An Axiom Forestalling Modal Collapse and its Application (in Object Theory)[*]

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Abstract

Linsky & Zalta (1994) argued that simplest quantified modal logic (SQML), with its fixed domain, can be given an actualist interpretation if the Barcan formula is interpreted to conditionally assert the existence of contingently nonconcrete objects. But SQML itself doesn't require the existence of such objects; in interpretations of SQML in which there is only one possible world, there are no contingent objects, nonconcrete or otherwise. I defend an axiom for SQML that will provably (a) force the domain to have the relevant objects and thereby (b) force the existence of more than one possible world, thereby forestalling modal collapse. I show that the new axiom can be justified by describing the theorems that can be proved when it is added to SQML. I further justify the axiom by the reviewing the theorems the axiom allows us to prove when we assume object theory ('OT'), in its latest incarnation, as a background framework. Finally, I consider the conclusions one can draw when we consider the new axiom in connection with *actualism*, as this view has been (re-)characterized in recent work.

[*]This paper includes a number of (edited) excerpts from different sections of an unpublished manuscript (cited as Zalta m.s.). These have been woven together here to present a more unified picture of a useful axiom in object theory.

[†]I'm grateful to Seyed Mousavian for organizing, and inviting me to contribute to, this *Festschrift* for Bernie Linsky. I first met Bernie when he spent his 1989–1990 sabbatical year at Stanford. We discovered that we were both intrigued by issues in modal logic, including the relationship between Lewis's view (that there *exist* possible objects that aren't actual) and the Meinongians view (that there *are* possible objects that don't existent). Our subsequent collaboration has yielded 6 papers. In several of those works, Bernie suggested a number of nice ways of defending and extending my theory of abstract objects. That theory wouldn't be where it is today if I hadn't had the benefit of his suggestions for understanding the theory's epistemology and extending its metaphysical application. It is my great pleasure to be able to contribute to this volume in his honor.

1 Introduction

In this paper, I discuss several issues in quantified modal logic (QML). I'll focus first on (the axioms and rules of) the simplest QML (SQML), which I take to be quantified S5 under a semantics without an accessibility relation on possible worlds but with a fixed domain of individuals. Later, I'll focus on object theory, which involves a further extension of SQML. To set the stage, I must first briefly (and somewhat freely) summarize the argument in Linsky & Zalta 1994 (hereafter 'L&Z'). In that paper, L&Z argued that there are two (incompatible) interpretations of SQML. Though we called these 'possibilist' and 'actualist' interpretations, the understanding of these terms have changed since L&Z was written and so we might more neutrally call the two interpretations 'Meinongian' and 'Quinean' (cf. Meinong 1904, Quine 1948).[1] The key task of L&Z was to show that the following two theorems of SQML were unobjectionable no matter which interpretation of the logic one adopted:

- Barcan (1946) Formula: $\Diamond\exists x\varphi \to \exists x\Diamond\varphi$ (BF$\Diamond$)
- Necessary 'Existence': $\forall x\Box\exists y(y=x)$ (NE)

The Meinongian interpretation regards the quantified formula $\exists x\varphi$ as asserting only that *there is* an x such that φ, without implying *there exists* an x such that φ. (To formally assert *there exists* an x such that φ, one would add an existence predicate and assert $\exists x(E!x \,\&\, \varphi)$.) Under the Meinongian interpretation of the quantifier, BF$\Diamond$ asserts only that if possibly something is φ, then something is possibly φ. When the antecedent is true for a particular formula φ, BF doesn't imply that something possibly such that φ *exists*. Rather, if one assumes that *there are contingently nonexisting objects*, then BF$\Diamond$ implies, given facts such as that b doesn't have a sister but might have, only that something (that doesn't exist) that might be b's sister. And NE only asserts that everything is necessarily identical to something, not that everything necessarily exists. On this interpretation, there are two kinds of contingent objects, those that contingently exist and those the contingently fail to exist. Using

[1]By calling the second interpretation 'Quinean', I'm not suggesting that Quine would accept the logic under this interpretation. I'm only invoking Quine's name because this interpretation of the logic is based on his understanding of the quantified formula $\exists x\varphi$. Quine would no doubt reject the two theorems named (BF$\Diamond$) and (NE) below, under this or any other interpretation.

this Meinongian interpretation of the quantifier, both Parsons 1980 and Zalta 1983 provided coherent accounts of nonexistent objects that are contingently nonexistent.

The Quinean interpretation of SQML required a different understanding of the quantified formula $\exists x\varphi$. By supposing that this formula asserts *there exists* an x such that φ, one can interpret BF$\Diamond$ as asserting: if possiby there exists something that is φ, then there exists something that is possibly φ. L&Z then suggested that if one assumes that *contingently nonconcrete objects* exist and are actual, then BF$\Diamond$ implies, given the datum that b doesn't have a sister but might have, only that there exists something actual that might be b's sister. Moreover, while NE, under this interpretation, asserts that everything necessarily exists, it doesn't assert that everything is necessarily concrete, for one may suppose that there are two kinds of actually existing contingent objects, the contingently concrete and the contingently nonconcrete.[2] I offered this alternative Quinean interpretation of my theory of abstract objects as a kind of Platonism that avoids Meinongianism, thereby avoiding a commitment to (contingently and necessarily) nonexistent objects (Zalta 1983, 51–52; 1988, 103).

L&Z found that the SQML was neutral between these two interpretations. They also argued that the Quinean interpretation was consistent with the definition of *actualism* used at the time, namely: everything there is, i.e., everything that exists, is actual. The existence of actual nonconcrete, but possibly concrete, objects is consistent with this definition. So SQML had both a possibilist (Meinongian) and an actualist (Quinean) interpretation.

Given this context, I'd like to discuss three issues. The first issue concerns the fact that in both of their interpretations of SQML, L&Z implicitly asserted a semantic principle to the effect that the fixed domain of individuals contains either contingently nonexisting objects (on the Meinongian interpretation) or contingently nonconcrete objects (on the Quinean interpretation). The issue about this is twofold: (a) SQML itself doesn't assert any axiom that requires the domain to include such objects, and (b) as far as standard modal semantics goes, there are legitimate semantic interpretations of SQML in which there is only one possible world in the semantic domain of primitive possible worlds, giving rise to 'modal collapse', where every formula and its necessitation are equivalent (i.e., where $\varphi \equiv \Box\varphi$ is true, for every φ). Kripke semantics (1959, 1963) only assumes a *non-empty* domain of possible worlds and those semantic interpretations that contain only one world foreclose the option of having contingently nonexistent or contingently nonconcrete objects, since they foreclose all contingencies. I address (both components of) this first issue in Section 2, where I add to the SQML an axiom that will (a) force the domain to have the relevant objects and (b) forestall modal collapse. The investigation will, therefore, be primarily proof-theoretic, with semantic principles mentioned only to forestall ambiguity.

[2] See Williamson 1998 for a somewhat different way of positioning this interpretation of QML.

The second issue is connected to the first, namely, an investigation into the ways in which the new axiom being proposed can be justified. Later in this investigation, I'll use object theory ('OT'), in its more recent incarnation, as a framework for conducting the investigation. The basic form of justification is that the new axiom allows one to prove interesting philosophical theorems in OT. For example, it allows us to (a) prove the existence of at least one contingently true proposition and one contingently false proposition, (b) prove the existence of a property that is contingently exemplified and contingently unexemplified, (c) prove the existence of at least two possible worlds, one of which isn't actual, and (d) prove the existence of discernible objects (these objects were recently shown, in Nodelman & Zalta 2024, to be a key to reconstructing Frege's Theorem without mathematical primitives or axioms).

The third and final issue I consider is how to think about the new axiom from the point of view of *actualism*, as this view has been recently characterized in Menzel 2020 and 2024. Menzel has developed new characterization of actualism and it would appear that the Quinean interpretation of SQML no longer satisfies the definition. I consider the issues this raises and then suggest reasons why one may legitimately view this conclusion as a positive result.

2 An Axiom That Forestalls Modal Collapse

For the remainder of this paper I shall adopt the Quinean interpretation of SQML, i.e., I'll use Quine's understanding of the formula $\exists x\varphi$ as asserting that *there exists* an x such that φ. And I'll use 'there exists' and 'there are' in the metalanguage interchangeably.

To talk about the existence of contingently nonconcrete objects within SQML, I'll need more expressive power than what's available in that logic. In particular, I'll assume that SQML has been extended to include:

- a distinguished primitive predicate $E!$ (*being concrete*), for which formulas of the form $E!x$ are to be read: x exemplifies *being concrete* (or x is concrete); intuitively, this predicate denotes a property whose extension varies from world to world.
- an actuality operator, $\mathcal{A}$, for which formulas of the form $\mathcal{A}\varphi$ are to be read: *it is actually the case that* φ (or actually, φ).
- 2nd-order quantification, for which formulas of the form $\forall F^n\varphi$ and $\exists F^n\varphi$ (for $n \geq 1$) are to be read: *every* n-ary relation F is such that φ and *there exists* an n-ary relation F such that φ.

Let us call the result of extending SQML with this additional expressive power SQML$^+$. A few words about these extensions are in order.

The concreteness predicate $E!$ allows us to distinguish the following kinds of objects:

$$x \text{ is } \textit{contingently concrete} \equiv_{df} E!x \,\&\, \Diamond\neg E!x$$

$$x \text{ is } \textit{contingently nonconcrete} \equiv_{df} \neg E!x \,\&\, \Diamond E!x$$

$$x \text{ is } \textit{necessarily concrete} \equiv_{df} \Box E!x$$

Our primary focus in this paper will be on the first two kinds of objects; we'll leave necessarily concrete objects, such as Spinoza's God, if it exists, for some other occasion. Moreover, we won't say that nonconcrete objects are 'abstract', but rather reserve 'abstract' for those objects that *couldn't* be concrete. Thus, we'll also sometimes distinguish abstract from ordinary objects as follows:

$$x \text{ is } \textit{abstract} \text{ (`}A!x\text{')} \equiv_{df} \neg\Diamond E!x$$

$$x \text{ is } \textit{ordinary} \text{ (`}O!x\text{')} \equiv_{df} \Diamond E!x$$

Though *being ordinary* includes the contingently concrete (because being concrete implies possibly being concrete), contingently nonconcrete, and necessarily concrete objects, we'll focus in this section on ordinary objects that aren't necessarily concrete.

The logic of the operator $\mathcal{A}$ plays an important role in what follows, but it suffices now to mention that the semantic principle governing the truth of $\mathcal{A}\varphi$ is simple: $\mathcal{A}\varphi$ is true if and only if φ is true at the distinguished world (of the model). Thus, we're assuming (a) that (in the interpretations of modal logic), there is a distinguished actual world $\boldsymbol{w}_\alpha$ (in the domain of possible worlds), and (b) 'actual' is not an indexical – in any (modal) context, the truth of $\mathcal{A}\varphi$ is determined by the truth of φ at the actual world. This semantics for $\mathcal{A}$ gives rise to an interesting logic and though we leave the details to a footnote, it does play a role in what follows.[3]

The logic of the 2nd-order quantifiers we've added to SQML is classical. We may, for our purposes, assume that for each n, $n \geq 1$, the quantifiers $\forall F^n$ and $\exists F^n$ range over a *fixed* domain of primitive n-ary relations ($n \geq 1$), so that the 2nd-order Barcan formula and its converse are both valid, i.e., for every n, $\forall F^n\Box\varphi \to \Box\forall F^n\varphi$ and $\Box\forall F^n\varphi \to \forall F^n\Box\varphi$. So the n-ary predicates of SQML$^+$ rigidly denote the primitive relations in the relevant domain, but at each possible world, these relations may have a

[3] The logic of actuality has 3 parts: a group of necessary axioms, a contingent axiom, and a rule of inference. The necessary axioms are: $\mathcal{A}$ commutes with $\neg$, distributes over $\to$, commutes with $\forall\alpha$, is idempotent, is necessarily true when true, and applies to every necessary truth. Formally:

$$\mathcal{A}\neg\varphi \equiv \neg\mathcal{A}\varphi$$
$$\mathcal{A}(\varphi \to \psi) \equiv (\mathcal{A}\varphi \to \mathcal{A}\psi)$$
$$\mathcal{A}\forall\alpha\varphi \equiv \forall\alpha\mathcal{A}\varphi$$
$$\mathcal{A}\varphi \equiv \mathcal{A}\mathcal{A}\varphi$$
$$\mathcal{A}\varphi \to \Box\mathcal{A}\varphi$$
$$\Box\varphi \equiv \mathcal{A}\Box\varphi$$

The contingent axiom is:

$$\mathcal{A}\varphi \to \varphi$$

The presence of this axiom requires that the Rule of Necessitation (Rule RN) be restricted – RN may not be applied to any line of a proof that is derived from this axiom. Since the axioms allow one to derive $\varphi \to \mathcal{A}\varphi$ from the contingent axiom, $\varphi \to \mathcal{A}\varphi$ is a contingent theorem and, thus, so is the further consequence $\mathcal{A}\varphi \equiv \varphi$.

The rule of inference for the logic of actuality is the Rule of Actualization, which in its simplest form asserts that if φ is a theorem, then $\mathcal{A}\varphi$ is a theorem. However, if wants to apply the rule when reasoning under assumptions or premises, then the rule states that if φ is derivable from a set of premises Γ, then $\mathcal{A}\varphi$ is derivable from $\mathcal{A}\Gamma$, i.e., from the set containing the actualizations of all the premises in Γ.

In thinking about how to best develop the logic of actuality, I've benefited from reading Hazen 1978, 1990, and Hazen, Rin, & Wehmeier 2013. But compare Glazier & Krämer 2024, which offers a different analysis of the actuality operator in QML.

different exemplification extension (i.e., the set of n-tuples of objects exemplifying the relation at that world). We'll rely on the fact that classical 2nd-order logic includes a comprehension principle for relations as an axiom schema.[4]

With SQML$^+$ as our background logic, we can now introduce, and discuss interesting consequences of, an axiom that not only forestalls modal collapse, but that grounds our modal intuitions about contingent objects and plays a role in proving a variety of other important philosophical claims. The axiom I'm proposing asserts: there might have been a concrete object that isn't actually concrete. Formally:

$$\Diamond\exists x(E!x \;\&\; \neg\mathcal{A}E!x) \tag{1}$$

This axiom doesn't assert that there exist concrete objects. *A fortiori*, it doesn't assert that there are concrete objects that exemplify a property but might not have, or that there are concrete objects that fail to exemplify a property but might have. Rather, it is designed to a weak way of capturing the contingency that the world exhibits with respect to what is concrete.

Of course, it should be easy to see why this axiom semantically forestalls modal collapse. Let's suppose, for the moment, that we've extended the standard semantics for SQML to a semantics for SQML$^+$ and that we can, in the metalanguage, talk about primitive possible worlds and about the exemplification extensions of properties. Then, for (1) to be true, the semantics of SQML$^+$ requires there to be at least two possible worlds, $\boldsymbol{w}_\alpha$ and $\boldsymbol{w}_1$, and an object, say a, that is in the exemplification extension of the property denoted by '$E!$' at $\boldsymbol{w}_1$, but not in the exemplification extension of this property at $\boldsymbol{w}_\alpha$. So, SQML$^+$ extended with (1) doesn't require that there be any concrete objects at the actual world. That's a good thing, since otherwise it would hard to call it an *a priori* principle of modal logic. (1) asserts only the *possibility* that there are concrete objects of a certain sort. Indeed, it *implies* the possible existence of contingently concrete objects, for if (1) is added as an axiom to the logic of SQML$^+$, the following becomes a theorem:

$$\vdash \Diamond\exists x(E!x \;\&\; \Diamond\neg E!x) \tag{2}$$

[4]This schema asserts, for any $n \geq 1$:

$$\exists F^n \Box\forall x_1 \ldots \forall x_n(F^n x_1 \ldots x_n \equiv \varphi, \text{ provided } F^n \text{ doesn't occur free in } \varphi.$$

This guarantees, for example, that every relation has a negation, that every two relations has a conjunction, etc.

(The proof is in Appendix A.) Intuitively, at any world $\boldsymbol{w}$ where there exists a concrete object, say a, that is not actually concrete, it follows that a is a contingently concrete object at $\boldsymbol{w}$.

Moreover, it should be clear that (1), if added as an axiom to SQML$^+$, lets us *derive* the existence of the contingently nonconcrete objects that were postulated semantically in L&Z. In other words, given (1) as an axiom, the following becomes a theorem of SQML$^+$:

$$\vdash \exists x(\neg E!x \;\&\; \Diamond E!x) \tag{3}$$

(The proof is in Appendix A.)[5] By adding (1) as an axiom, we strengthen SQML$^+$ so that the semantic principle assumed in L&Z becomes derivable in the logic.

Moreover, (1) implies that (4) for some properties F, there are objects that possibly exemplify F even though they (actually) don't, and (5) for some properties F, there are objects that possibly don't exemplify F even though they (actually) do. We've put '(actually)' in parenthesis to indicate that the theorem holds both with the actuality operator and without, though one must appeal to the contingent axiom for actuality to prove the second of the pair. Formally, we have:

$$\begin{aligned} &\vdash \exists F\exists x(\Diamond Fx \;\&\; \mathcal{A}\neg Fx) \\ &\vdash \exists F\exists x(\Diamond Fx \;\&\; \neg Fx) \end{aligned} \tag{4}$$

$$\begin{aligned} &\vdash \exists F\exists x(\Diamond\neg Fx \;\&\; \mathcal{A}Fx) \\ &\vdash \exists F\exists x(\Diamond\neg Fx \;\&\; Fx) \end{aligned} \tag{5}$$

(The proofs are in Appendix A.)

These consequences of (1) allow us formulate, in the language of SQML$^+$ itself, partial answers to questions about so-called 'mere possibilia' (we will give fuller answers in Section 3). The questions arise in connection with data that typically involve concreteness-entailing properties, i.e., properties F such that $\Box\forall x(Fx \to E!x)$. The data has the form $\neg\exists xFx \& \Diamond\exists xFx$ and, in SQML$^+$, the second conjunct of such data implies that $\exists x\Diamond Fx$. For example, since *being a diamond* and *being a donkey* are concreteness-entailing properties, data such as:

[5]The proof appeals to the contingent axiom $\mathcal{A}\varphi \to \varphi$, and so one may not apply Rule RN and conclude that the theorem is necessarily true. And we *shouldn't* be able to derive its necessity, for (1) doesn't require that there exist contingently nonconcrete objects at every possible world! Recall the simple model we developed in the previous paragraph; there were no contingently nonconcrete objects at $\boldsymbol{w}_1$, but only one contingently concrete object.

- There are no million carat diamonds but there might have been.
- There are no talking donkeys but there might have been.

imply, respectively that there exist objects that might have been million carat diamonds and that there exist objects that might have been talking donkeys. In answer to the question, "What kind of objects serve as a witness to such existential claims?", (1) grounds the reply: contingently nonconcrete objects. In the case of the above data, these would be objects that aren't million carat diamonds or talking donkeys but which have the modal properties *possibly being a million carat diamond* or *possibly being a talking donkey*. Of course, these data don't definitively imply that the witnesses are nonconcrete; one might still point to an existing diamond and claim that though it isn't a million carats, it could have been, or point to some particular donkey and claim that though it doesn't talk it might have.

But there is more definitive data that implies the existence of contingently nonconcrete objects. For example, if b doesn't have a sister but might have, it is reasonable to suggest that one cannot point out an existing concrete person who might have been b's sister (Kripke 1972 [1980, 112–114]). Given certain facts about the necessity of one's origins, there is no existing, concrete person (not even a first cousin of b) that could have been b's sister. Accordingly, the following would constitute data:

- b doesn't have a sister but might have, and no concrete object could have been b's sister.

$$\neg\exists xSxb \;\&\; \Diamond\exists xSxb \;\&\; \neg\exists x(E!x \;\&\; \Diamond Sxb) \tag{6}$$

Similarly, if there are no aliens but might have been, then one might legitimately hold the view that there is no concrete object that might have been an alien. These kinds of cases provide a more definitive reason for adopting (1), since it sets up the response, in the language of SQML$^+$, that the witnesses to the existential claims of the form $\exists x\Diamond Fx$ (when F is concreteness-entailing) are contingently nonconcrete objects that exist, fail to be F, but have the modal property *possibly being* F. In Section 3.2.2, we'll see how to give an even fuller answer, by deriving the world-theoretic truth conditions for these claims in OT itself.

Moreover, (1) and the theorems we can derive from it provide us with answers to the puzzle about iterated modalities (McMichael 1983). Consider the claim:

- Daughterless person c might have a daughter who might become president.

$$\neg\exists yDyc \;\&\; \Diamond\exists x(Dxc \;\&\; \Diamond Px) \tag{7}$$

The second conjunct of (7) implies $\exists x\Diamond(Dxc \;\&\; \Diamond Px)$, by BF$\Diamond$. But the *daughter of* relation D necessarily implies the concreteness of its relata, i.e., that $\Box\forall x\forall y(Dxy \rightarrow E!x \;\&\; E!y)$. So (1) clears the ground for claiming, in the language of SQML$^+$, that the witness to the existential claim $\exists x\Diamond(Dxc\&\Diamond Px)$ is a contingently nonconcrete object that exemplifies the modal property of *possibly: being a sister of c and possibly being president*. We'll see later that in OT, which includes the resources of the relational λ-calculus, this property can be formally represented as $[\lambda x\Diamond(Sxc\&\Diamond Px)]$.

The foregoing suggests that (1) extends SQML$^+$ with an axiom that (a) forestalls modal collapse, (b) asserts the existence of the objects described semantically in L&Z's (Quinean) interpretation of the simplest QML, and (c) provides a framework for answering questions about the modal reality underlying natural language data in the language of SQML$^+$ itself. Of course, (1) also works well for those who prefer the Meinongian interpretation of the quantifier and who accept that there are contingently nonexisting objects, for then (1) asserts that possibly, there exist objects that don't actually exist. (Recall that, under the Meinongian interpretation, $\exists x(E!x \;\& \ldots)$ may be read as "there exists an x such that φ", and so I've used that to read $\Diamond\exists x(E!x \;\&\; \neg\mathcal{A}E!x)$.) And so by BF$\Diamond$, there are objects that possibly exist even though they don't actually exist.

In what follows, though, I describe other reasons for adopting (1). I will work within *object theory* (hereafter 'OT'), which is an extension of SQML$^+$. Each interesting philosophical theorem that (1) helps us to prove in OT provides a reason. Moreover, once we define and prove the existence of possible worlds, we'll be able to derive, *in the language of OT*, the world-theoretic truth conditions of modal claims and thus derive that the contingently concrete objects implied by (1) are nonconcrete at the actual world but concrete at other possible worlds.

3 Applications of the New Axiom in OT

In Section 3.1, I'll first describe the latest version of OT, as found in Zalta m.s., and this will make it clear how OT extends SQML$^+$. Then, in Section 3.2, I'll describe a number of theorems of OT in which (1) plays a central role. I'll limit myself to just those theorems of OT in which (1) plays a direct or central role, thereby omitting theorems that (recursively) depend on them.

3.1 The Latest Development of OT

The most recent and fully developed version of OT, which includes (1), can be described as follows (Zalta m.s.). However, those already familiar with earlier versions of OT can skip this subsection, with very little loss of understanding.

3.1.1 The Language of OT

OT is expressible in a language that extends SQML$^+$ with:

- an additional mode of predication, $x_1 \dots x_n F^n$ ('$x_1, \dots, x_n$ *encode* F^n') that and leaves the (logic of the) classical mode of predication, $F^n x_1 \dots x_n$ ('$x_1, \dots, x_n$ *exemplify* F^n') completely intact;
- 0-ary relation variables ($p, q, \dots$), and relation constants ($p_i, q_i, \dots$) for $i \geq 1$;
- complex individual terms of the form $\imath\nu\varphi$ (i.e., definite descriptions, interpreted rigidly), where ν is any individual variable; and
- complex n-ary relation terms ($n \geq 0$) of the form $[\lambda \nu_1, \dots, \nu_n\, \varphi]$ (i.e., λ-expressions, where the ν_i are distinct individual variables), interpreted relationally not functionally.

Both kinds of complex terms may fail to denote and so OT uses a negative free logic for those terms. Identity is not a primitive of (SQML$^+$ or) OT.

To state the axioms of OT, we need not only the definitions of $A!x$ and $O!x$ given above in Section 2, but also definitions of the conditions under which it can be said that an individual or a relation exists (this is especially important given that there are non-denoting terms in the language) and definitions of the conditions under which it can be said that individuals or relations are identical. These definitions reveal that the existence and identity of objects and relations have been reduced to predication and quantification in a modal context (Zalta forthcoming). We use $\tau\downarrow$ to express existence; for example, $\imath xPx\downarrow$ asserts "the x that exemplifies P exists", and $[\lambda x\, \neg Px]\downarrow$ asserts "(the property) being an x that fails to exemplify P exists". However, when $\tau\downarrow$ holds, we often say, in the metalanguage, that τ denotes (or has a denotation), or is logically proper, or is significant. Though the symbol $\downarrow$ may be unfamiliar, we will use the familiar symbol $=$ when defining identity conditions for individuals and relations.

The definitions for existence and identity are given by cases, informally as follows (see Appendix B for the formal versions):

- An individual exists just in case it exemplifies a property
- An n-ary ($n \geq 1$) relation exists relation exists just in case there are n objects that *encode* it.
- A proposition p exists just in case the propositional property *being such that p* ($[\lambda x\, p]$) exists.
- Individuals are identical just in case they are ordinary objects that necessarily exemplify the same properties or abstract objects that necessarily encode the same properties.
- Properties are identical just in case they are necessarily encoded by the same objects.
- n-ary relations ($n \geq 2$) are identical just in case identical properties result when the relations are plugged up by $n-1$ objects in the same way; propositions are identical just in case their corresponding propositional properties are identical.

So relation and proposition identity reduces to property identity.

3.1.2 The Axioms and Rules of OT

The axioms and rules of OT govern the primitive and defined expressions, and are, for the most part, the ones you would expect for those expressions. Exceptions are noted below, and the axioms for the *encoding* mode of predication will be made explicit. OT uses:

- classical propositional logic and classical predicate logic for constants, variables, and λ-expressions of the form $[\lambda x_1 \dots x_n\, \varphi]$ in which none of the x_i occur as one of the arguments of an encoding formula somewhere in φ. However, a negative free logic applies to all other λ-expressions and to definite descriptions; true atomic formulas imply that all of the primary terms of the formula have a denotation;

- unrestricted substitution of identicals;
- classical S5 modal logic, extended with the new axiom (1);
- the logic for the actuality operator $\mathcal{A}$ described in footnote 3; the Rule of Necessitation can't be applied to theorems derived from the contingent axiom $\mathcal{A}\varphi \to \varphi$;
- a classical axiom for definite descriptions, adjusted only to reflect that descriptions are interpreted rigidly, namely:

  $$y = \imath x\varphi \equiv \forall x(\mathcal{A}\varphi \equiv x = y),$$

  which asserts that y is identical to the x such that φ if and only if all and only xs actually such that φ are identical to y; and
- the free logic of the λ-calculus, interpreted relationally, in which including α- and β-Conversion are conditioned on $[\lambda x_1 \ldots x_n\, \varphi]{\downarrow}$ (η-Conversion need not be conditioned);[6] in addition, an axiom of OT asserts that if $[\lambda x_1 \ldots x_n \varphi]$ denotes and $\Box\forall x_1 \ldots \forall x_n(\varphi \equiv \psi)$, then $[\lambda x_1 \ldots x_n\, \psi]$ denotes.

The axioms for encoding are as follows (we omit the axiom governing n-ary encoding formulas since we only plan to make use of unary encoding formulas):

- $O!x \to \neg\exists F\, xF$ (8)
- $xF \to \Box xF$ (9)
- $\exists x(A!x \,\&\, \forall F(xF \equiv \varphi))$, provided x doesn't occur free in φ (10)

The first asserts that ordinary objects fail to encode properties. The second asserts that if x encodes a property, it does so necessarily; i.e., encoding is rigid. The third is an unrestricted comprehension principle abstract objects. For a fuller account of the axioms, see Zalta m.s.

[6] η-Conversion applies to 'elementary' λ-expressions of the form $[\lambda x_1 \ldots x_n\, Fx_1 \ldots x_n]$ and asserts simply that $[\lambda x_1 \ldots x_n\, Fx_1 \ldots x_n] = F$, for any $n \geq 0$. The free variable F can be instantiated by any n-ary relation constant, variable, or denoting λ-expression.

The definitions and axioms imply that every formula denotes a proposition, that identity is reflexive, symmetric, and transitive, that identity (both for individuals and relations) is necessary when it holds, and a number of other theorems that we won't pause to describe here. However, it is important to mention that the principle of β-Conversion governing λ-expressions, yields a comprehension principle for n-ary relations ($n \geq 0$) as a theorem schema:

> $\exists F^n \Box\forall x_1 \ldots \forall x_n(F^n x_1 \ldots x_n \equiv \varphi)$, provided F^n is not free in φ and none of the x_i occur as an argument in an encoding formula somewhere in φ

When we combine this theorem schema with the formal definitions for the identity of relations given in Appendix B, i.e., (28), (29), and (30), the result is a hyperintensional theory of relations. The claim that relations F and G are necessarily equivalent doesn't imply that they are identical, i.e., $\Box\forall x_1 \ldots \forall x_n(Fx_1 \ldots x_n \equiv Gx_1 \ldots x_n)$ doesn't imply that $F = G$.

3.2 How the New Axiom Functions in OT

Since OT's modal logic includes (1), it has all the consequences of this axiom described in Section 2. Thus, OT forestalls modal collapse and implies: the possible existence of contingently concrete objects (2); the existence of contingently nonconcrete objects (3); that there are properties that some objects contingently exemplify (4); and that there properties that some objects contingently fail to exemplify (5). It thus provides a framework for demonstrating how the modal properties of contingently nonconcrete objects explain the various sorts of data about so-called 'mere possibilia'.

However, as noted earlier, the addition of (1) to OT yields a proof of a number of interesting philosophical theorems. We discuss these below. Once possible worlds are defined and their existence proved, we show that one can derive the world-theoretic truth conditions of modal claims and derive that the contingently concrete objects implied by (1) are nonconcrete at the actual world but concrete at other possible worlds.

3.2.1 Theorems Provable With the New Axiom

There are three groups of new theorems provable in OT with the help of (1). The first group extends the theory of properties and propositions (we're ignoring n-ary relations, $n \geq 2$, for simplicity). A second group extends the theory of possible worlds. And a third group extends the theory of natural numbers.

To see how (1) in OT extends the theory of properties and propositions, a word of motivation is in order. Of course it is easy to *apply* OT by adding particular assertions such as "Aristotle is a philosopher but might not have been" ($Pa \,\&\, \Diamond\neg Pa$). This would guarantee that there is continently true proposition, namely, Pa, and guarantee that there is a contingently false proposition, namely, $\neg Pa$. But the question of deeper interest is, how can one prove the existence of a contingently true proposition and a contingently false proposition from first principles, *without* applying OT by adding the data? This is of interest because 2nd-order SQML (i.e., SQML with comprehension for relations), doesn't imply that there are any contingent propositions, given that it has interpretations under which there is modal collapse. Nor does it imply the existence of any contingently exemplified and contingently unexemplified properties (or relations).

But SQML$^+$ and OT both do, since they include (1). We'll focus just on OT, in which both of the following are theorems:

- There exists a contingently true and a contingently false proposition.
  $\vdash \exists p(p \,\&\, \Diamond\neg p) \,\&\, \exists p(\neg p \,\&\, \Diamond p)$ (11)
- There exists a property that some object contingently exemplifies and there exists a property that some object contingently fails to exemplify.
  $\vdash \exists F\exists x(Fx \,\&\, \Diamond\neg Fx) \,\&\, \exists F\exists x(\neg Fx \,\&\, \Diamond Fx)$ (12)

Appendix C contains formalizations and proof sketches.

The second group of new theorems extends the theory of possible worlds previously developed within OT (Zalta 1993). The new theorems presuppose familiarity with that earlier work, for it includes the object-language definitions needed to derive the basic axioms of possible world theory as theorems. So, before we describe the new theorems—presented in (16) – (19) below—here is a brief sketch of the earlier work. A *situation* is defined as any abstract object that encodes only propositional properties of the form *being such that* p ('$[\lambda y\, p]$'); s *makes* p *true*, or p is *true in* s ('$s \models p$') whenever s encodes $[\lambda y\, p]$; a *possible world* is any situation s such that might be such that all and only true propositions are true in s, and a situation s is *actual* just in case every proposition true in s is true. The formal definitions are provided in a footnote.[7] Since possible worlds are defined as situations, the formal expression '$w \models p$' becomes an instance of the definition $s \models p$; we may therefore read $w \models p$ as p is *true at* w. In what follows, $\models$ always takes the smallest scope, so that $s \models p \equiv p$ is to be parsed as $(s \models p) \equiv p$ and not as $s \models (p \equiv p)$.

From these definitions, the basic theorems of world theory were derived (Zalta 1993, 414–419). Here are some of the basic theorems that will play a role in what follows:

- There is a unique possible world that is actual.
  $\vdash \exists! s(PossibleWorld(s) \,\&\, Actual(s))$ '$\boldsymbol{w}_\alpha$'
- A proposition is true if and only if it is true at the actual world.
  $\vdash p \equiv \boldsymbol{w}_\alpha \models p$ (13)
- A proposition is possibly true iff there exists a possible world at which p is true.
  $\vdash \Diamond p \equiv \exists w(w \models p)$ (14)
- A proposition is necessarily true iff it is true at all possible worlds.
  $\vdash \Box p \equiv \forall w(w \models p)$ (15)

[7] Formally:

$$Situation(x) \equiv_{df} A!x \,\&\, \forall F(xF \rightarrow \exists p(F = [\lambda y\, p]))$$

$$s \models p \equiv_{df} s[\lambda x\, p]$$

$$PossibleWorld(s) \equiv_{df} \Diamond\forall p(s \models p \equiv p)$$

$$Actual(s) \equiv_{df} \forall p(s \models p \rightarrow p)$$

See Zalta 1993, 410–411, 413–414, for additional details.

Note that we didn't make use of an actuality operator in the definition of $Actual(s)$. So on the above definition, $Actual(s)$ doesn't imply $\Box Actual(s)$. To see why, intuitively consider an actual situation s at some other possible world w and consider the fact that it encodes the same propositions no matter what world you consider (by (9)). Then, s makes p true and p is false at w, then it is not the case that every proposition s makes true is true at w. But with an actuality operator, one could define $Actual(s)$ as $\forall p(s \models p \rightarrow \mathcal{A}p)$. Under this definition, any actual situation s will be actual no matter what the modal context, by the rigidity of encoding axiom (9 and facts about the actuality operator).

We've chosen to keep the original definition in this paper, but as we'll see below, one does have to make some allowances – certain facts about the actual world will not be necessitatable, and this should be expected. See, for example, (13) below and the discussion that follows.

In more recent work, it is shown that possible worlds are closed under the connectives and quantifiers, and modally closed (Zalta m.s.).[8]

Note that since $\boldsymbol{w}_\alpha$ is defined in terms of a rigid definite description, (13) is derived with the help of the contingent axiom $\mathcal{A}\varphi \to \varphi$. This is not a flaw: clearly we don't want to be able to necessitate (13), since if p is, say, true at the actual world and we then consider a world, say w_1, where p is false, the biconditional claim $p \equiv \boldsymbol{w}_\alpha \models p$ is false at w_1 (the left side is false while the right side is true). If one wants a version of (13) that is necessitatable, then one need only consider the theorem $\mathcal{A}p \equiv \boldsymbol{w}_\alpha \models p$.

The addition of (1) to OT let's us extend this theory of possible worlds with the following important theorems:

- If there is a contingently true proposition, then there exists a non-actual possible world.
  $$\vdash \exists p(p \,\&\, \Diamond\neg p) \to \exists w(\neg Actual(w)) \tag{16}$$
- If there is a contingently false proposition, then there exists a non-actual possible world.
  $$\vdash \exists p(\neg p \,\&\, \Diamond p) \to \exists w(\neg Actual(w)) \tag{17}$$
- There is at least one non-actual possible world.
  $$\vdash \exists w \neg Actual(w) \tag{18}$$
- At least two possible worlds exist.
  $$\vdash \exists w \exists w'(w \neq w') \tag{19}$$

Given these theorems, we don't have to apply OT by adding specific data of the form $\neg p \,\&\, \Diamond p$ or $p \,\&\, \Diamond\neg p$ in order to *prove* the existence of non-actual possible worlds. Theorem (14) already guarantees that the addition of such data requires the existence of worlds that are distinct from the actual world (exercise). But by extending OT with (1), we have a theoretical justification for the existence of non-actual possible worlds without having to appeal to any specific contingent truth, or specific contingent falsehood, and thus no need to add additional, applied predicates. From the *possibility* that there are concrete objects that aren't actually concrete, the existence of non-actual possible worlds is guaranteed and the fact that there are at least two possible worlds can be derived in the metaphysics. This doesn't have to be stipulated in the semantics.

Finally, there is a third group of theorems that become provable with (1), indeed, ones that are essential to the derivation of second-order Peano Arithemtic in OT. I shall not go into great detail here, but rather summarize recently published work in Nodelman & Zalta 2024. In that paper, we noted that Frege's theory of natural numbers doesn't generalize well when formulated in a modal context, since it yields different natural numbers at different possible worlds.[9] To avoid this result, we adapted Frege's methods by introducing an actuality operator into the definition of $Numbers(x,G)$ so that the same group of natural cardinals constitute the natural numbers at every possible world. We then used (1) as a replacement for the modal axiom adopted in Zalta 1999.[10] Using (1), we established that OT implies that *discernible* objects exist, where we defined a discernible object x ('$D!x$') to be one such such that, for every object y distinct from x, some property distinguishes y from x.[11] Some of the key theorems proved with the help of (1) are:

[8] For closure under the primitive connectives and quantifiers, the theorems are:

$$w \models \neg p \equiv \neg w \models p$$

$$w \models (p \to q) \equiv w \models p \to w \models q$$

$$w \models \forall\alpha\varphi \equiv \forall\alpha(w \models \varphi)$$

And for modal closure, OT implies that possible worlds satisfy the definition:

$$ModallyClosed(s) \equiv_{df} \forall p((Actual(s) \Rightarrow p) \to s \models p)$$

where $\varphi \Rightarrow \psi$ is defined as $\Box(\varphi \to \psi)$. This definition, contributed by Uri Nodelman, differs from the one used in Zalta 1993; it is more general.

[9] At each possible world, the equivalence classes of equinumerous properties will differ, and since Frege abstracts the numbers from such equivalence classes, his methods would yield different natural numbers at different possible worlds.

[10] The modal axiom used in Zalta 1999 asserted: if there is a natural number n that numbers the Gs, then there might have been a concrete object distinct from all the actual Gs. Axiom (1) is considerably weaker than this.

[11] Formally, using $D!$ to represent the property of being discernible, we define:

$$D!x \equiv_{df} \forall y(y \neq x \to \exists F \neg(Fx \equiv Fy))$$

It may be of some interest to know that in OT, it is a theorem that there are distinct *abstract* objects that are *indiscernible*; i.e.,

$$\exists x \exists y(A!x \,\&\, A!y \,\&\, x \neq y \,\&\, \forall F(Fx \equiv Fy))$$

The existence of such objects implies that the standard definition of equinumerosity fails to be an equivalence relation. In Zalta 1999, the workaround was to develop a theory of Frege numbers as objects that can count only ordinary objects, all of which are discernible. However, since Frege's theory assumes that there aren't indiscernibles in the domain, one can fully preserve Frege's number theory in OT by defining the equinumerosity of F and G with respect to the discernible objects. Then one can recapture Frege's number theory so that numbers can count any discernible objects, whether ordinary or abstract, that fall under a property.

- Discernible objects exist.
  $\vdash \exists x D!x$

- The *predecessor* relation $\mathbb{P}$ is not an empty relation.
  $\vdash \exists x \exists y \mathbb{P}xy$

- Natural cardinals are discernible.
  $\vdash \mathit{NaturalCardinal}(x) \rightarrow D!x$

- Every natural number has a successor.
  $\vdash \mathbb{N}x \rightarrow \exists y(\mathbb{N}y \,\&\, \mathbb{P}xy)$

See Nodelman & Zalta 2024 for the details.

3.2.2 The Truth Conditions for Modal Claims Derived

To prepare us for the discussion of the possibilism-actualism debate in the next section, note that we can easily derive, in OT itself, the world-theoretic facts implied by a datum such as (6). (These facts were described semantically in L&Z, not derived from general principles and primitive modal facts.) By the theorems of world theory described earlier, the three conjuncts of (6) respectively imply:

$$\boldsymbol{w}_\alpha \models \neg\exists y Syb$$

$$\exists w(w \models \exists x Sxb)$$

$$\boldsymbol{w}_\alpha \models \neg\exists x(E!x \,\&\, \Diamond Sxb)$$

The first and third follow from (6) by (13) and the second follows from (6) by (14).

Given the further assumption that, necessarily, the relata of the *sister of* relation S are concrete, it then becomes relatively straightforward to derive from (6) and this further assumption that:

> There is, at the actual world, a contingently non-concrete object that isn't a sister of b but which is, at some other possible world, concrete and a sister of b.

That is, where the further assumption is represented as:

$$\Box\forall x\forall y(Sxy \rightarrow E!x \,\&\, E!y) \tag{20}$$

there is a derivation that serves as a witness to the following derivability claim:

$$(6),(20) \vdash \exists x\big(\boldsymbol{w}_\alpha \models (\neg E!x \,\&\, \Diamond E!x \,\&\, \neg Sxb) \,\&\, \exists w(w \neq \boldsymbol{w}_\alpha \,\&\, w \models (E!x \,\&\, Sxb))\big) \tag{21}$$

(The derivation is in Appendix C.)

Moreover, one can derive, as a theorem (in the language) of OT, the possible-worlds truth conditions of *iterated* modalities. Earlier, we considered the datum that daughterless person c might have a daughter who might become President. Where D represents the *daughter of* relation, we represented the fact that *c might have a daughter who might become president* as:

(φ_1) $\Diamond\exists x(Dxc \,\&\, \Diamond Px)$

The possible world analysis of this claim is expressible in OT as:

(ψ_1) $\exists w\exists x\big((w \models Dxc) \,\&\, \exists w'(w' \models Px)\big)$

That is, for some possible world w, there is an object x such that (i) *x is a daughter of c* at w and (ii) at some possible world w', *x is president*. Now it is straightforward to show, in OT, that:

$$\vdash \varphi_1 \equiv \psi_1 \tag{22}$$

(The proof is in Appendix C.) Note also that since it follows from φ_1 that $\exists x\Diamond(Dxc \,\&\, \Diamond Px)$, we may infer, for an arbitrary witness to this claim, say d, that d has the modal property $[\lambda x\, \Diamond(Dxc \,\&\, \Diamond Px)]d$, by β-Conversion. And by establishing, in the usual way, that d is a contingently nonconcrete object, it becomes clear that contingently nonconcrete objects have modal properties that make them suitable as truthmakers for iterated modality claims.

If a further argument is needed to show that ψ_1 provides *truth conditions* for φ_1 without our having to postulate any semantically primitive possible worlds, then first consider the fact that, in OT, the principle of β-Conversion is:

$$[\lambda x_1 \ldots x_n\, \varphi]\downarrow \rightarrow ([\lambda x_1 \ldots x_n\, \varphi]y_1 \ldots y_n \equiv \varphi^{y_1,\ldots,y_n}_{x_1,\ldots,x_n}) \qquad (n \geq 0)$$

This asserts: if *being $x_1,\ldots,x_n$ such that φ* exists, then objects $y_1,\ldots,y_n$ exemplify this property if and only if $y_1,\ldots,y_n$ are such that φ. Now the 0-ary case of β-Conversion is:

$$[\lambda\, \varphi]\!\downarrow\; \rightarrow ([\lambda\, \varphi] \equiv \varphi])$$

But since it is a theorem of OT that $[\lambda\, \varphi]\!\downarrow$ for every φ, the following claim becomes provable:

$$[\lambda\, \varphi] \equiv \varphi \tag{23}$$

In Zalta 2014, it was argued that this theorem constitutes a theory of *truth*, since it has the natural reading: that-φ *is true* if and only if φ. That is, the notion of *exemplification* used in the reading of n-ary β-Conversion becomes the notion of *truth* in the reading of 0-ary λ-Conversion.[12]

Now, given (23) as a theory of truth, we know $[\lambda\, \varphi_1] \equiv \varphi_1$, i.e., that-$\varphi_1$ is true if and only if φ_1. Moreover, we've established $\varphi_1 \equiv \psi_1$ as theorem (22). So it follows that $[\lambda\, \varphi_1] \equiv \psi_1$, which we may faithfully read as:

> *That possibly c has a daughter who possibly become president* is true if and only if for some possible world w, there is an object x such that (a) *x is a daughter of c* at w and (b) for some possible world w', *x is president* at w'.

These are clearly *truth* conditions even though we aren't using Tarski's semantic account of truth. Tarski's semantic account still requires an account of truth for propositions and it is the account of truth in OT that we are using to state metaphysical, not semantic, truth conditions for modal claims. By identifying *truth* as 0-ary predication and giving a theory of possible worlds, our theory of truth is a metaphysical one, and doesn't rest on any semantic notions or the mathematical notions typically assumed when doing semantics. This point will become relevant in the final section.

[12] When producing a reading of the biconditional $[\lambda\, \varphi] \equiv \varphi$ in Zalta 1988 (59) and 1993 (408), I had not yet realized that the condition on the left of the biconditional sign had be read as a sentence and that 'exemplifies' reduces to 'is true' in the 0-ary case. Menzel 1993 (117) nearly expresses the point; he doesn't explicitly read the 0-ary case of β-Conversion as "$[\lambda\, \varphi]$ is true if and only if φ", but he does say, speaking semantically about β-Conversion, that "[i]n the limiting case where $n = 0$, a 0-place term standing alone will suffice: The 0-place predication $[\lambda\, \varphi]$ is true iff the proposition P is denotes is true". I want to be explicit, however, that 0-ary β-Conversion, $[\lambda\, \varphi] \equiv \varphi$, expresses a theory of *truth* in the object language. The point was extended in Zalta 2014 to suggest that the Tarski T-schema, understood propositionally, is in fact a tautology; one simply gives a truth-functional reading of λ-expressions in which the λ doesn't bind any variables. This yields a new class of tautologies, as these are defined classically, of which the Tarski T-schema is one.

4 The Possibilism-Actualism Debate

4.1 Menzel's Definition of Possibilism and Actualism

Menzel (2024, Section 2.2) distinguishes possibilism and actualism on the basis of the work in Menzel 2020. The claim we've labeled above as (3) plays a central role in his discussion. If we use the predicate $E!$ instead of Menzel's predicate $C!$ for *being concrete*, then he defines:

> **Possibilism**
> There exist contingently nonconcrete objects.
> $\exists x(\neg E!x \;\&\; \Diamond E!x)$ (3)
>
> **Actualism**
> Contingently nonconcrete objects couldn't possibly exist.
> $\neg\Diamond\exists x(\neg E!x \;\&\; \Diamond E!x)$.

This seems correct – the original actualists eschewed "mere possibilia" and a careful reading of their work suggests that they had intended to avoid any commitment to even the possibility of contingently nonconcrete objects. Unfortunately, their definition of actualism as "Everything there is, i.e., everything that exists, is actual" didn't do the job, since one could assert the *actual* existence of contingently nonconcrete objects and comply with the demands of actualism as they had defined it. But Menzel's new definition of actualism directly excludes even the possibility of the contingently nonconcrete.

Given this new definition of possibilism, both SQML$^+$ and OT qualify as possibilist. When (1) is added to SQML$^+$ and OT, both imply (3) as a theorem and, as such, are possibilist as defined above. Interestingly, though, the theorem isn't necessitatable, as noted in footnote 5 and as evident in the proof of (3) in Appendix A.[13]

Now in contexts such as SQML$^+$ and OT, where we have the expressive power provided by an actuality operator, possibilism, i.e., (3), implies, by the logic of actuality ($\mathcal{A}\varphi \equiv \varphi$), that:

[13] This raises an interesting difference between the present conception of possibilism and Menzel's (2020, p. 1987) characterization of Williamson's possibilism as $\Diamond\exists x(\neg E!x \;\&\; \Diamond E!x)$ (again using $E!$ instead of C for *being nonconcrete*). Williamson's version, i.e., $\Diamond$(3), follows immediately from (3) by the T$\Diamond$ principle. And by the 5 principle, we can infer $\Box\Diamond$(3). But as we've seen, the derivation of (3) from (1) relies on the contingent axiom for actuality $\mathcal{A}\varphi \rightarrow \varphi$. So the conclusions that $\Diamond$(3) and $\Box\Diamond$(3) both have to be marked as claims derived from a contingency. So, in SQML$^+$ and OT, both (3), $\Diamond$(3), and $\Box\Diamond$(3) are all derivable, though all are derived using the contingent axiom schema $\mathcal{A}\varphi \rightarrow \varphi$.

$\mathcal{A}\exists x(\neg E!x \,\&\, \Diamond E!x)$
It is actually the case that there exists something that's both nonconcrete and possibly concrete.

Even though this asserts the actual existence of contingently nonconcrete objects, anyone who takes on board Menzel's definition of possibilism would reject the suggestion that this result turns the theory into an actualist one. So it now becomes clear where the focus of the debate has to be, namely, on philosophical merits of possibilism v. actualism. And the foregoing work shows that, in evaluating possibilism, one has to evaluate the merits of the modal axiom (1) that implies (3). We've seen that by adding (1) as an axiom to OT, there are a number of interesting, philosophical consequences. In the remainder of this paper, I don't plan to argue against the reasons that philosophers have put forward in defense of actualism, but will instead spend some time arguing that (1) and (3) have an additional, important feature that can't be easily preserved by an understanding of modality that doesn't endorse the contingently nonconcrete.

4.2 The Challenges for Actualism as Now Defined

The additional, important feature of (1) and (3) is that they provide a foundation for, and theory of, modal truth and not just a *representation* of modal truth. I will spell out this first by contrasting actualist theories that accept possible worlds but eschew the existence of contingently nonconcrete objects, and then by constrasting actualist theories that additionally eschew the existence of possible worlds.[14]

Axiom (1) and its consequence (3) set the stage for arguing that the truth of the modal data we've been discussing is grounded in the nature of contingently nonconcrete objects and that the logical relationships between possible worlds and propositions depend on this fact. To see why, consider that the second conjunct of (6) implies $\exists x\Diamond Sxb$, and if we consider an arbitrary witness to this claim, say, e, then from the fact that $\Diamond Seb$, it follows that $[\lambda x\, \Diamond Sxb]e$, i.e., e has the modal property of possibly being b's sister. Since we've seen how to use (1) and its consequence (3) to establish that e is contingently nonconcrete, we can derive from (6) that a contingently nonconcrete object has a modal property. Similarly, with (7). We saw that any witness to the second conjunct of (7) has the property *possibly: being a daughter of c and possibly being president*. In OT, we can formulate this property as $[\lambda x\, \Diamond(Sxc \,\&\, \Diamond Px)]$. If d is such a witness and thereby exemplifies this property, then we can derive from (7) that a contingently nonconcrete object has a 'iterated' or 'nested' modal property.

[14]It should be clear that there are actualists who accept possible worlds, when the latter are understood as abstract objects of a certain sort (e.g., Prior 1967 , Plantinga 1974, Fine 1977, Adams 1974, Stalnaker 2012, Williamson 2013. Indeed, one could accept OT and its theory of possible worlds (Zalta 1993) without accepting (1. For an argument to this effect, see Menzel and Zalta 2014.

Moreover, we have seen that having these modal properties is provably equivalent to world-theoretic facts that can be specified object-theoretically instead of semantically. We saw this in our discussion of the truth conditions of (φ_1) as (ψ_1). Thus, the contingently nonconcrete provably have the right nature: exemplifying the modal properties attributed to them in primitive modal facts is equivalent to the right world-theoretic truth conditions for those modal facts.

By contrast, any semantic representation of contingently nonconcrete objects by way of mathematical objects, e.g., as primitive set-theoretic elements of the domain of Tarski models or as nodes in a mathematical graph, don't have the modal properties that contingently nonconcrete objects have. Metaphysically, such mathematical objects as set-theoretic elements and nodes in a graph, don't have such modal properties as *possibly having a sister* or *possibly: being someone's daughter and possibly being president*. The only relevant modal properties that such mathematical objects exemplify are that they *might represent* or *model* some part of modal reality. But that is not the same as giving us a theory of modal reality or a theory of the truth conditions of modal facts. Modal possibility claims such as (6) and (7) are not about nodes in a graph, but about the properties that ordinary things like you and me have in other possible worlds. Models of modal reality are not theories of modal reality.

Furthermore, if an actualist additionally denies the existence of possible worlds, even as abstract entities, and instead rests only with mathematical points in a semantics, then again we are presented with a representation or model of the truth conditions of modal facts, not a theory of those truth conditions. Mathematical points aren't possible worlds; they don't have the right (modal) properties. But in OT, possible worlds *have in the encoding sense* the right properties: some worlds *are* (i.e., encode being) such that there are talking donkeys; some worlds *are* (i.e., encode being) such that there are million carat diamonds; some worlds *are* (i.e., encode being) such that there is an x that is b's sister; and some worlds w *are* (i.e., encode being) such that there is an x who is a daughter of c and such that at some world w', x is president of c at w'. The possible worlds of this general theory are not otiose: they *have*, in an important sense, the exactly the right properties.

I therefore suggest that we reject the idea that Tarski's semantic conception of truth provides any metaphysical insight about the truth of modal facts. Only a properly formulated modal metaphysics can do that. The alethic properties of atomic propositions aren't explained by a model, but rather by a theory that offers a conceptual framework that integates truth, modality, and possible worlds. Our understanding of modal claims and the theory of possible worlds have to be developed together, and each nourishes the other, as captured by the Fundamental Theorem (14).

If we are to have a genuine theory of modal truth, then we can't simply rest with a representation of modal truth that consists of mathematical objects standing in some abstract relations. And this point becomes intensified when we consider iterated modality claims, for to represent such claims without possible worlds and contingently nonconcrete objects, one has to postulate a number of interrelated set-theoretical or graph structures, and elements or nodes of those structures, so as to structurally mirror the modal properties being asserted by the data. If this is intended to be a genuine theory of modal truth, then such an actualist view becomes Pythagorean, i.e., it requires that: (a) modal reality consist of primitive mathematical objects, (b) that mathematical axioms are among the fundamental truths of metaphysics, and (c) that mathematical objects have modal properties that account for our modal beliefs. OT avoids these results; it recognizes that there are objects (contingently concrete and contingently nonconcrete) which are characterized by their modal properties, and that these modal properties can be given a world-theoretic understanding as part of general theory of possible worlds that makes use of a primitive modal operator.

So the benefits of (1) don't include just the facts that it yields both *bona fide* truth makers for (arbitrarily nested) modal claims and proofs of important philosophical theorems. It also avoids the concern that a semantic representation of modal truth in terms of mathematical objects is not a theory of modal truth.

Appendix A: Proofs in SQML$^+$

(2) By the laws of actuality (see footnote 3), it is a theorem that $\Box(\mathcal{A}\neg\varphi \equiv \neg\mathcal{A}\varphi)$. So by the Rule of Substitution, we can infer $\Diamond\exists x(E!x \,\&\, \mathcal{A}\neg E!x)$ from (1). But it is also a theorem of actuality that $\mathcal{A}\varphi \rightarrow \Diamond\varphi$.[15] So by classical modal reasoning, we can infer from our last result that $\Diamond\exists x(E!x \,\&\, \Diamond\neg E!x)$.[16]

(3) By BF$\Diamond$, it follows from (1) that $\exists x\Diamond(E!x \,\&\, \neg\mathcal{A}E!x)$. Suppose a is such an object, so that we know $\Diamond(E!a \,\&\, \neg\mathcal{A}E!a)$. Then since a possibly true conjunction implies that each conjunct is possible, we know both $\Diamond E!a$ and $\Diamond\neg\mathcal{A}E!a$. Now independently, the laws of actuality imply $\Box(\mathcal{A}\neg\varphi \equiv \neg\mathcal{A}\varphi)$. So by the Rule of Substitution, $\Diamond\neg\mathcal{A}E!a$ implies $\Diamond\mathcal{A}\neg E!a$. But by the logic of actuality, it is a theorem that $\Diamond\mathcal{A}\neg E!a \rightarrow \mathcal{A}\neg E!a$.[17] So $\mathcal{A}\neg E!a$. Then by the (contingent) axiom for the logic of actuality, it follows that $\neg E!a$. So we've established $\neg E!a \,\&\, \Diamond E!a$, i.e., a is contingently nonconcrete. ⋈

(4) We start with the first form of the theorem. By BF$\Diamond$, it follows from (1) that $\exists x\Diamond(E!x \,\&\, \neg\mathcal{A}E!x)$. Suppose a is such an object, so that we know $\Diamond(E!a \,\&\, \neg\mathcal{A}E!a)$. It follows that both $\Diamond E!a$ and $\Diamond\neg\mathcal{A}E!a$. But since $\Box(\neg\mathcal{A}\varphi \equiv \mathcal{A}\neg\varphi)$, we can infer $\Diamond\mathcal{A}\neg E!a$ from $\Diamond\neg\mathcal{A}E!a$. But $\Diamond\mathcal{A}\neg E!a$

[15] To see this, first establish the following fact about the actuality operator:

> *Lemma*. $\Box\varphi \rightarrow \mathcal{A}\varphi$
>
> *Proof*. The T schema $\Box\varphi \rightarrow \varphi$ is an axiom, and so are its closures. Hence, $\mathcal{A}(\Box\varphi \rightarrow \varphi)$ is an axiom. Since $\mathcal{A}$ distributes over the conditional, it follows that $\mathcal{A}\Box\varphi \rightarrow \mathcal{A}\varphi$. Independently, the axiom $\Box\varphi \equiv \mathcal{A}\Box\varphi$ implies $\Box\varphi \rightarrow \mathcal{A}\Box\varphi$. So by hypothetical syllogism, $\Box\varphi \rightarrow \mathcal{A}\varphi$.

So, as an instance of our Lemma, we know $\Box\neg\varphi \rightarrow \mathcal{A}\neg\varphi$. By contraposition, $\neg\mathcal{A}\neg\varphi \rightarrow \neg\Box\neg\varphi$. But it is a theorem that $\mathcal{A}\varphi \rightarrow \neg\mathcal{A}\neg\varphi$. So by biconditional syllogism, $\mathcal{A}\varphi \rightarrow \neg\Box\neg\varphi$. Hence, by definition of $\Diamond$, $\mathcal{A}\varphi \rightarrow \Diamond\varphi$.

[16] The reasoning involves the K$\Diamond$ principle: if we prove that $\exists x(E!x \,\&\, \mathcal{A}\neg E!x)$ ($= \varphi$) necessarily implies $\exists x(E!x \,\&\, \Diamond\neg E!x)$ ($= \psi$), then from the K$\Diamond$ principle, i.e., $\Box(\varphi \rightarrow \psi) \rightarrow (\Diamond\varphi \rightarrow \Diamond\psi)$, we obtain the desired conclusion $\Diamond\exists x(E!x \,\&\, \Diamond\neg E!x)$. And it is easy to see that $\Box(\varphi \rightarrow \psi)$. Assume φ, and suppose a is a witness, so that we know $E!a \,\&\, \mathcal{A}\neg E!a$. Then since the laws of actuality include $\mathcal{A}\varphi \rightarrow \Diamond\varphi$, we may conclude $E!a \,\&\, \Diamond\neg E!a$, i.e., ψ. So by conditional proof, $\varphi \rightarrow \psi$, and by Rule RN, $\Box(\varphi \rightarrow \psi)$.

[17] Recall from footnote 3 that it is axiomatic that $\mathcal{A}\varphi \rightarrow \Box\mathcal{A}\varphi$. So, take a negated formula as an instance, and we obtain $\mathcal{A}\neg\varphi \rightarrow \Box\mathcal{A}\neg\varphi$ as a valid schema. By contraposition, this becomes $\neg\Box\mathcal{A}\neg\varphi \rightarrow \neg\mathcal{A}\neg\varphi$, i.e., $\Diamond\neg\mathcal{A}\neg\varphi \rightarrow \neg\mathcal{A}\neg\varphi$. But since $\neg\mathcal{A}\neg\varphi$ is necessarily equivalent to $\mathcal{A}\varphi$, our conditional reduces to $\Diamond\mathcal{A}\varphi \rightarrow \mathcal{A}\varphi$. The conditional in the text is therefore a valid instance of this theorem: $\Diamond\mathcal{A}\neg E!a \rightarrow \mathcal{A}\neg E!a$.

implies $\mathcal{A}\neg E!a$. And by the contingent axiom governing $\mathcal{A}$, it follows that $\neg E!a$. So we've established, without appealing to a contingent axiom:

(ϑ) $\Diamond E!a \,\&\, \mathcal{A}\neg E!a$

However, if apply an appropriate instance of the contingent theorem $\mathcal{A}\varphi \rightarrow \varphi$ to the second conjunct, then we can also conclude:

(ξ) $\Diamond E!a \,\&\, \neg E!a$

From (ϑ), we can infer, by $\exists$I, $\exists F\exists x(\Diamond E!x \,\&\, \mathcal{A}\neg E!x)$. This can be necessitated. However, from (ξ), though we can infer $\exists F\exists x(\Diamond E!x \,\&\, \neg E!x)$, this result can't be necessitated, since it was derived from a contingency. ⋈

(5) We start with the first form of the theorem. Consider the negation of $E!$, i.e., $\overline{E!}$, and the fact that $\overline{E!}$ obeys the law $\Box\forall x(\overline{E!}x \equiv \neg E!x)$, i.e., $\Box\forall x(\neg\overline{E!}x \equiv E!x)$. Now again apply BF$\Diamond$ to (1), to obtain $\exists x\Diamond(E!x \& \neg\mathcal{A}E!x)$. Then by the Rule of Substitution, we may infer:

$$\exists x\Diamond(\neg\overline{E!}x \,\&\, \neg\mathcal{A}\neg\overline{E!}x)$$

Suppose a is such an object, so that we know $\Diamond(\neg\overline{E!}a \,\&\, \neg\mathcal{A}\neg\overline{E!}a)$. Then we know both $\Diamond\neg\overline{E!}a$ and $\Diamond\neg\mathcal{A}\neg\overline{E!}a$. But the second of these implies, by the fact that $\Box(\neg\mathcal{A}\neg\varphi \equiv \mathcal{A}\varphi)$ and the Rule of Substitution, that $\Diamond\mathcal{A}\overline{E!}a$. But this last result implies $\mathcal{A}\overline{E!}a$, since $\Diamond\mathcal{A}\varphi \rightarrow \mathcal{A}\varphi$. Note that if we appeal to the contingent axiom for actuality, it follows that $\overline{E!}a$. But without appealing to the contingent axiom, we've established $\Diamond\neg\overline{E!}a \,\&\, \mathcal{A}\overline{E!}a$. Hence, $\exists F\exists x(\Diamond\neg Fx \,\&\, \mathcal{A}Fx)$. This result is necessitable. But we've also established $\Diamond\neg\overline{E!}a \,\&\, \overline{E!}a$ by contingent means. So though we can conclude $\exists F\exists x(\Diamond\neg Fx \,\&\, Fx)$, we can't necessitate this result. ⋈

Appendix B: Existence and Identity Defined

In the definitions below, the free object-language variables in the definientia and definienda are to be regarded as metavariables. The use of object-language variables facilitates readability. But they must be regarded as metavariables given the presence of non-denoting complex terms (descriptions and λ-expressions) in OT, so that the definitions can be instantiated even by non-denoting terms.

The following definitions capture the ones stated in the text. We use $\downarrow$ to assert existence:

$$x\!\downarrow\ \equiv_{df}\ \exists F Fx \tag{24}$$

$$F^n\!\downarrow\ (n \geq 1)\ \equiv_{df}\ \exists x_1 \ldots \exists x_n(x_1 \ldots x_n F^n) \tag{25}$$

$$p\!\downarrow\ \equiv_{df}\ [\lambda x\, p]\!\downarrow \tag{26}$$

$$x = y\ \equiv_{df}\ (O!x \,\&\, O!y \,\&\, \Box\forall F(Fx \equiv Fy)) \vee (A!x \,\&\, A!y \,\&\, \Box\forall F(xF \equiv yF)) \tag{27}$$

$$F^1 = G^1\ \equiv_{df}\ F\!\downarrow \,\&\, G\!\downarrow \,\&\, \Box\forall x(xF \equiv xG) \tag{28}$$

$$\begin{aligned} F^n = G^n\ \equiv_{df}\ & F\!\downarrow \,\&\, G\!\downarrow \,\&\, \\ & \forall y_1 \ldots \forall y_{n-1}([\lambda x\, Fxy_1 \ldots y_{n-1}] = [\lambda x\, Gxy_1 \ldots y_{n-1}] \,\&\, \\ & [\lambda x\, Fy_1xy_2 \ldots y_{n-1}] = [\lambda x\, Gy_1xy_2 \ldots y_{n-1}] \,\&\ldots\&\, \\ & [\lambda x\, Fy_1 \ldots y_{n-1}x] = [\lambda x\, Gy_1 \ldots y_{n-1}x]) \end{aligned} \tag{29}$$

$$p = q\ \equiv_{df}\ p\!\downarrow \,\&\, q\!\downarrow \,\&\, [\lambda x\, p] = [\lambda x\, q] \tag{30}$$

See Zalta forthcoming and the chapter titled "The Language" in Zalta m.s., for further discussion of these definitions.

Appendix C: Proofs in OT

(11) First we prove the existence of a contingent proposition, i.e., a proposition p such that $\Diamond p \,\&\, \Diamond\neg p$. Then we use that fact to prove the existence of a contingently true and a contingently false proposition. Let q_0 be the proposition $\exists x(E!x \,\&\, \neg\mathcal{A}E!x)$. We already know $\Diamond q_0$ by the new axiom (1). And to see $\Diamond\neg q_0$, note that we can easily establish that $\neg\mathcal{A}\exists x(E!x \,\&\, \neg\mathcal{A}E!x)$, by the following argument:

> Assume, for reductio, $\mathcal{A}\exists x(E!x \,\&\, \neg\mathcal{A}E!x)$. Then by the logic of actuality, $\exists x\mathcal{A}(E!x \,\&\, \neg\mathcal{A}E!x)$. Suppose a is such an object, so that we know $\mathcal{A}(E!a \,\&\, \neg\mathcal{A}E!a)$. Then again by the logic of actuality, $\mathcal{A}E!a \,\&\, \mathcal{A}\neg\mathcal{A}E!a)$. But the second conjunct is necessarily equivalent to $\neg\mathcal{A}\mathcal{A}E!a$, which is necessarily equivalent to $\neg\mathcal{A}E!a$. Contradiction.

But, it then follows from $\neg\mathcal{A}\exists x(E!x \,\&\, \neg\mathcal{A}E!x)$ that $\mathcal{A}\neg\exists x(E!x \,\&\, \neg\mathcal{A}E!x)$. And since $\mathcal{A}\varphi \rightarrow \Diamond\varphi$, we may infer $\Diamond\neg\exists x(E!x \,\&\, \neg\mathcal{A}E!x)$, i.e., $\Diamond\neg q_0$.

Now that we know that contingent propositions exist, we prove that there is a contingently true proposition, i.e., $\exists p(p \,\&\, \Diamond\neg p)$, and a contingently false one, i.e., $\exists p(\neg p \,\&\, \Diamond p)$. Without loss of generality, it suffices to show that there is a contingently true proposition, since the negation of any witness will provably be contingently false. We've just established there is a contingent proposition, and so again let q_0 be such, so that we know:

(ϑ) $\Diamond q_0 \,\&\, \Diamond\neg q_0$

Then we establish that $\exists p(p \,\&\, \Diamond\neg p)$ by cases from the tautology $q_0 \vee \neg q_0$. Assume q_0. From this and the second conjunct of (ϑ), we have $q_0 \,\&\, \Diamond\neg q_0$. So, $\exists p(p \,\&\, \Diamond\neg p)$. Now assume $\neg q_0$. From this and the first conjunct of (ϑ), we have $\neg q_0 \,\&\, \Diamond q_0$. But now consider the negation of q_0 ('$\overline{q_0}$'), where this is defined to be $\neg q_0$. Then since it is provable from this definition, without appeal to contingencies, that $\neg q_0 \equiv \overline{q_0}$ and that $q_0 \equiv \neg\overline{q_0}$, these biconditionals are necessary and so the two sides of each biconditional are substitutable for one another. If we perform the substitutions simultaneously, then $\neg q_0 \,\&\, \Diamond q_0$ becomes $\overline{q_0} \,\&\, \Diamond\neg\overline{q_0}$. Generalizing on $\overline{q_0}$, it follows that $\exists p(p \,\&\, \Diamond\neg p)$. ⋈

(**12**) Again, we show only that there exists a property that is exemplified but possibly not, since the negation of the witness to this claim can be straightforwardly used to prove that there exists a property that is unexemplified but possibly exemplified. In the following, let Q_p be the property of being such that p, where p is a variable ranging over propositions:

$$Q_p =_{df} [\lambda z\, p]$$

In OT, $Q_p\!\downarrow$, since the variable bound by the λ doesn't occur in encoding position in the matrix. So, by the definition of Q_p, β-Conversion, Rule RN, and GEN we know:

(ϑ) $\forall p \forall x \Box(Q_p x \equiv p)$

Now by (11), we know that there are contingently true propositions. Let p_1 be such a proposition, so that we know $p_1 \,\&\, \Diamond\neg p_1$. Then consider Q_{p_1}, which we know exists. Here is a sketch of the remainder of the proof:

(A) Show: $p_1 \vdash Q_{p_1} y$. For this, however, it suffices to show $p_1 \to Q_{p_1} y$. So assume p_1. Then by (ϑ), $\forall x \Box(Q_{p_1} x \equiv p_1)$. Hence $\Box(Q_{p_1} y \equiv p_1)$, and by the T schema, $Q_{p_1} y \equiv p_1$. So $Q_{p_1} y$.

(B) Show: $\Diamond\neg p_1 \vdash \Diamond\neg Q_{p_1} y$. Similarly, it suffices to show $\Diamond\neg p_1 \to \Diamond\neg Q_{p_1} y$. By instantiating p_1 and y into (ϑ), we know $\Box(Q_{p_1} y \equiv p_1)$. *A fortiori*, $\Box(Q_{p_1} y \to p_1)$. This implies $\Box(\neg p_1 \to \neg Q_{p_1})$. Hence by K$\Diamond$, $\Diamond\neg p_1 \to \Diamond\neg Q_{p_1}$.

(C) Infer from (A) and (B):

$$p_1 \,\&\, \Diamond\neg p_1 \vdash Qy \,\&\, \Diamond\neg Qy$$

by using the principle:

If $\varphi \vdash \psi$ and $\chi \vdash \theta$, then $\varphi \,\&\, \psi \vdash \chi \,\&\, \theta$

(D) By $\exists$I, we independently know: $Qy \,\&\, \Diamond\neg Qy \vdash \exists F \exists x(Fx \,\&\, \Diamond\neg Fx)$

(E) Hence from (C) and (D) it follows by hypothetical syllogism that:

$$p_1 \,\&\, \Diamond\neg p_1 \vdash \exists F \exists x(Fx \,\&\, \Diamond\neg Fx)$$

(F) It follows by $\exists$E that:

$$\exists p(p \,\&\, \Diamond\neg p) \vdash \exists F \exists x(Fx \,\&\, \Diamond\neg Fx)$$

(G) But by (11), we've established $\exists p(p \,\&\, \Diamond\neg p)$, and so it follows from (F) that $\exists F \exists x(Fx \,\&\, \Diamond\neg Fx)$. ⋈

(**16**) Assume $\exists p(p \,\&\, \Diamond\neg p)$ and let p_1 be an arbitrary such proposition, so that we know both p_1 and $\Diamond\neg p_1$. If we then instantiate (14) to $\neg p_1$, we know $\Diamond\neg p_1 \equiv \exists w(w \models \neg p_1)$. From this and the second conjunct of our assumption, it follows that $\exists w(w \models \neg p_1)$. Let w_1 be such a possible world, so that we know $w_1 \models \neg p_1$. Now suppose, for reductio, that $Actual(w_1)$. Then by definition of *Actual*, every proposition true at w_1 is true. Hence $\neg p_1$. Contradiction. So $\neg Actual(w_1)$ and, hence, $\exists w \neg Actual(w)$. ⋈

(**17**) (Exercise)

(**18**) This follows from (11) and either (16) or (17). ⋈

(**19**) Since $\exists! w Actual(w)$, suppose w_1 is such a possible world, so that we know $Actual(w_1)$, among other things. Independently, by (18), we know $\exists w \neg Actual(w)$. Let w_2 be such a possible world, so that we know $\neg Actual(w_2)$. Since w_1 is actual and w_2 is not, it follows that $w_1 \neq w_2$. Hence $\exists w \exists w'(w \neq w')$. ⋈

(**21**) Our assumptions are:

(6) $\neg\exists x Dxb \,\&\, \Diamond\exists x Dxb \,\&\, \neg\exists x(E!x \,\&\, \Diamond Dxb)$

(20) $\Box\forall x\forall y(Sxy \rightarrow E!x \,\&\, E!y)$

We have to show:

$$(\vartheta)\ \exists x\big(\boldsymbol{w}_\alpha \models (\neg E!x \,\&\, \Diamond E!x \,\&\, \neg Dxb) \,\&\, \exists w(w \neq \boldsymbol{w}_\alpha \,\&\, w \models (E!x \,\&\, Dxb))\big)$$

From the second conjunct of (6), we know $\exists x \Diamond Dxb$. Suppose a is such an object, so that we know $\Diamond Dab$. By the third conjunct of (6), it then follows that $\neg E!a$, since the third conjunct tells us that no concrete object is possibly a sister of b. Now by applying CBF to (20) and instantiating the result, we know:

$$(\xi)\ \Box(Dab \rightarrow E!a \,\&\, E!b)$$

Two important things follow from (ξ). First, by the K$\Diamond$ schema, (ξ) implies $\Diamond Dab \rightarrow \Diamond(E!a \,\&\, E!b)$ and since the antecedent holds by the definition of a, it follows *a fortiori* that $\Diamond E!a$. Second, by applying the T schema to (ξ), then the already established fact that $\neg E!a$ implies $\neg Dab$. So we've established $\neg E!a \,\&\, \Diamond E!a \,\&\, \neg Dab$. Hence, by a (contingent) theorem of world theory (13), this conjunction holds at the actual world and so it follows that:

$$(\omega)\ \boldsymbol{w}_\alpha \models (\neg E!a \,\&\, \Diamond E!a \,\&\, \neg Dab)$$

Since a will be our witness to it (ϑ), we need only show $\exists w\big(w \neq \boldsymbol{w}_\alpha \,\&\, w \models (E!a \,\&\, Dab)\big)$. We know $\Diamond Dab$ by the definition of a, and so it follows by a fundamental theorem of world-theory (14) that $\exists w(w \models Dab)$. Suppose w_1 is such a possible world, so that we know $w_1 \models Dab$. Independently, by the dual of (14), i.e., (15), (ξ) implies $\forall w(w \models Dab \rightarrow E!a \& E!b)$. Hence:

$$(\zeta)\ w_1 \models (Dab \rightarrow E!a \,\&\, E!b)$$

Since worlds are closed under the connectives (see footnote 8), it follows *a fortiori* from $w_1 \models Dab$ and (ζ) that $w_1 \models E!a$. Again since worlds are closed under the connectives, it follows that $w_1 \models (E!a \,\&\, Dab)$. Moreover, since we know both $\boldsymbol{w}_\alpha \models \neg E!a$ and $w_1 \models E!a$, it follows by a theorem of world theory that $w_1 \neq \boldsymbol{w}_\alpha$. Hence $\exists w(w \neq \boldsymbol{w}_\alpha \,\&\, w \models (E!a \,\&\, Dab))$, which is all that it remained to show. ⋈

(22) The following, 'modally strict' proof includes no appeals to a contingency:

$$
\begin{array}{lll}
\Diamond\exists x(Dxc \,\&\, \Diamond Px) & & \\
\quad\equiv & \exists x\Diamond(Dxc \,\&\, \Diamond Px) & \text{BF } (\rightarrow),\ \text{CBF } (\leftarrow) \\
\quad\equiv & \Diamond(Dbc \,\&\, \Diamond Pb) & \text{witness } (\rightarrow),\ \exists\text{I } (\leftarrow) \\
\quad\equiv & \exists w\big(w \models (Dbc \,\&\, \Diamond Pb)\big) & \text{by (14)} \\
\quad\equiv & w_1 \models (Dbc \,\&\, \Diamond Pb) & \text{witness } (\rightarrow),\ \exists\text{I } (\leftarrow) \\
\quad\equiv & w_1 \models Dbc \,\&\, w_1 \models \Diamond Pb & w \text{ closed under } \& \\
\quad\equiv & w_1 \models Dbc \,\&\, \exists w(w \models \Diamond Pb) & \text{by } \exists\text{I } (\rightarrow),\ \text{witness } (\leftarrow) \\
\quad\equiv & w_1 \models Dbc \,\&\, \Diamond\Diamond Pb & \text{by (14)} \\
\quad\equiv & w_1 \models Dbc \,\&\, \Diamond Pb & \text{by } 4\Diamond\ (\rightarrow),\ \text{T}\Diamond\ (\leftarrow) \\
\quad\equiv & w_1 \models Dbc \,\&\, \exists w'(w' \models Pb) & \text{by (14)} \\
\quad\equiv & \exists x\big((w_1 \models Dxc) \,\&\, \exists w'(w' \models Px)\big) & \exists\text{I } (\rightarrow),\ \text{witness } (\leftarrow) \\
\quad\equiv & \exists w\exists x\big((w \models Dxc) \,\&\, \exists w'(w' \models Px)\big) & \text{by } \exists\text{I } (\rightarrow),\ \text{witness } (\leftarrow)
\end{array}
$$

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