On Anselm’s Ontological Argument in Prosligion II*

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Abstract

Formulations of Anselm’s ontological argument have been the subject of a number of recent studies. After examining these studies, the authors respond to criticisms that have surfaced in reaction to their earlier papers, identify a more refined representation of Anselm’s argument on the basis of new research, and compare their representation, which analyzes that than which none greater can be conceived as a definite description, to a representation that analyzes it as an arbitrary name.

In 1991, we argued that Anselm needed 2 premises, a minimal condition on the greater than relation, and a definition of God, to give a valid argument for God’s existence. In 2011, we showed how computational investigations established that one of the premises and the definition of God from the 1991 paper, were sufficient, just by themselves, to validly conclude God’s existence. In the analysis section of our 2011 paper, we offered a number of observations about the differences between the original 1991 formulation and the simplified 2011 representation of the argument.

The goal in what follows is to: (1) respond to criticisms that have surfaced in reaction to our papers of 1991 and 2011, (2) identify a somewhat improved representation of Anselm’s argument, and (3) show that this new version is immune to the criticisms of the earlier version (even if one were to grant that the criticisms are valid) and (4) show that Anselm’s term that than which nothing greater can be conceived is more accurately analyzed as a definite description than as an arbitrary name. In connection with (1), we address the criticism, that the version in Oppenheimer & Zalta 2011 is question-begging, by investigating this notion a bit more carefully. Along the way, we point out how certain criticisms of that work insufficiently attend to the analysis and observations we made in that paper concerning the simplified version of the argument.

1 Review

In preparation for the discussion of (1) – (3), here is a brief summary of our 1991 and 2011 papers. In our 1991 paper, we proposed a reading of the ontological argument on which Anselm needed two nonlogical premises and a meaning postulate about the greater than relation. To state these premises we used a first-order logic with primitive definite descriptions that is classical with respect to its treatment of constants but free for descriptions. In this logic, the quantifier ‘∃’ was not assumed to be existentially loaded (it simply means “there is”). To this logic we added an existence predicate ‘EI’, a conceivability predicate ‘C’, and a 2-place greater than predicate ‘G’. Let us again use φ1 to represent the formula:

(φ1) Cx & ¬∃y(Gyx & Cy)

This formula asserts: x is conceivable and such that nothing greater is conceivable. We then formulated the two nonlogical premises and meaning postulate as follows:

Premise 1: ∃xφ1

Meaning Postulate: ∀x∀y(Gxy ∨ Gyx ∨ x = y)

Premise 2: ¬EI∃xφ1 → ∃y(Gyxφ1 & Cy)

Premise 1 asserts merely that there is a conceivable object such that nothing greater is conceivable.1 The Meaning Postulate for greater than asserts merely that greater than is a connected relation, i.e., for any two

1 As noted above, ∃xφ does not imply ∃x(φ & EIφ).

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distinct objects, either the first is greater than the second or the second is greater than the first. From the Meaning Postulate, we derived a lemma (Lemma 2) that uses the uniqueness quantifier \( \exists!x \phi \) (defined in the usual way) and that asserts \( \exists!x \phi_1 \rightarrow \exists!x \phi_2 \). Given Lemma 2, Premise 1 implies that there exists a unique conceivable object such that nothing greater is conceivable. By the classical principles governing definite descriptions, it follows that the description \( \exists!x \phi_1 \) has a denotation, i.e., that \( \exists y(y = \exists!x \phi_1) \). This justified Anselm’s use of the definite description \( \text{that than which nothing greater can be conceived (id quo majus cogitari non potest) in his reasoning in } \text{Proslogion II} \). Moreover, from this conclusion and Premise 2, the ontological argument easily proceeds to the conclusion that God exists, if given the definition that God (‘g’) is, by definition, \( \exists!x \phi_1 \). The proof cites logical theorems governing definite descriptions, which are explained in the 1991 paper.²

We subsequently discovered (2011), by computational means, that the conclusion that God exists can be derived solely from Premise 2 and the definition of God. Again, the proof cites logical theorems governing descriptions, which are explained in the 2011 paper.³ From this brief review, it is clear why the title and focus of our 2011 paper had to be on (a) how Premise 1 and the Meaning Postulate for greater than are redundant given the strength of Premise 2, and (b) the discovery of this redundancy by computational means. But our paper also contained a number of observations and some discussion of the soundness of the argument; this analysis, and in particular, our doubts about Premise 2 and our view about how one might tweak the 1991 argument to avoid the redundancies, seem to have been overlooked by the critics. In particular, in Section 4 of our 2011 paper, we compared the original 1991 version with the simplified 2011 version and in Section 5, we concluded our paper with reasons for objecting to Premise 2.

We show, in what follows, that these observations and reasons already anticipate and address some of the criticisms that have been raised about our work.³ This is not to say that no valid criticisms have been raised. We agree with the section of Garbacz’s 2012 paper (Section 3), where he raises some genuine methodological issues about our translation of the 1991 argument into Prover9 notation. He noted, for example, that our use of the constant ‘g’ implicitly validated Premise 1 simply by representing the definition that g is identical to the conceivable x such that none greater can be conceived.

We are happy to acknowledge these points from Section 3 of Garbacz’s paper. In 2009–2010, while doing research for our 2011 paper, we were in the early stages of learning how to use resolution-based automated reasoning systems to represent statement schemata for a first-order, natural deduction logic extended with definite descriptions. There are a number of subtleties that we had not recognized and so our methodology was not as refined as it might have been. We’ve therefore revised our webpage on the computational investigation of the ontological argument to reflect our enhanced understanding of how to implement the 1991 premises in a first-order reasoning environment. The new representation no longer defines ‘g’ by deploying a creative definition. We’ve acknowledged Garbacz’s concerns on that webpage.⁵ Nevertheless, despite these flaws, we deployed the automated reasoning engine with enough sophistication to discover that Premise 2 makes Premise 1 and the Meaning Postulate redundant.⁶ This hadn’t been noticed in the 20 years subsequent to the

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<td>( \exists! x \phi )</td>
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<td>from (5), by Premise 2</td>
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<td>( \neg \exists y(Gyx \phi_1 \land Cy) )</td>
<td>from (4), by &amp;E</td>
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<td>8</td>
<td>( E \exists! x \phi_1 )</td>
<td>from (5), (6), and (7), by Reductio</td>
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<td>( E g )</td>
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⁴A notable exception is Parent 2015, who recognized that we were suspicious and, indeed, critical of the strength of Premise 2 in our 2011 paper.

⁵See http://mally.stanford.edu/cm/ontological-argument/.

⁶It is worth mentioning work by Blumson (2017), who implemented our 1991 and 2011 representations of the argument in the higher-order reasoning system Isabelle/HOL.
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publication of our 1991 paper. As we shall see, however, other criticisms raised in Garbacz’s paper are not as effective and we shall address these in Section 3 below. We first address the charge of begging the question, however, which was raised by Rushby.

2 Does the Argument Beg the Question?

Rushby (2018, 1475–78, 1484) claims that the version of the argument in our 1991 paper begs the question. He introduces his claim by saying (1475):

In the next section, I introduce a strict definition of “begging the question” and show that a rendition of the Argument due to Oppenheimer and Zalta[19] is vulnerable to this charge.

His subsequent argument has two parts: first there is a definition of begging the question and then there is the claim that an analogue of Premise 2 begs the question according to that definition. We challenge both the definition and the claim that Premise 2 begs the question.

Here is Rushby’s definition of begging the question, where he uses the equality sign = as a biconditional sign (2018, 1476):

... if C is our conclusion, Q our “questionable” premise (which may be a conjunction of simpler premises) and P our other premises, then Q begs the question if C is equivalent to Q, assuming P: i.e.,

P ⊢ C = Q.

Note here that Rushby has defined what it is for a premise to beg the question. Rushby then says (2018, 1478):

...the premise Greater1 begs the question under the other assumptions of the formalization ... . Given that we have proved God_re from Greater1 and vice-versa, we can easily prove they are equivalent. Thus, in the definition of “begging the question” given earlier, C here is God_re, Q is Greater1 and P is the rest of the formalization (i.e., ExUnd, the definition of God?, and the predicate subtype trichotomous? asserted for >).

In this passage Greater1 refers to the sentence (2018, 1477):

Greater1: AXIOM FORALL x: (NOT re?(x) => EXISTS y: y > x)

where “The uninterpreted predicate re? identifies those beings that exist ‘in reality’” (2018, 1477). This is just a more general version of our Premise 2, since Premise 2 asserts specifically that if xφ1 doesn’t exist, then there is a conceivable thing that is greater. So Rushby’s argument, if good, would imply that Premise 2 is question-begging.

Let’s turn first to his definition of begging the question. Of course, it should be observed that the fallacy of begging the question traditionally applies to arguments, not to premises, yet for some reason, Rushby takes it to apply to premises.7 Let’s put this aside for the moment, since we’ll later argue that begging the question is a charge that can be leveled only against an argument relative to a dialogical context. Let’s focus for now solely on Rushby’s definition of begging the question. We think it is easy to show that his definition categorizes premises that clearly don’t beg the question as ones that do. We can undermine his formal definition using a purely formal example. For consider the following valid argument:

Fa ≡ Gb
Gb ≡ Rd
∴ Fa ≡ Rd

In this argument a biconditional conclusion follows from two biconditional premises. The argument is non-question begging—both premises play a role in the derivation of the conclusion. Moreover, the following, related argument is also valid and non-question begging:

Fa ≡ Gb
Gb ≡ Rd
Fa
∴ Rd

But according to Rushby’s definition, the premise Fa is question-begging in this last argument. That’s because from the first two premises Fa ≡ Gb and Gb ≡ Rd, we can derive Fa ≡ Rd. That is, Fa ≡ Gb, Gb ≡ Rd ⊢ Fa ≡ Rd. Now, in Rushby’s definition, let C be the conclusion Rd,

Parent (2015, 477) suggests that we would accept that Premise 2 is question-begging, for he says “Nevertheless, O&Z go on to give an independent case that (P) [Premise 2] is either false or question-begging, and as things currently stand, their verdict strikes me as correct”. But it is important to correct this misattribution, since otherwise it counts against what we say below. Though we did develop (2011, Section 5) reasons against adopting Premise 2 we didn’t claim there that Premise 2 is question-begging or that the argument with Premise 2 is question-begging.
let $Q$ be the premise $Fa$, and let $P$ (the other premises) stand for the first two premises. Then the derivation that we just identified, namely, $Fa \equiv Gb, Gb \equiv Rd \vdash Fa \equiv Rd$, would become represented in Rushby’s notation as $P \vdash C \equiv Q$. Thus, according to Rushby, $Fa$ begs the question. But it clearly doesn’t. Nor does $Rd$ beg the question in the argument:

\[
\begin{align*}
Fa & \equiv Gb \\
Gb & \equiv Rd \\
Rd
\end{align*}
\]

This is not question-begging even though $Fa \equiv Gb, Gb \equiv Rd \vdash Fa \equiv Rd$. Again, Rushby’s schema, when applied, wrongly entails that the premise $Rd$ begs the question. One needn’t provide an interpretation of the formal claims to see that they constitute a counterexample to Rushby’s definition – any non-trivial interpretation will do.

It should also be noted here that Rushby ignores the fact that the definition we use in developing the 1991 argument, i.e., $g =_{df} \exists x \phi_1$, is crucial in deriving the conclusion $Eg$. This definite description doesn’t play a role in Rushby’s representation of our 1991 argument. So Rushby’s formalization isn’t faithful to our 1991 version. It may be that his formalization is subject to the charge of begging the question, but our 1991 version is not.

A better definition of question-begging is one that applies to arguments as a whole. Indeed, it applies to arguments relative to a dialogical situation.\(^8\) Though we don’t have a replacement definition to offer, we take it that, to a first approximation, an argument $A$ begs the question if and only if (1) $A$ is valid, and (2) in the dialogical situation in which $A$ is presented, the arguer is not entitled to one of the premises (even if only for the sake of the argument). This, at least, clearly explains why the valid argument $P \vdash P$ is question-begging: in any dialogical situation in which the arguer is presenting an argument for $P$, they are not entitled to the premise $P$ even if only for the sake of the argument. For if one were entitled to $P$ (even if only for the sake of the argument), one wouldn’t need an argument, since the premise is sufficient and no further argument is called for.

Using this rough definition, we see no reason to think that the 1991 version is question-begging. Moreover, a proponent of the 2011 version, with its single Premise 2, does not beg the question. For the conclusion additionally requires the definition $g =_{df} \exists x \phi_1$, which according to the standard theory of definitions is equivalent to adding a premise. Moreover, it should be noted that even though the definition $g =_{df} \exists x \phi_1$ introduces a conservative extension of the premises used in both the 1991 and 2011 versions, the fact that the definition is conservative is justified by the corresponding premises—and requires such justification.\(^9\) Without such a justification, the definition would either be empty (in a free logic for individual constants) or creative (in a logic free for definite descriptions but not for individual constants)! These points have been insufficiently appreciated in the debate surrounding the argument and they are key to the understanding of the logic underlying the argument.

Finally, it is unlikely that Anselm was presenting the ontological argument in a dialogical situation in which he was confronting an atheist. So, if we were to suppose that the interlocutor is a theist, there is reason to think that the arguer would be entitled to all the premises in all the versions of the argument we’ve presented. We emphasize here that we are not defending the soundness of the argument. Let’s suppose, for the purposes of this paper, that the argument is being presented in a dialogical situation where the interlocutor is an atheist or agnostic. We still think that one is entitled to use the premises of all the versions of the argument we’ve put forward, if only for the sake of the argument. This doesn’t require that the premises be true, but only that using all of the premises doesn’t defeat the arguer’s purpose in the situation of arguing for a conclusion. So, one can continue the debate after the argument is presented by considering whether the premises are true or false. And that is where we think the focus should be with respect to the ontological argument as we’ve presented it, not on whether the argument begs the question. Indeed, we think that this way of looking at the matter acknowledges that there is, in the background, a question about the significance of the definition of God ($g =_{df} \exists x \phi_1$) in the argument. This is crucial to the conclusion (and given some theories of definition, has the status of a premise), since it stands in need of justification. So we take

\(^8\)See the literature on the dialectical/dialogical approach to fallacies, for instance Hamblin 1970 and Walton 1994.

\(^9\)In particular, the introduction of the definition is justified by showing that $\exists! x \phi_1$ follows from the premises. Once this unique existence claim has been established, one is entitled to give the uniquely existing object a name.
it to be a mistake to focus just on the premises when alleging that the argument is question begging.

3 Other Criticisms of Premise 2

Since Premise 2 makes Premise 1 and the Meaning Postulate for greater than otiose, the revised version of the argument presented in 2011 has a single premise (Premise 2) and a single conclusion (E!g). In fact, it also crucially requires a definition and we’ll get to this in a moment. Our 2011 version was criticized in Garbacz 2012, who claims, in Section 2 of his paper, that Premise 2 is equivalent to the conclusion that God exists and so using it in this argument is ‘epistemically inefficient’ (2012, 588). His criticism is developed on the basis of a certain understanding of our paper that he puts forward, namely, “The main contribution of Oppenheimer and Zalta 2011 is a discovery that premises Premise 1 and Connectedness are obsolete, i.e., that you can reach the same conclusion assuming only Premise 2” (Garbacz 2012, 586). But in developing his criticisms, Garbacz missed certain crucial facts about the 2011 argument and points addressing his concerns about Premise 2 that were made in that paper.

Let’s first look at exactly what Garbacz says. He notes, on pp. 586–587, that Premise 2 is equivalent to the claim that E!xφ1. That is, Garbacz notes that:

(A) E!xφ1 ≡ (¬E!xφ1 → ∃y(Gyxφ1 & Cy))

Clearly, Garbacz is correct about this. The right-to-left direction of the above biconditional was established by our 2011 version of the argument. And the left-to-right direction follows by propositional logic: from p it follows that ¬p → q.

However, Garbacz uses this fact as part of his argument to the conclusion that “Premise 2 cannot be used in any proper argument for the existence of God” (2012, 586). His first reason is that “It [Premise 2] not only implies that God exists but it is (logically) equivalent to the latter claim”, where the ‘latter claim’ refers to ‘God exists’. But the equivalence (A) above doesn’t allow one to infer that Premise 2 is logically equivalent to the claim that God exists. For Garbacz’s claim to be true, Premise 2 would have to be logically equivalent to the claim E!g. It is not; that is, the claim E!g is not equivalent to the claim on the left side of equivalence (A): it is not equivalent to ‘the x such that φ1 exists’. Rather, for E!g and E!xφ1 to be equivalent, you need the definition g =df ixφ1. And this is a non-trivial part of the argument. One can’t introduce this definition into the argument until it is justified. And the justification can be given by noting that the intermediate conclusion E!xφ1 implies that the definite description is well-defined. Specifically, if you examine the 2011 argument reproduced in footnote 3 above, you will see that the intermediate conclusion E!xφ1 occurs on line 8; this implies, by Russell’s theory of descriptions, that the definite description is well-defined (i.e., has a unique denotation). So to reach the conclusion that God exists from Premise 2, you need the non-trivial step of justifying and introducing the definition of God as the x such that φ1.

We think this shows that Premise 2 is not (logically) equivalent to the claim that God exists. Thus far, though, we’ve only shown that Premise 2 doesn’t imply that God exists. But the converse implication also fails: the claim that God exists doesn’t imply Premise 2. Again, to draw the inference from the claim that God exists to Premise 2, one must first convert E!g to E!xφ1 by the definition of God. Then one could correctly claim, as Garbacz does, that the latter is equivalent to Premise 2. But one can’t thereby conclude that God exists (E!g) implies Premise 2 without appealing to, and justifying, the definition of God. And that can’t be done in this direction; there is no justification for stipulating that g =df ixφ1 simply on the basis of E!g.

This observation undermines Garbacz’s first reason for suggesting that our 2011 version is ‘epistemically inefficient’. For if the question is ‘Does God exist?’ and the formal representation of that claim is E!g, then Premise 2, despite its power, does not imply E!g without the definition of God.

But Garbacz gives two other reasons why Premise 2 shouldn’t be used in the ontological argument (587–58), namely, that the consequent of Premise 2 is logically inconsistent and so the claim E!xφ1 becomes equivalent to a tautology. But clearly, the logical inconsistency of the consequent of Premise 2 is not obvious and it takes some reasoning (indeed a diagonal argument) and the logic of definite descriptions to show that one can derive a contradiction from the assumption that ∃y(Gyxφ1 & Cy).

This is precisely how Anselm’s argument gets its purchase—he coupled a consequent that subtly implies a contradiction to the antecedent ¬E!xφ1.

We suggest that these reasons Garbacz offers against Premise 2 don’t
go beyond the reservations we already expressed about this premise in 2011. The fourth observation in Section 4 (2011, 346) included the lines:

\[ \neg E!\exists y (Gyx \land C_y) \]  

... it is interesting to note that one can (i) abandon the definition of God as \( \exists x \phi_1 \), (ii) generalize Premise 2 to the claim that \( \neg E!x \rightarrow \exists y (Gyx \land C_y) \), and still (iii) develop a valid argument to the conclusion that anything that satisfies \( \phi_1 \) exemplifies existence.

And in Section 5 (2011, 348), we gave an extended argument against Premise 2. The last paragraph of our paper (2011, 348–349) included the following lines:

Thus, arguments ... above show that the defender of the ontological argument needs independent support for two claims: that the definite description denotes and that Premise 2 is true. Our 1991 analysis of the argument is still relevant, since it shows how the ontological arguer could justify Anselm's use of the definite description. [Footnote 14: Given the argument outlined above against Premise 2, a defender of Anselm might consider whether the ontological argument can be strengthened by using our original formulation as in 1991, but with the general form of Premise 2 discussed earlier: \( \neg E!x \rightarrow \exists y (Gyx \land C_y) \). The justification of this more general premise may not be subject to the same circularity that infects the justification of Premise 2 (though, of course, it may have problems of its own).] The present analysis shows why the use of the definite description needs independent justification. Consequently, though the simplified ontological argument is valid, Premise 2 is questionable and to the extent that it lacks independent justification, the simplified argument fails to demonstrate that God exists. The use of computational techniques in systematic metaphysics has illuminated the relationship between Premise 2 of the ontological argument and the conclusion that God exists.

It is clear from these passages not only that we gave reasons why Premise 1 and the connectedness of greater than are not obsolete, but also that we questioned Premise 2. Thus, we anticipated the conclusion of the extended argument that Garbacz develops in Section 2 of his paper, where he concludes that (589):

...Thus, it is little wonder that one can dispense with Premise 1 and Connectedness in Oppenheimer and Zalta’s [1991] ontological argument as their logical contribution (to this argument) is covered by Premise 2. Alas, this third premise seems to be too strong to be an acceptable basis for any argument for the existence of God.

Garbacz here makes it seem that we were advocating for Premise 2 in our paper. While the specific criticisms he puts forward do offer some additional reasons for not accepting Premise 2 (though see below), we did not argue in our paper that, in a proper reconstruction of the argument, Premise 2 should replace Premise 1 and the Meaning Postulate.

Indeed, in the fourth observation and the final paragraph (footnote 14) quoted above from our 2011 paper, we suggested that the 1991 paper should have weakened Premise 2 so that it becomes the following universal claim:

Premise 2': \( \forall x (\neg E!x \rightarrow \exists y (Gyx \land C_y)) \)

This asserts, in essence, that if a thing fails to exist then something greater than it can be conceived. We were pointing out here that Premise 1, the Meaning Postulate, and Premise 2' yield a perfectly good argument for God's existence without appealing to Premise 2. The resulting argument is a variant of the argument in the 1991 paper—once one establishes that the description \( \exists x \phi_1 \) denotes, one can instantiate it into Premise 2' to obtain the original Premise 2. And then the reductio proceeds in the same manner as in the 1991 paper. We lay out the revised argument with the weaker Premise 2' in a footnote.10 This revised version of the argument goes exactly like the original 1991 argument except at the point where Premise 2 is invoked.

We think it is easy to see that Premise 2' alone does not yield the intermediate conclusion that \( E!x \phi_1 \), nor does any pairwise combination of Premise 1, Premise 2', and the Meaning Postulate for greater than. The

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| 11 | On Anselm’s Ontological Argument in Proslogion II | Paul E. Oppenheimer and Edward N. Zalta | 12 |
reader might wish to use automated reasoning tools to establish these last claims, as we shall not argue for them further here.\textsuperscript{11}

Indeed, to forestall an objection to Premise 2', we can weaken it even further, so that it asserts: for any \( x \), if \( x \) is a conceivable thing such that nothing greater is conceivable and \( x \) fails to exist, then something greater than \( x \) can be conceived. Formally:

\[
\text{Premise 2'}: \forall x ((\phi_1 \land \neg E!x) \rightarrow \exists y (Gyx \land Cy))
\]

This would forestall the objection that Premise 2' is consistent with, and even licenses, the view that an existing conceivable evil thing is greater than a nonexistent one. A variant of the 1991 ontological argument still goes through with Premise 2'\textsuperscript{′′}:

\begin{enumerate}
\item \( \exists x \phi_1 \) Premise 1
\item \( \exists y (y = ix \phi_1) \) from (1), by Lemma 2 (1991)
\item \( Cix \phi_1 \land \neg \exists y (Gyx \phi_1 \land Cy) \) from (3), by Desc. Thm. 1 (1991)
\item \( (Cix \phi_1 \land \neg \exists y (Gyx \phi_1 \land Cy) \land \neg E!ix \phi_1) \rightarrow \exists y (Gyx \phi_1 \land Cy) \)
\item \( \neg E!ix \phi_1 \) Assumption, for \textit{Reductio}
\item \( Cix \phi_1 \land \neg \exists y (Gyx \phi_1 \land Cy) \land \neg E!ix \phi_1 \) from (4), (6)
\item \( \exists y (Gyx \phi_1 \land Cy) \) from (5), (7)
\item \( \neg \exists y (Gyx \phi_1 \land Cy) \) from (4), by \&E
\item \( E!ix \phi_1 \) from (6), (7), and (8), by \textit{Reductio}
\item \( E!g \) from (9), by the definition of \( g \)
\end{enumerate}

This represents our best analysis of the argument; had we seen the issue with Premise 2 in 1991, we would have eliminated Premise 2 in favor of Premise 2'\textsuperscript{′′}. Note here how Premise 1 and the Meaning Postulate for \textit{greater than} still play a role. The latter is needed for the derivation of Lemma 2, which moves us from the former to the claim that there is a unique \( x \) such that \( \phi_1 \). These are key to the justification of the use of the description \( ix \phi_1 \), which, in turn, is needed to instantiate Premise 2'\textsuperscript{′′} to obtain line 5. In free logic, one may not instantiate a term into a universal claim until one has established that the term has a denotation.

Consequently, by substituting either Premise 2' or, preferably, Premise 2'\textsuperscript{′′} for Premise 2 in the 1991 argument, we obtain a \textit{valid} argument in which all of the premises are needed to derive the intermediate conclusion \( E!ix \phi_1 \).\textsuperscript{12} In either case, none of the premises are redundant and the argument doesn’t beg the question.\textsuperscript{13}

### 4 Does Anselm Use a Definite Description?

In a recent paper, Eder \& Ramharter (2015) also recognize that question-begging is not an accurate charge to bring against any of the versions of the ontological argument of the kind that we’ve been discussing.\textsuperscript{14} But they suggest that the regimentation of the argument using a definite description doesn’t properly capture the reasoning in \textit{Proslogion} II. While our versions of the argument represent Anselm’s term \textit{that which nothing greater can be conceived} as a definite description, Eder \& Ramharter argue that it is not. They introduce two abbreviations and put them in boldface: \textit{id quo} abbreviates ‘\textit{id quo maius cogitari non potest}’ (‘that than which nothing greater can be conceived’), and \textit{aliquid quo} abbreviates ‘\textit{aliquid quo nihil maius cogitari potest}’ (‘something than

\textsuperscript{12}Note how our reconstruction using Premise 2'\textsuperscript{′′} differs from Garbacz’s proposed weakening of Premise 2 (2012, 591). His proposed weakening asserts that, for any \( x \), if it is not the case that both \( \phi_1 \) and \( E!x \), then there is something \( y \) such that (a) \( y \) is greater than \( x \), (b) \( \phi_1 \), and (c) \( x \) is conceivable. Formally, \( \forall x (\neg (\phi_1 \land E!x) \rightarrow \exists y (Gyx \land \neg \phi_1 \land Cy)) \). We think Premise 2'\textsuperscript{′′} is a more elegant weakening of Premise 2; \( \phi_1 \) is not required in both the antecedent and the consequent to derive the desired conclusion. And the antecedent of Premise 2'\textsuperscript{′′} considers only the case of a \( \phi_1 \) object that doesn’t exist (the antecedent of Garbacz’s proposed weakening considers the cases where either \( x \) isn’t such that none greater can be conceived or \( x \) doesn’t exist). Modulo the definition of God (see below), Premise 2'\textsuperscript{′′} is sufficient to yield the existence of God given Premise 1 and the connectedness of \textit{greater than}.

\textsuperscript{13}To see this with respect to the argument that cites Premise 2', notice two things: (1) line 5 depends on Premise 1 and the Meaning Postulate for \textit{greater than} even though those weren’t cited as the justification. But these two principles are needed to conclude that the description has a denotation and can thereby be instantiated, via the free logic version of \textit{\&E}, into Premise 2'\textsuperscript{′′}. (2) When one assumes, for reductio, at line 6, that \( \neg E!ix \phi_1 \), one cannot then reach line 7 from Premise 2'\textsuperscript{′′} alone; one must additionally have Premise 1 and the Meaning Postulate for \textit{greater than} in order to establish the second and third conjuncts of line 7, which come from line 4.

\textsuperscript{14}They say (2015, fn 4, 2797):

Sometimes, proofs of the existence of God are accused of being question-begging, but this critique is untenable. It is odd to ask for a deductive argument whose conclusion is not contained in the premises. Logic cannot pull a rabbit out of the hat.
which nothing greater can be conceived’). Then they say (p. 2802):

Whether or not a reconstruction of Anselm’s argument is valid may crucially depend on whether \textit{id quo} has to be understood as a definite description. But we think that it is not just that we do not \textit{have to} understand \textit{id quo} as a definite description, but that we \textit{should} not.\footnote{Their position has recently been endorsed in Campbell 2018 (55).} For one thing, if \textit{id quo} had to be read as a definite description, Anselm would be committed to presupposing the uniqueness\footnote{In this quote, their footnote numbers are preserved in square brackets.} of \textit{aliquid quo} already in Chap. II, which seems to be in conflict with the fact that only in Chap. III does Anselm mention God’s uniqueness for the first time.\footnote{For one thing, if \textit{id quo} had to be read as a definite description, Anselm would be committed to presupposing the uniqueness of \textit{aliquid quo} already in Chap. II, which seems to be in conflict with the fact that only in Chap. III does Anselm mention God’s uniqueness for the first time.\footnote{Their position has recently been endorsed in Campbell 2018 (55).} Rather, it seems to us that Anselm is using this diction only as a device to refer back to \textit{something} ‘than which nothing greater can be conceived’. In other words, we think that Anselm’s \textit{id quo} is best understood as an auxiliary name, which is used to prove something from an existence assumption.\footnote{Their position has recently been endorsed in Campbell 2018 (55).}}

\[\text{Rule EI}\]

Assume that the constant symbol \(c\) does not occur in \(\phi, \psi\) or \(\Gamma\), and that \(\Gamma, \phi \vdash \psi\). Then \(\Gamma, \exists x \phi \vdash \psi\), and there is a deduction of \(\psi\) from \(\Gamma\) and \(\exists x \phi\) in which \(c\) does not occur.

But more importantly, the suggestion that \textit{id quo} is being used by Anselm as an arbitrary name can’t be sustained given their representation of the argument. To see the structure of our objection to this suggestion, note that when a number theorist introduces an arbitrary name for a prime number by saying ‘let \(\alpha\) be an arbitrary prime’, they may go on to reason about \(\alpha\) using the known principles of number theory, including known principles about primes. But what they cannot do is, at some point in the argument, assert new principles (i.e., new axioms or premises) governing \(\alpha\). This understanding is made clear by the classical rule for using arbitrary names within an argument to derive the conclusion from some premises. The classical rule requires that one choose an arbitrary name that doesn’t already occur in the premises (or in the existentially quantified premise or in the conclusion). In a standard logic text, e.g., Enderton (2001, 124), Rule EI is stated as follows, where \(\Gamma\) is a set of premises, \(\exists x \phi\) is an existentially quantified premise, \(\psi\) is the conclusion, and \(\phi^c\) is the result of substituting \(c\) for the free occurrences of \(x\) in \(\phi\):

\[\text{Rule EI}\]

Assume that the constant symbol \(c\) does not occur in \(\phi, \psi\) or \(\Gamma\), and that \(\Gamma, \phi^c \vdash \psi\). Then \(\Gamma, \exists x \phi \vdash \psi\), and there is a deduction of \(\psi\) from \(\Gamma\) and \(\exists x \phi\) in which \(c\) does not occur.

Thus, the rule says if you can derive \(\psi\) from some premises \(\Gamma\) in which \(c\) doesn’t occur and from the fact that \(c\) is such that \(\phi\), then you can derive \(\psi\) from \(\Gamma\) and the claim \(\exists x \phi\). One guarantees that the constant \(c\) is arbitrary by requiring that it be new to the proof, i.e., that no other information about the particular name \(c\) occurs in the premises. This implies that if one uses an arbitrary name \(c\) as an instance of \(\exists x \phi\), one can’t then introduce new premises or assumptions that govern \(c\).

But Eder & Ramharter’s analysis of the argument seems to violate Rule EI in a number of ways. First, their conclusion contains the arbitrary name. In the formal proof in their paper (p. 2813), they introduce constant \(g\) as the arbitrary constant (instead of the more cumbersome \textit{id quo}). From the principles \textit{ExUnd} (\(\exists x Gx\)) and \textit{Def C-God} (\(Gx \iff \neg \exists y (y > x)\)) they conclude \(\exists y \neg \exists x (x > y)\) and then say “let \(g\) be such that \(\neg \exists x (x > g)\)” Then they assume \(\neg E!g\) for reductio, reach a contradiction, and so conclude \(E!g\). But this conclusion of the argument contains the arbitrary name. The reasoning doesn’t really conform to Rule EI; the conclusion of the argument doesn’t seem to have the right
form.

Let’s suppose that they can reformulate the argument to address this issue. The second problem is that it appears that they have used premises in their argument in which the arbitrary name occurs, in violation of the rule that the arbitrary constant can’t occur in any premises used in the derivation. To match up their argument with the requirements of Rule EI, take $\exists x \phi$ of Rule EI to be a premise with *aliquid quo* as the subject and take the arbitrary constant $c$ of Rule EI to be *id quo* or $g$. Then Rule EI says that if you can derive God’s existence ($\psi$) from some premises $\Gamma$ in which *id quo* doesn’t occur and from the premise introducing *id quo* as an arbitrary *aliquid quo*, you can derive God’s existence from just $\Gamma$ and the premise which has *aliquid quo* as the subject.

But the argument put forward by Eder & Ramharter violates this rule because the premises they use to derive God’s existence are not free of the arbitrary name *id quo*. Just consider the fact that on p. 2813, they introduce the arbitrary constant $g$, then define a new notion ($F_{EI}(F)$) by means of a biconditional in which $g$ occurs, and then use that definition to derive facts about $g$. The definition in question is:

$$F_{EI}(F) :\iff Fg \lor F = E!$$

Note first that the definiendum appears to depend on the choice of $g$, but this isn’t acknowledged by indexing the definiendum to $g$. Note second that a definition is equivalent to a biconditional axiom, and so an appeal to this definition in their proof becomes an appeal to a premise in which the arbitrary name occurs, something that is explicitly ruled out by Rule EI. Because the definition uses the arbitrary name in the definiens, there is a real question here about the propriety of such a definition; inspection of the literature on the theory of definition suggests that such a definition hasn’t been well studied, if at all, as a legitimate logical method. Of course, it may be that the authors can discharge the definition so that no premise involving the arbitrary name is used in violation of Rule EI. For example, they might be able to (a) use a definition in which a variable replaces the arbitrary name, (b) prove theorems about the notion defined, and (c) appeal to those theorems when reasoning with the arbitrary name in the ontological argument.

For example, instead of the above, they could offer the following definition, indexing the definiendum to $x$ and $E!$:

$$F_{x,E!}(F) :\iffFx \lor F = E!$$

With this definition, they might be able to prove theorems about the defined notion $F_{x,E!}(F)$ that hold for arbitrary $x$. Then, when reasoning in the ontological argument with respect to the arbitrarily chosen object $g$, they could instantiate these theorems to $g$ without the theorems counting as premises that would violate Rule EI.

We think, therefore, that the argument formulated in their paper has to be reformulated much more carefully, to make sure that their reasoning with arbitrary names is valid. But, again, for the sake of argument, let’s grant them that *id quo* is an arbitrary name and that they’ve developed reasoning with this name that is valid according to Rule EI. The final problem for their representation is the fact that the name of God never makes an appearance in the argument. Examination of their formal representation shows that they introduce (p. 2808) the label *God!* to stand for the formal claim $\exists x(Gx & E!x)$ (“there is a $x$ such that $x$ is a God and $x$ exists”), where $Gx$ is defined by the statement Def C-God identified above. Then on p. 2813, they say “Now that everything is in place, we are in a position to prove *God!* as follows.”

Putting aside the fact that they use both second-order and third-order logic in the argument, the problem is that the formal representation doesn’t show that Proslogion II has an argument for the existence of God. Nowhere is the name of God introduced into the argument. As we’ve seen, the constant $g$ is not a name of God, but rather an arbitrary name which they use to represent ‘*id quo maius cogitari non potest!*’. Thus, the conclusion of their argument, $E!g$, doesn’t use a name of God. So once you grant them that the phrase ‘*that than which nothing greater can be conceived*’ is an arbitrary name and not a description, they have only established a fact about an arbitrarily chosen object of the kind nothing

\[\begin{align*}
\text{(**) \ \forall x \exists y! \forall F(F(F) \iff Fx)}
\end{align*}\]

We don’t see any textual justification or other reason for thinking that higher-order machinery is needed for Anselm’s argument in Proslogion II.
greater is conceivable, namely, that such an object exists. The conclusion of the argument should be that God exists, not that \( g \) or \( \text{id quo} \) exists.

We don’t think it would be a good response to suggest: since uniqueness isn’t discussed until \textit{Proslogion III}, Anselm can’t conclude that God exists until that next chapter. Such a response won’t work for the following reason. In the \textit{opening} of \textit{Proslogion II}, Anselm directly uses the name ‘God’ (= ‘Deus’) and the vocative case for ‘Lord’ (= ‘Domine’ = vocative case of ‘Dominus’). So Anselm clearly takes the conclusion of the argument to apply to God. And that is how Eder & Ramharter understand \textit{Proslogion II}. They say (2015, p. 2800):

Having established in Chap. II that God exists in reality from the assumption that God exists at least in the understanding, Anselm proceeds in Chap. III by proving it is inconceivable that God does not exist.\(^{[10]}\)

So Eder & Ramharter themselves agree that in \textit{Prologion II}, there is an argument that establishes something about God, and not just about some arbitrarily chosen object such that nothing greater can be conceived.

Indeed, we can’t accept the concluding clause of Eder & Ramharter’s claim (quoted earlier) that (p. 2802):

... if \( \text{id quo} \) had to be read as a definite description, Anselm would be committed to presupposing the uniqueness\(^{[19]}\) of \( \text{aliquid quo} \) already in Chap. II, which seems to be in conflict with the fact that only in Chap. III does Anselm mention God’s uniqueness for the first time.\(^{[20]}\)

Their evidence for the concluding clause, given in footnote 20, is to quote Anselm as saying, in \textit{Proslogion III}, “In fact, everything else there is, except You alone, can be thought of as non existing. You, alone then, ...” (Anselm [MW, 2008, 88]). But this hardly counts as a statement that God is a unique thing such that nothing greater can be conceived. Here Anselm is saying only that God uniquely has existence in the highest degree, and this is a claim that plays no role in the ontological argument, as far as we can tell. This is why we don’t accept their conclusion that our account, which uses a definite description for the argument in \textit{Proslogion II}, “seems to be in conflict with the fact that only in Chap. III does Anselm mention God’s uniqueness for the first time.” By using ‘God’ as a proper name in \textit{Proslogion II}, he is already presupposing uniqueness, and that presupposition, together with Premise 1 and the meaning postulate for greater than, justifies his move from \( \text{aliquid quo} \) to \( \text{id quo} \), as suggested by our representation of the argument.

We conclude that one can not so easily dismiss the suggestion that ‘\text{id quo maius cogitari non potest}’ is used as a definite description in \textit{Proslogion II}. At present, Eder & Ramharter’s suggestion that \( \text{id quo} \) is being used as an arbitrary name leads to the list of problems just outlined. They would have to make a much stronger case before one should be willing to accept this analysis.

5 Conclusion

The work we’ve done in this paper leads us to conclude that the analysis of \textit{Proslogion II} using a definite description still has a lot to offer those trying to understand Anselm’s ontological argument for the existence of God. To our way of thinking, the interesting questions concern the truth of the premises and the justification of the definition of God. Given what we’ve now learned, the premises in question are Premise 1, Premise 2′, the Meaning Postulate for greater than, and the definition of God: \( g =_{df} \forall \phi \forall x \phi \).

In our paper of 2007, we argued that Premise 1 is the real culprit in the argument. We tried to show that Premise 1 is too strong because it yields the existence of an object that \textit{exemplifies} the property of being a conceivable thing such that nothing greater is conceivable. We argued that Anselm’s subsidiary argument for Premise 1 involves two assumptions: (1) that the mere understanding of the phrase ‘conceivable thing such that nothing greater is conceivable’ requires one to grasp an intensional object, and (2) any such intensional object has to exemplify the property \textit{being a conceivable thing such that nothing greater is conceivable}. We then challenged the second assumption, on the grounds that the intentionality [with-a-t] involved in understanding the phrase only requires that the intensional [with-an-s] object (which is thereby grasped) \textit{encode} the property \textit{being a conceivable thing such that nothing greater is conceivable}. Here we appealed to the notion of encoding used in the theory of abstract objects (Zalta 1983, 1988).

Interestingly, this is a point of contact with the work of Eder & Ramharter’s paper, since their principle \textbf{Realization} (reported in footnote 17) is a kind of comprehension principle that underlies Anselm’s assertion that there is something in the understanding such that nothing
greater is conceivable. Eder & Ramharter write:

So, bearing in mind that first-order quantifiers are ranging over objects existing in the understanding, *Realization* seems plausible. It appears to be an analytic truth that any consistent set of (primitive, positive) conditions is realized by some object in the understanding. This seems to be confirmed by passages like (II.8), where Anselm claims that ‘whatever is understood is in the understanding’.\(^{[41]}\) Bearing in mind that by ‘understanding something’ Anselm means understanding what its properties are, we can see that whenever we conceive of a certain set of (non-contradictory) properties, this set gives rise to an object that exists in the understanding—and this is just what *Realization* says. So even though Anselm does not state *Realization* explicitly, we think that it is implicit in how Anselm thinks about objects.

The authors here are placing a lot of weight on this third-order principle (*Realization*) and it isn’t clear to us that we should accept that Anselm is committed to this principle simply on the basis of the fact that he takes whatever is understood to be in the understanding. Nor is it clear to us that “by ‘understanding something’ Anselm means understanding what its properties are”. We don’t see any textual evidence for this claim. But more importantly, they take *Realization* to be an integral premise of the ontological argument.

By contrast, our 2007 paper shows that whereas the comprehension principle for intensional objects might help us to see why Anselm thought Premise 1 is true, such a principle doesn’t need to be added as a premise in the ontological argument itself. It might be needed to justify Premise 1, but it doesn’t make an appearance in *Proslogion* II. To formulate the ontological argument, one shouldn’t need, as a premise, that for any primitive condition on properties, there is an object that *exemplifies* just the properties satisfying that condition. But this is what *Realization* intuitively asserts. Thus, the work in Oppenheimer & Zalta 2007 bears on this question, and we suggest that a question for further study is to focus on Premise 1 and the implicit comprehension principle that Anselm must be relying upon to conclude that it is true.

References


