}_1$;
  - $\mathbf{D}$ is the union of two mutually disjoint sets $\mathbf{A}$ and $\mathbf{O}$ such that $\mathbf{A}$ is nonempty;
  - $\mathbf{Op}$ is a set of logical operations **neg**, **cond**, **univ**, **nec**, **vac**, **plug** described more fully below;
  - $\mathbf{ex}$ is a total function on $\mathbf{W} \times \mathbf{P}$ that maps $\mathbf{W} \times \mathbf{P}_0$ into $\{0, 1\}$ and $\mathbf{W} \times \mathbf{P}_1$ into $\wp(\mathbf{D})$;
  - $\mathbf{en}$ maps $\mathbf{P}_1$ into $\wp(\mathbf{A})$ in such a way that, for distinct $\mathbf{a}_1, \mathbf{a}_2 \in \mathbf{A}$, there is a $\mathbf{p}_1 \in \mathbf{P}_1$ such that $\{\mathbf{a}_1, \mathbf{a}_2\} \not\subseteq \mathbf{en}(\mathbf{p}_1)$;
  - $\mathbf{V}$ maps each term of our language to a member of $\mathbf{D}$, each 0-place predicate to a member of $\mathbf{P}_0$, and each 1-place predicate to a member of $\mathbf{P}_1$.

Constraints on Exemplification Extensions

Where $\mathbf{r}_i, \mathbf{s}_i \in \mathbf{P}_i$, for $0 \leq i \leq 1$:

- $\mathbf{ex}(\mathbf{w}, \mathbf{plug}(\mathbf{r}_1, \mathbf{a})) = 1$ iff $\mathbf{a} \in \mathbf{ex}(\mathbf{w}, \mathbf{r}_1)$
- $\mathbf{ex}(\mathbf{w}, \mathbf{neg}(\mathbf{r}_0)) = 1 - \mathbf{ex}(\mathbf{w}, \mathbf{r}_0)$
  $\mathbf{ex}(\mathbf{w}, \mathbf{neg}(\mathbf{r}_1)) = \mathbf{D} \setminus \mathbf{ex}(\mathbf{w}, \mathbf{r}_1)$
- $\mathbf{ex}(\mathbf{w}, \mathbf{cond}(\mathbf{r}_0, \mathbf{s}_0)) = \mathit{max}\{1 - \mathbf{ex}(\mathbf{w}, \mathbf{r}_0), \mathbf{ex}(\mathbf{w}, \mathbf{s}_0)\}$
  $\mathbf{ex}(\mathbf{w}, \mathbf{cond}(\mathbf{r}_1, \mathbf{s}_1)) = (\mathbf{D} \setminus \mathbf{ex}(\mathbf{w}, \mathbf{r}_1)) \cup \mathbf{ex}(\mathbf{w}, \mathbf{s}_1)$
- $\mathbf{ex}(\mathbf{w}, \mathbf{univ}(\mathbf{r}_1)) = 1$ iff $\mathbf{ex}(\mathbf{w}, \mathbf{r}_1) = \mathbf{D}$
- $\mathbf{ex}(\mathbf{w}, \mathbf{nec}(\mathbf{r}_0)) = \mathit{min}\{\mathbf{ex}(\mathbf{w}', \mathbf{r}_0) \mid \mathbf{w}' \in \mathbf{W}\}$
  $\mathbf{ex}(\mathbf{w}, \mathbf{nec}(\mathbf{r}_1)) = \bigcap\{\mathbf{ex}(\mathbf{w}', \mathbf{r}_1) \mid \mathbf{w}' \in \mathbf{W}\}$
- $\mathbf{ex}(\mathbf{w}, \mathbf{vac}(\mathbf{r}_0)) = \begin{cases} \mathbf{D} & \text{if } \mathbf{ex}(\mathbf{w}, \mathbf{r}_0) = 1 \\ \emptyset & \text{otherwise} \end{cases}$

Denotations of λ-expressions

Given an interpretation $\mathcal{I}$ and variable assignment $\boldsymbol{f}$, we define $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}$ as usual, with the following special conditions for λ-expressions:

- $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \rho]) = \boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}(\rho)$, for 0-place predicates ρ
  $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda \nu\, \pi\nu]) = \boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}(\pi)$, for 1-place predicates π
- $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \pi\tau]) = \mathbf{plug}(\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}(\pi), \boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}(\tau))$
- $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \neg\varphi]) = \mathbf{neg}(\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \varphi]))$
  $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda \nu\, \neg\varphi]) = \mathbf{neg}(\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda \nu\, \varphi]))$, if ν occurs free in φ
- $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \varphi \rightarrow \psi]) = \mathbf{cond}(\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \varphi]), \boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \psi]))$
  $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda \nu\, \varphi \rightarrow \psi]) = \mathbf{cond}(\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda \nu\, \varphi]), \boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda \nu\, \psi]))$,
  if ν is free in $\varphi \rightarrow \psi$
- $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \forall\nu\varphi]) = \mathbf{univ}(\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda \nu\, \varphi]))$
- $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \Box\varphi]) = \mathbf{nec}(\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \varphi]))$
  $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda \nu\, \Box\varphi]) = \mathbf{nec}(\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda \nu\, \varphi]))$, if ν is free in φ
- $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda \nu\, \varphi]) = \mathbf{vac}(\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}([\lambda\, \varphi]))$, if ν is not free in φ.

Truth in an Interpretation

- An assignment $\boldsymbol{f}$ satisfies a formula φ w.r.t. world $\mathbf{w}$ under an interpretation $\mathcal{I}$, written $\models_{\mathcal{I},\mathbf{w},\boldsymbol{f}} \varphi$, is defined:
  - $\models_{\mathcal{I},\mathbf{w},\boldsymbol{f}} \rho$ iff $\mathbf{ex}(\mathbf{w}, \boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}(\rho)) = 1$
  - $\models_{\mathcal{I},\mathbf{w},\boldsymbol{f}} \pi\tau$ iff $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}(\tau) \in \mathbf{ex}(\mathbf{w}, \boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}(\pi))$.
  - $\models_{\mathcal{I},\mathbf{w},\boldsymbol{f}} \tau\pi$ iff $\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}(\tau) \in \mathbf{en}(\boldsymbol{d}_{\mathcal{I},\boldsymbol{f}}(\pi))$.
  - . . . (and so on, in the usual way)
- A formula φ is true w.r.t $\mathbf{w}$ under $\mathcal{I}$, written $\models_{\mathcal{I},\mathbf{w}} \varphi$, iff every assignment $\boldsymbol{f}$ is such that $\models_{\mathcal{I},\mathbf{w},\boldsymbol{f}} \varphi$.
- A formula φ is true under $\mathcal{I}$, written $\models_{\mathcal{I}} \varphi$, iff $\models_{\mathcal{I},\mathbf{w}^*} \varphi$

The Simplest Non-Trivial Model: Two Worlds

- $\mathbf{W} = \{\mathbf{w_0}, \mathbf{w_1}\}$
- $\mathbf{w}^* = \mathbf{w_0}$
- $\mathbf{P} = \mathbf{P}_0 \cup \mathbf{P}_1$, where $\mathbf{P}_0 = \{\mathbf{p}_0, \overline{\mathbf{p}_0}, \mathbf{q}_0, \overline{\mathbf{q}_0}\}$ and $\mathbf{P}_1 = \{\mathbf{p}_1, \overline{\mathbf{p}_1}, \mathbf{q}_1, \overline{\mathbf{q}_1}\}$; and
- $\mathbf{D} = \mathbf{A} \cup \mathbf{O}$, where $\mathbf{O} = \emptyset$ and $\mathbf{A} = \wp(\mathbf{P}_1)$
- $\mathbf{Op}$ is as specified below
- $\mathbf{ex}(\mathbf{w}, \mathbf{p}_0) = 1$ and $\mathbf{ex}(\mathbf{w}, \overline{\mathbf{p}_0}) = 0$, for $\mathbf{w} \in \mathbf{W}$
  $\mathbf{ex}(\mathbf{w}_0, \mathbf{q}_0) = \mathbf{ex}(\mathbf{w}_1, \overline{\mathbf{q}_0}) = 1$; $\mathbf{ex}(\mathbf{w}_1, \mathbf{q}_0) = \mathbf{ex}(\mathbf{w}_0, \overline{\mathbf{q}_0}) = 0$
  $\mathbf{ex}(\mathbf{w}, \mathbf{p}_1) = \mathbf{D}$ and $\mathbf{ex}(\mathbf{w}, \overline{\mathbf{p}_1}) = \emptyset$, for $\mathbf{w} \in \mathbf{W}$
  $\mathbf{ex}(\mathbf{w}_0, \mathbf{q}_1) = \mathbf{ex}(\mathbf{w}_1, \overline{\mathbf{q}_1}) = \mathbf{D}$; $\mathbf{ex}(\mathbf{w}_1, \mathbf{q}_1) = \mathbf{ex}(\mathbf{w}_0, \overline{\mathbf{q}_1}) = \emptyset$
- $\mathbf{en}(\mathbf{r}) = \{\mathbf{a} \in \mathbf{A} \mid \mathbf{r} \in \mathbf{a}\}$
- $\mathbf{V}(A!) = \mathbf{p}_1$

Collapsing Properties and Propositions

Key: At each $\mathbf{w}$, all abstract objects exemplify the same properties

- $\mathbf{plug}(\mathbf{r}_1, \mathbf{a}) = \mathbf{r}_0$, for all $\mathbf{a} \in \mathbf{D}$
- $\mathbf{neg}(\mathbf{r}_i) = \overline{\mathbf{r}_i}$, for $\mathbf{r}_i \in \{\mathbf{p}_i, \mathbf{q}_i\}$
  $\mathbf{neg}(\overline{\mathbf{r}_i}) = \mathbf{r}_i$, for $\mathbf{r}_i \in \{\mathbf{p}_i, \mathbf{q}_i\}$
- $\mathbf{cond}(\mathbf{p}_i, \mathbf{r}_i) = \mathbf{r}_i$, for $\mathbf{r}_i \in \mathbf{P}_i$
  $\mathbf{cond}(\overline{\mathbf{p}_i}, \mathbf{r}_i) = \mathbf{p}_i$, for $\mathbf{r}_i \in \mathbf{P}_i$
  $\mathbf{cond}(\mathbf{q}_i, \mathbf{p}_i) = \mathbf{cond}(\mathbf{q}_i, \mathbf{q}_i) = \mathbf{p}_i$
  $\mathbf{cond}(\mathbf{q}_i, \overline{\mathbf{p}_i}) = \mathbf{cond}(\mathbf{q}_i, \overline{\mathbf{q}_i}) = \overline{\mathbf{q}_i}$
  $\mathbf{cond}(\overline{\mathbf{q}_i}, \mathbf{p}_i) = \mathbf{cond}(\overline{\mathbf{q}_i}, \overline{\mathbf{q}_i}) = \mathbf{p}_i$
  $\mathbf{cond}(\overline{\mathbf{q}_i}, \overline{\mathbf{p}_i}) = \mathbf{cond}(\overline{\mathbf{q}_i}, \mathbf{q}_i) = \mathbf{q}_i$
- $\mathbf{univ}(\mathbf{r}_1) = \mathbf{r}_0$, for $r \in \mathbf{P}$
- $\mathbf{nec}(\mathbf{p}_i) = \mathbf{p}_i$
  $\mathbf{nec}(\mathbf{q}_i) = \mathbf{nec}(\overline{\mathbf{q}_i}) = \mathbf{nec}(\overline{\mathbf{p}_i}) = \overline{\mathbf{p}_i}$
- $\mathbf{vac}(\mathbf{r}_0) = \mathbf{r}_1$

Summary

- The smallest *non-trivial* model of the object-theoretic principles used in (and needed for the terms used in) the proof of the Lewis Principle:
  - 2 possible worlds
  - 4 properties
  - 4 propositions
  - 16 abstract objects
- But if our metaphysics is correct, the two worlds can be identified as two of the abstract objects, namely, the ones corresponding to the smallest model elements:
  - $\{\mathbf{vac}(\mathbf{p}_0), \mathbf{vac}(\mathbf{q}_0)\} = \{\mathbf{p}_1, \mathbf{q}_1\}$
  - $\{\mathbf{vac}(\mathbf{p}_0), \mathbf{vac}(\overline{\mathbf{q}_0})\} = \{\mathbf{p}_1, \overline{\mathbf{q}_1}\}$

General Epistemological Consequences

- One can endorse (the principles needed to prove) the Lewis Principle without being committed to a large ontology.
- Object theory with propositions adds 16 abstract objects to the smallest non-trivial models of second-order quantified modal logic with propositions, but eliminates the two worlds.
- We obtain a large ontology only when we add the data.
- So to justify the existence of a large number of possible worlds, we don't have to justify them individually.
- Instead, we justify the principles needed to prove the Lewis Principle, and then add our modal beliefs, since the latter, plus $\Diamond p \;\&\; \neg p$, guarantees the existence of a distinct non-actual possible world for each such distinct p.

Specific Justification of Principles

- Note 1: Since logics are easier to justify epistemologically, consider whether the small models suggest our system looks more like a logic than a metaphysics.
- Progression: non-empty (trivial) or 2-element (non-trivial) domain of worlds $\rightarrow$ 2 (4) properties and 2 (4) propositions (or $\rightarrow$ a small domain of abstract objects.
- The existence principle for the latter, **OC**, can be reformulated as an abstraction principle:

(**Ab**) $\iota x(A!x \,\&\, \forall F(xF \equiv \varphi))G \equiv \varphi^G_F$, x not free in φ

- The Lewis Principle is derivable from the system with **Ab** replacing **OC**. (See the paper.)
- **Ab** has some claim to being logical, if it is analogous to λ-conversion (one level up) and λ-conversion is logical.
- We suggest: treat our system as the minimum commitments of a logic required to systematize both our modal beliefs and the meaning of those beliefs (as quantifications over worlds).

Final Observations

- We've systematized our modal inferential practices and our philosophical understanding of those practices.
- We've grounded the meaningfulness of unanalyzed modal beliefs and possible world talk in the inferential roles this kind of talk plays in our discourse: the meaning of 'world' is a (objectified) pattern within that discourse.
- This treats meanings (i.e., the abstracta) as the patterns arising from systematic use of language.
- So the Lewis Principle is justified by principles we can accept on the grounds that they systematize our practices. No special faculty to access them is needed other than the faculty to understand language and inferences.
- Lewis Principle becomes a logical theorem in a logic of worlds, and as a theorem, implies only a small ontology.

Bibliography

- Fitelson, B. and E. Zalta, 2007, 'Steps Towards a Computational Metaphysics', *Journal of Philosophical Logic*, 36/2 (April): 227–247.
- Kripke, S., 1963, "Semantical Considerations on Modal Logic," *Acta Philosophica Fennica*, 16: 83–94.
- Kripke, S., 1959, "A Completeness Theorem in Modal Logic," *Journal of Symbolic Logic*, 24: 1–15.
- Lewis, D., 1986, *On the Plurality of Worlds*, Oxford: Blackwell.
- Zalta, E., 1993, 'Twenty-Five Basic Theorems in Situation and World Theory', *Journal of Philosophical Logic*, 22: 385–428.
- Zalta, E., 1983, *Abstract Objects: An Introduction to Axiomatic Metaphysics*, Dordrecht: D. Reidel.