On Anselm’s Ontological Argument in *Proslogion* II*

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Abstract

Formulations of Anselm’s ontological argument have been the subject of a number of recent studies. We examine these studies in light of Anselm’s text and (a) respond to criticisms that have surfaced in reaction to our earlier representations of the argument, (b) identify and defend a more refined representation of Anselm’s argument on the basis of new research, and (c) compare our representation of the argument, which analyzes that than which none greater can be conceived as a definite description, to a representation that analyzes it as an arbitrary name. Along the way, we point out how certain criticisms of our 2011 paper insufficiently attend to the analysis and observations we made in that paper.

1 Review

We begin by reviewing the relevant lines of *Proslogion* II (the line numbers in what follows refer to those in the Appendix, which quotes the J. Barnes’s 1972 translation). On line 2, Anselm takes God to be something than which nothing greater can be imagined. Then in lines 3–7, he argues that there is such a thing, on the grounds that even the fool understands the phrase “something than which nothing greater can be imagined”. The preliminary conclusion at line 8 is that there is in the understanding something than which nothing greater can be imagined. Then starting with line 8, he develops an argument to an intermediate conclusion (on line 12), namely, that something than which nothing greater can be imagined exists in reality.

So, there is a puzzle: where is the conclusion that everyone agrees is contained in *Proslogion* II, namely, that God exists? One might suggest that it follows from the intermediate conclusion on line 12 in the context of lines 1 and 2, where Anselm is directing his remarks to God. But with a little work, we can see why this doesn’t really work. Let $C$ be the property being conceivable, let $G$ be the greater than relation, and consider the formula that we used in our 1991 paper:

$$(\phi_1) \ Cx \land \neg \exists y (Gyx \land Cy)$$

Then, line 2 of *Proslogion* II is the claim that God is such an object, since the antecedent of ‘you’ in line 2 is ‘Lord’ (‘Domine’) in line 1. So line 2 seems to assert, where ‘$g$’ is a constant or name of God:
Cg & ¬∃y(Ggy & Cy)  \tag{L2}

But this can’t be used with the intermediate conclusion on line 12 to conclude that God exists. The intermediate conclusion on line 12 is:

\[ \exists x(\phi_1 \& E!x) \tag{L12} \]

I.e., there is something (a) that is conceivable, (b) such that nothing greater is conceivable, and (c) that exists. The conclusion that God exists (E!g) doesn’t validly follow from (L2) and (L12).\(^1\)

So, is there a valid argument to the conclusion that God exists in \textit{Proslogion II}? In our paper of 1991, we attempted to show that there is, and that all Anselm really needs to derive God’s existence, in addition to what he says in \textit{Proslogion II} is, (a) to regard the phrase “that than which nothing greater can be conceived” (\textit{id quo maius cogitari non potest}) as a definite description, and (b) the assumption that \textit{greater than} is a connected relation. The following summary of our 1991 argument explains why.

In 1991, we started with a 1st-order logic extended with primitive definite descriptions of the form \(ix\phi\). We assumed that the classical laws of quantification theory govern constants and variables, but that a free logic governs definite descriptions. We also assumed that quantified formulas are not existentially loaded: we read \(\exists x\phi\) as “there is an \(x\) such that \(\phi\)” (not as “there exists an \(x\) such that \(\phi\)”). And we defined the uniqueness quantifier (\(\exists!x\phi\)) in the usual way. We also used an existence predicate: \(E!\). So a formula of the form \(\exists x(E!x \& \phi)\) asserts “there is an \(x\) such that that \(x\) exists and is such that \(\phi\)”, i.e., \(there\ exists\ an\ \(x\)\ such\ that\ \phi\).

Then we used \(\phi_1\) (identified above) to represent the claim: \(x\) is conceivable and such that nothing greater than \(x\) is conceivable. We then

\(^1\)In the absence of any assumptions about \textit{greater than}, we can see that this inference is invalid by considering a model in which there are distinct, equally great objects, say \(a\) and \(b\), such that both are conceivable and such that none greater are conceivable. And suppose, in this model, that \(a\) exists (i.e., makes the claim ‘Elu’ true), \(b\) doesn’t exist (i.e., makes the claim ‘¬Elb’ true), and the denotation of God (i.e., the denotation of ‘g’) is the object \(b\). Then, in this model, L2 is true (since \(b\) is conceivable and such that none greater is conceivable and \(g = b\), it follows that \(g\) is conceivable and such that none greater is conceivable); L12 is true (since something, namely, \(a\), is conceivable and such that nothing greater is conceivable, and that exists); and \textit{God exists} (i.e., E!g) is false (since ‘¬Elb’ and \(g = b\)). So, God’s existence doesn’t yet follow and we’re still at a loss as to how to see \textit{Proslogion II} as containing an ontological argument for the existence of God.

proposed a formalization of the ontological argument in \textit{Proslogion II} on which Anselm used two nonlogical premises and the previously mentioned meaning-postulate about the \textit{greater than} relation:

Premise 1: \(\exists x\phi_1\)

Meaning Postulate: \(\forall x\forall y(Gxy \lor Gyx \lor x = y)\)

Premise 2: \(¬E!x\phi_1 \rightarrow \exists y(Gyix\phi_1 \& Cy)\)

Premise 1 captures the preliminary conclusion at line 8 of \textit{Proslogion II}; it asserts merely that \textit{there} is a conceivable object such that nothing greater is conceivable. The Meaning Postulate for \textit{greater than} asserts merely that \textit{greater than} is a connected relation, i.e., for any two distinct objects, either the first is greater than the second or the second is greater than the first. From the Meaning Postulate, we derived a lemma (Lemma 2): \(\exists x\phi_1 \rightarrow \exists!x\phi_1\). In other words, the mere connectedness of \textit{greater than} allows us to infer, from Premise 1, that there is a unique conceivable thing such that nothing greater is conceivable. By the classical principles governing definite descriptions, it follows that the description \(ix\phi_1\) has a denotation, i.e., that \(\exists y(y = ix\phi_1)\). This fact justifies Anselm’s use of the definite description that \textit{than which nothing greater can be conceived} (\textit{id quo majus cogitari non potest}) in his reasoning in \textit{Proslogion II}. Moreover, from this conclusion and Premise 2, the ontological argument easily proceeds to the conclusion that God exists, if given the definition that God (‘g’) is, by definition, \(ix\phi_1\). In other words, with the connectedness of \textit{greater than}, Anselm can strengthen line 2 of \textit{Proslogion II}; instead of the claim that God is an object such that \(\phi_1\), line 2 can be strengthened to the identity statement that God \textit{is} (identical to) the conceivable thing such that nothing greater can be conceived and so be treated as a definition.\(^2\)

The actual reasoning in the argument cites logical theorems governing definite descriptions, which are explained in the 1991 paper.\(^3\) But, the conclusion of the argument is that \textit{God exists} (E!g) and so this represent-
tation explains why it appears that Proslogion II contains an ontological argument for the existence of God.

We subsequently discovered (2011), by computational means, that the conclusion that God exists can be derived solely from Premise 2 and the definition of God. Again, the proof cites logical theorems governing descriptions, which are explained in the 2011 paper. From this brief review, it is clear why the title and focus of our 2011 paper had to be on (a) how Premise 1 and the Meaning Postulate for greater than are redundant given the strength of Premise 2, and (b) the discovery of this redundancy by computational means. But our paper also contained a number of observations and some discussion of the soundness of the argument; this analysis, and in particular, our doubts about Premise 2 and our view about how one might tweak the 1991 argument to avoid the redundancies, seem to have been overlooked by the critics. In particular, in Section 4 of our 2011 paper, we compared the original 1991 version with the simplified 2011 version and in Section 5, we concluded our paper with reasons for objecting to Premise 2.

We show, in what follows, that these observations and reasons already anticipate and address some of the criticisms that have been raised about how one might tweak the 1991 argument to avoid the redundancies, as refined as it might have been. We’ve therefore revised our webpage on the computational investigation of the ontological argument to reflect our enhanced understanding of how to implement the 1991 premises in a first-order reasoning environment. The new representation no longer defines ‘g’ by deploying a creative definition. We’ve acknowledged Garbacz’s concerns on that webpage.

Nevertheless, despite these flaws, we deployed the automated reasoning engine with enough sophistication to discover that Premise 2 makes Premise 1 simply by representing the definition that g is identical to the conceivable x such that none greater can be conceived.

We agree with the section of Garbacz’s 2012 paper (Section 3), where he raises some genuine methodological issues about our translation of the 1991 argument into Prover9 notation. He noted, for example, that in the context of Prover9, our use of the constant ‘g’ implicitly validated Premise 1 simply by representing the definition that g is identical to the conceivable x such that none greater can be conceived.

In the next section, I introduce a strict definition of “begging the question” and show that a rendition of the Argument due to Op-

\[
\begin{align*}
1. \exists x \phi_1 & \quad \text{Premise 1} \\
2. \exists x \phi_1 & \quad \text{from (1), by Lemma 2 (1991)} \\
3. \exists y (y = x \phi_1) & \quad \text{from (2), by Description Thm 1 (1991)} \\
4. C_1 \exists x \phi_1 \land \neg \exists y (Gyx \phi_1 \land C_y) & \quad \text{from (3), by Description Thm 2 (1991)} \\
5. \neg E_1 \exists x \phi_1 & \quad \text{Assumption, for Reductio} \\
6. \exists y (Gyx \phi_1 \land C_y) & \quad \text{from (5), by Premise 2} \\
7. \neg \exists y (Gyx \phi_1 \land C_y) & \quad \text{from (4), by \&E} \\
8. E_1 \exists x \phi_1 & \quad \text{from (5), (6), and (7), by Reductio} \\
9. E_l g & \quad \text{from (8), by the definition of } g
\end{align*}
\]

4In tabular form:

\[
\begin{align*}
1. \neg E_1 \exists x \phi_1 & \quad \text{Assumption, for Reductio} \\
2. \exists y (Gyx \phi_1 \land C_y) & \quad \text{from (1), by Premise 2 and MP} \\
3. Ghx \phi_1 \land C_h & \quad \text{from (2), by } E \exists, ‘h’ \text{ arbitrary} \\
4. Ghx \phi_1 & \quad \text{from (3), by \&E} \\
5. \exists y (y = x \phi_1) & \quad \text{from (4), by Dese. Thm. 3 (2011)} \\
6. C_1 \exists x \phi_1 \land \neg \exists y (Gyx \phi_1 \land C_y) & \quad \text{from (5), by Dese. Thm. 2 (1991)} \\
7. \neg \exists y (Gyx \phi_1 \land C_y) & \quad \text{from (6), by \&E} \\
8. E_1 \exists x \phi_1 & \quad \text{from (1), (2), (7), by Reductio} \\
9. E_l g & \quad \text{from (8), by the definition of } g
\end{align*}
\]

5A notable exception is Parent 2015, who recognized that we were suspicious and, indeed, critical of the strength of Premise 2 in our 2011 paper.

2 Does the Argument Beg the Question?

Rushby (2018, 1475–78, 1484) claims that the version of the argument in our 1991 paper begs the question. He introduces his claim by saying (1475):

In the next section, I introduce a strict definition of “begging the question” and show that a rendition of the Argument due to Op-

6See http://mally.stanford.edu/cm/ontological-argument/.

7It is worth mentioning work by Blumson (2017), who implemented our 1991 and 2011 representations of the argument in the higher-order reasoning system Isabelle/HOL.
penheimer and Zalta\textsuperscript{[19]} is vulnerable to this charge.

His subsequent argument has two parts: first there is a definition of 	extit{begging the question} and then there is the claim that an analogue of Premise 2 begs the question according to that definition. We challenge both the definition and the claim that Premise 2 begs the question.

Here is Rushby’s definition of 	extit{begging the question}, where he uses the equality sign = as a biconditional sign (2018, 1476):

\[
\ldots \text{if } C \text{ is our conclusion, } Q \text{ our “questionable” premise (which may be a conjunction of simpler premises) and } P \text{ our other premises, then } Q \text{ begs the question if } C \text{ is equivalent to } Q, \text{ assuming } P: \text{i.e., } P \vdash C = Q.
\]

Note here that Rushby has defined what it is for a premise to beg the question. Of course, it should be observed that the fallacy of begging the question traditionally applies to arguments, not to premises, yet for some reason, Rushby takes it to apply to premises.\textsuperscript{8} Let’s put this aside for the moment, since we’ll later argue that 	extit{begging the question} is a charge that can be leveled only against an argument relative to a dialogical context. Let’s focus for now solely on Rushby’s definition of 	extit{begging the question}.

We think it is easy to undermine his definition by showing that it categorizes premises and arguments that clearly don’t beg the question as ones that do. We do this by considering the following two arguments, neither of which is question-begging.

\[
\begin{align*}
A & \equiv B \\
B & \equiv C \\
\therefore & C \equiv A
\end{align*}
\]

\[
\begin{align*}
\vdash & C \equiv A \\
\therefore & C
\end{align*}
\]

In the first argument, a biconditional conclusion follows from two biconditional premises. The argument is non-question begging—both premises play a role in the derivation of the conclusion. In the second argument, we derive a conclusion C by adding the premise A to the first argument.

\textsuperscript{8}Parent (2015, 477) suggests that we \textit{would} accept that Premise 2 is question-begging, for he says “Nevertheless, O&Z go on to give an independent case that (P) [Premise 2] is either false or question-begging, and as things currently stand, their verdict strikes me as correct”. But it is important to correct this misattribution, since otherwise it counts against what we say below. Though we did develop (2011, Section 5) reasons against adopting Premise 2 we didn’t claim there that Premise 2 is question-begging or that the argument with Premise 2 is question-begging.

But according to Rushby’s definition, the premise A in the second argument is question-begging. To see this, let us assign elements of the second argument as values to the variables in Rushby’s definition of question-begging: let C be the conclusion, let Q be the premise A, and let P (the other premises) stand for the first two premises. Then plugging in these values, Rushby’s definition makes the following claim about the second argument:

\[
\text{If } C \text{ is our conclusion, } A \text{ our “questionable” premise (which may be a conjunction of simpler premises) and } P \text{ our other premises } (A \equiv B \text{ and } B \equiv C), \text{ then } A \text{ begs the question if } C \text{ is equivalent to } A, \text{ assuming } P, \text{i.e., } P \vdash C \equiv A, \text{i.e., if } A \equiv B, B \equiv C \vdash C \equiv A.
\]

Thus, Rushby’s definition of question-begging leads to the incorrect conclusion that the premise A in the second argument begs the question, since the first argument does indeed show that from the premises A \equiv B and B \equiv C, we can derive the conclusion C \equiv A.

Of course, it may be that our 1991 representation of the ontological argument is question-begging, but one can’t establish this from reasoning that makes use of Rushby’s incorrect definition. A better definition of question-begging is one that applies to arguments as a whole. Indeed, it applies to arguments relative to a dialogical situation.\textsuperscript{9} We don’t have an uncontroversial definition of question-begging to offer in this paper. But we can say, to a first approximation, that our versions of the ontological argument do not assume what they are trying to prove, and do not help themselves to information to which they are not entitled (in the dialogical context). So if this is what is meant by begging the question, our version of the argument doesn’t do so.

Moreover, to a second approximation, we might say that an argument A begs the question if and only if (1) A is valid, and (2) in the dialogical situation in which A is presented, the arguer is not entitled to one of the premises (even if only for the sake of the argument). This, at least, explains why the valid argument P \vdash P is question-begging: in any dialogical situation in which the arguer is presenting an argument for P, they are not entitled to the premise P even if only for the sake of the argument. For if one were entitled to P (even if only for the sake of the argument), one wouldn’t need an argument, since the premise is sufficient.

\textsuperscript{9}See the literature on the dialectical/dialogical approach to fallacies, for instance Hamblin 1970 and Walton 1994.
and no further argument is called for.\textsuperscript{10} Using this rough definition, then, we see no reason to think that the 1991 version is question-begging.

We emphasize here that we are not defending the soundness of the ontological argument. But we’re happy to suppose, for the purposes of this paper, that the argument is being presented in a dialogical situation where the interlocutor is an atheist or agnostic.\textsuperscript{11} We still think that one is entitled to use the premises of all the versions of the argument we’ve put forward in such situations, if only for the sake of the argument. This doesn’t require that the premises be true, but only that using all of the premises doesn’t defeat the arguer’s purpose in the situation of arguing for a conclusion. Of course, one can continue the debate after the argument is presented by considering whether the premises are true or false. And that is where we think the focus should be with respect to the ontological argument as we’ve presented it, not on whether the argument begs the question. Indeed, in the next section, we’ll see that one can regard the definition of God \((g =_{df} ixφ₁)\) as an additional premise needed for the argument. The addition of another needed premise makes it even harder to conclude that the argument begs the question.

\section{Other Criticisms of Premise 2}

Since Premise 2 makes Premise 1 and the Meaning Postulate for greater than otiose, the revised version of the argument presented in 2011 has a single premise (Premise 2) and a single conclusion \((E!g)\). The 2011 version was criticized in Garbacz 2012, who claimed, in Section 2 of his paper, that Premise 2 is equivalent to the conclusion that God exists and so using it in this argument is ‘epistemically inefficient’ (2012, 588). His criticism was developed on the basis of a certain understanding of our paper that he

\textsuperscript{10}Note that this is one of the reasons why the ontological argument from Premise 2 alone is unsatisfactory. As we noted at the end of our 2011 paper, one cannot argue for Premise 2 by asserting: assume the description \(ixφ₁\) denotes and that what it denotes exists; then the antecedent of Premise 2 is false, since it asserts \(¬E!xφ₁\), in which case Premise 2 is true. Such an argument for Premise 2 helps itself to the claim that \(E!xφ₁\), something which a theist is not entitled to when arguing for the existence of God.

\textsuperscript{11}The Fool mentioned in \textit{Proslogion II} could be seen as an atheist interlocutor. Gaunilo, who is not mentioned in \textit{Proslogion II}, could also be seen as representing an atheist or agnostic point of view. But Gaunilo’s reply to Anselm was written after the \textit{Proslogion} was circulated, and so can’t be considered, strictly speaking, as part of the dialogical context internal to the \textit{Proslogion}.

puts forward, namely, “The main contribution of Oppenheimer and Zalta 2011 is a discovery that … Premise 1 and \textit{Connectedness} are obsolete, i.e., that you can reach the same conclusion assuming only Premise 2” (Garbacz 2012, 586). But in developing his criticisms, Garbacz missed certain crucial facts about the 2011 argument; we included some observations about Premise 2 in that paper that addressed his concerns.

Let’s first look at exactly what Garbacz says. He notes, on pp. 586–587, that Premise 2 is equivalent to the claim that \(E!xφ₁\). That is, Garbacz notes that:

\[(A) \ E!xφ₁ \equiv (¬E!xφ₁ \rightarrow ∃y(Gyφ₁ & Cy))\]

Clearly, Garbacz is correct about this. The right-to-left direction of the above biconditional was established by our 2011 version of the argument. And the left-to-right direction follows by propositional logic: from \(p\) it follows that \(¬p \rightarrow q\).

However, Garbacz uses this fact as part of his argument to the conclusion that “Premise 2 cannot be used in any proper argument for the existence of God” (2012, 586). His first reason is that “It [Premise 2] not only implies that God exists but it is (logically) equivalent to the latter claim”, where the ‘latter claim’ refers to ‘God exists’. But the equivalence (A) above doesn’t allow one to infer that Premise 2 is logically equivalent to the claim that God exists. For Garbacz’s claim to be true, Premise 2 would have to be logically equivalent to the claim \(E!g\). It is not; that is, the claim \(E!g\) is not equivalent to the claim on the left side of equivalence (A); it is not equivalent to ‘the \(x\) such that \(φ₁\) exists’. Rather, for \(E!g\) and \(E!xφ₁\) to be equivalent, you need the definition \(g =_{df} ixφ₁\). And this is a non-trivial part of the argument. One can’t introduce this definition into the argument until it is justified. And the justification can be given by noting that the intermediate conclusion \(E!xφ₁\) implies that the definite description is well-defined. Specifically, if you examine the 2011 argument reproduced in footnote 4 above, you will see that the intermediate conclusion \(E!xφ₁\) occurs on line 8; this implies, by Russell’s theory of descriptions, that the definite description is well-defined (i.e., has a unique denotation). So to reach the conclusion that God exists from Premise 2, you need the non-trivial step of justifying and introducing the definition of ‘God’ as the \(x\) such that \(φ₁\).

We think this shows that Premise 2 is not (logically) equivalent to the claim that God exists. Thus far, though, we’ve only shown that Premise 2
doesn’t imply that God exists. But the converse implication also fails: the claim that God exists doesn’t imply Premise 2. Again, to draw the inference from the claim that God exists to Premise 2, one must first convert $E!g$ to $E!lxφ₁$ by the definition of God. Then one could correctly claim, as Garbacz does, that the latter is equivalent to Premise 2. But one can’t thereby conclude that God exists ($E!g$) implies Premise 2 without appealing to, and justifying, the definition of God. And that can’t be done in this direction; there is no justification for stipulating that $g =_df lxφ₁$ simply on the basis of $E!g$.

This observation undermines Garbacz’s first reason for suggesting that our 2011 version is ‘epistemically inefficient’. For if the question is ‘Does God exist?’ and the formal representation of that claim is $E!g$, then Premise 2, despite its power, does not imply $E!g$ without the definition of God.

But Garbacz gives another other reason why Premise 2 shouldn’t be used in the ontological argument (587–58), namely, that the consequent of Premise 2 is logically inconsistent and so the claim $E!lxφ₁$ becomes equivalent to a tautology. But, clearly, the logical inconsistency of the consequent of Premise 2 is not obvious and it takes both some reasoning (indeed a diagonal argument) and the logic of definite descriptions to show that one can derive a contradiction from the assumption that $∃y(Gyxφ₁ & Cy)$. This is precisely how Anselm’s argument gets its purchase—he coupled a consequent that subtly implies a contradiction to the antecedent $¬E!lxφ₁$.

Moreover, this reason that Garbacz offers against Premise 2 doesn’t go beyond the reservations we had already expressed about this premise in 2011. The fourth observation in Section 4 (2011, 346) included the lines:

... it is interesting to note that one can (i) abandon the definition of God as $lxφ₁$, (ii) generalize Premise 2 to the claim that $¬E!x → ∃y(Gyx & Cy)$, and still (iii) develop a valid argument to the conclusion that anything that satisfies $φ₁$ exemplifies existence.

And in Section 5 (2011, 348), we gave an extended argument against Premise 2. The last paragraph of our paper (2011, 348–349) included the following lines:

Thus, arguments ... above show that the defender of the ontological argument needs independent support for two claims: that the definite description denotes and that Premise 2 is true. Our 1991 analysis of the argument is still relevant, since it shows how the ontological arguer could justify Anselm’s use of the definite description. [Footnote 14: Given the argument outlined above against Premise 2, a defender of Anselm might consider whether the ontological argument can be strengthened by using our original formulation as in 1991, but with the general form of Premise 2 discussed earlier: $¬E!x → ∃y(Gyx & Cy)$. The justification of this more general premise may not be subject to the same circularity that infects the justification of Premise 2 (though, of course, it may have problems of its own).] The present analysis shows why the use of the definite description needs independent justification. Consequently, though the simplified ontological argument is valid, Premise 2 is questionable and to the extent that it lacks independent justification, the simplified argument fails to demonstrate that God exists. The use of computational techniques in systematic metaphysics has illuminated the relationship between Premise 2 of the ontological argument and the conclusion that God exists.

It is clear from these passages not only that we gave reasons why Premise 1 and the connectedness of greater than are not obsolete, but also that we questioned Premise 2. Thus, we anticipated the conclusion of the extended argument that Garbacz develops in Section 2 of his paper, where he concludes that (589):

...Thus, it is little wonder that one can dispense with Premise 1 and Connectedness in Oppenheimer and Zalta’s [1991] ontological argument as their logical contribution (to this argument) is covered by Premise 2. Alas, this third premise seems to be too strong to be an acceptable basis for any argument for the existence of God.

Garbacz here makes it seem that we were advocating for Premise 2 in our paper. While the specific criticisms he puts forward do offer some additional reasons for not accepting Premise 2 (though see below), we did not argue in our paper that, in a proper reconstruction of the argument, Premise 2 should replace Premise 1 and the Meaning Postulate.

Indeed, in the fourth observation and the final paragraph (footnote 14) quoted above from our 2011 paper, we suggested that the 1991 paper should have weakened Premise 2 so that it becomes the following universal claim:
Premise 2': \( \forall x (\neg E!x \rightarrow \exists y(Gyx & Cy)) \)

This asserts, in essence, that if a thing fails to exist then something greater than it can be conceived. We pointed out that Premise 1, the Meaning Postulate, and Premise 2' yield a perfectly good argument for God’s existence without appealing to Premise 2. The resulting argument is a variant of the argument in the 1991 paper—once one establishes that the description \( \phi_1 \) denotes, one can instantiate it into Premise 2' to obtain the original Premise 2. And then the reductio proceeds in the same manner as in the 1991 paper, i.e., the revised version of the argument goes exactly like the original 1991 argument except at the point where Premise 2 is invoked.

We think it is easy to see that Premise 2' alone does not yield the intermediate conclusion that \( E!x\phi_1 \), nor does any pairwise combination of Premise 1, Premise 2' and the Meaning Postulate for greater than. The reader might wish to use automated reasoning tools to establish these last claims, as we shall not argue for them further here.\(^{12}\)

However, to forestall an objection to Premise 2', we can weaken it even further, so that it asserts: for any \( x \), if \( x \) is a conceivable thing such that nothing greater is conceivable and \( x \) fails to exist, then something greater than \( x \) can be conceived. Formally:

Premise 2'': \( \forall x ((\phi_1 & \neg E!x) \rightarrow \exists y(Gyx & Cy)) \)

This would forestall the objection that Premise 2' is consistent with, and even licenses, the view that an existing conceivable evil thing is greater than a nonexistent one. A variant of the 1991 ontological argument still goes through with Premise 2'':

1. \( \exists x\phi_1 \)
   Premise 1
2. \( \exists x\phi_1 \)
   from (1), by Lemma 2 (1991)
3. \( \exists y(y = ix\phi_1) \)
   from (2), by Desc. Thm. 1 (1991)
4. \( E!ix\phi_1 \)
   from (3) and Premise 2'', by \( \forall E \)
5. \( Cix\phi_1 & \neg \exists y(Gyx\phi_1 & Cy) & \neg E!ix\phi_1 \rightarrow \exists y(Gyx\phi_1 & Cy) \)
   from (3) and Premise 2'', by \( \forall E \)
6. \( \neg E!ix\phi_1 \)
   Assumption, for Reductio
7. \( Cix\phi_1 & \neg \exists y(Gyx\phi_1 & Cy) & \neg E!ix\phi_1 \)
   from (5), (6), by \&I
8. \( \exists y(Gyx\phi_1 & Cy) \)
   from (4), (7), by MP
9. \( \neg \exists y(Gyx\phi_1 & Cy) \)
   from (5), by \&E
10. \( E!ix\phi_1 \)
    from (6), (7), and (8), by Reductio
11. \( E!y \)
    from (9), by the definition of \( y \)

This represents our best analysis of the argument; had we seen the issue with Premise 2 in 1991, we would have eliminated Premise 2 in favor of Premise 2''. Note that neither Premise 2' nor Premise 2'' has a consequent that is a contradiction — the consequent of the conditionals embedded in the universal claims doesn’t invoke \( \forall x\phi_1 \), and as such, a contradiction can’t be derived. This undermines an objection Garbacz raised against Premise 2, which we discussed earlier. Note here how Premise 1 and the Meaning Postulate for greater than still play a role. The latter is needed for the derivation of Lemma 2, which moves us from the former to the claim that there is a unique \( x \) such that \( \phi_1 \). These are key to the justification of the use of the description \( \forall x\phi_1 \) which, in turn, is needed to instantiate Premise 2'' to obtain line 5. In free logic, one may not instantiate a term into a universal claim until one has established that the term has a denotation.

Consequently, by substituting Premise 2'' for Premise 2 in the 1991 argument, we obtain a valid argument in which all of the premises are needed to derive the intermediate conclusion \( E!ix\phi_1 \).\(^{13}\) None of the premises are redundant and the argument doesn’t beg the question.\(^{14}\)

\(^{12}\)See Parent 2015 (478) for another way of weakening Premise 2.

\(^{13}\)Note how our reconstruction using Premise 2'' differs from Garbacz's proposed weakening of Premise 2 (2012, 591). His proposed weakening asserts that, for any \( x \), if it is not the case that both \( \phi_1 \) and \( E!x \), then there is something \( y \) such that (a) \( y \) is greater than \( x \), (b) \( \phi_1 \), and (c) \( x \) is conceivable. Formally, \( \forall x(\neg (\phi_1 & E!x) \rightarrow \exists y(Gyx & \phi_1 & Cy) \)) We think Premise 2'' is a more elegant weakening of Premise 2; \( \phi_1 \) is not required in both the antecedent and the consequent to derive the desired conclusion. And the antecedent of Premise 2'' considers only the case of a \( \phi_1 \) object that doesn’t exist (the antecedent of Garbacz’s proposed weakening considers the cases where either \( x \) isn’t such that none greater can be conceived or \( x \) doesn’t exist). Modulo the definition of God (see below), Premise 2'' is sufficient to yield the existence of God given Premise 1 and the connectedness of greater than.

\(^{14}\)To see this with respect to the argument that includes Premise 2'', notice two things: (1) line 5 depends on Premise 1 and the Meaning Postulate for greater than even though those weren’t cited as the justification. But these two principles are needed to conclude that the description has a denotation and can thereby be instantiated, via the free logic version of \( \forall E \), into Premise 2''. (2) When one assumes, for reductio, at line 6, that \( \neg E!ix\phi_1 \), one cannot then reach line 7 from Premise 2'' alone; one must additionally have Premise 1 and the Meaning Postulate for greater than in order to establish the second and third conjuncts of line 7, which come from line 4.
4 Does Anselm Use a Definite Description?

In a recent paper, Eder & Ramharter (2015) also recognize that question-begging is not an accurate charge to bring against any of the versions of the ontological argument of the kind that we’ve been discussing. But they suggest that the regimentation of the argument using a definite description doesn’t properly capture the reasoning in Proslogion II. While our versions of the argument represent Anselm’s term that than which nothing greater can be conceived as a definite description, Eder & Ramharter argue that it is not. They introduce two abbreviations and put them in boldface: id quo abbreviates ‘id quo maius cogitari non potest’ (‘that than which nothing greater can be conceived’), and aliquid quo abbreviates ‘aliquid quo nihil maius cogitari potest’ (‘something than which nothing greater can be conceived’). Then they say (p. 2802):

Whether or not a reconstruction of Anselm’s argument is valid may crucially depend on whether id quo has to be understood as a definite description. But we think that it is not just that we do not have to understand id quo as a definite description, but that we should not.[18] For one thing, if id quo had to be read as a definite description, Anselm would be committed to presupposing the uniqueness[19] of aliquid quo already in Chap. II, which seems to be in conflict with the fact that only in Chap. III does Anselm mention God’s uniqueness for the first time.[20] Rather, it seems to us that Anselm is using this diction only as a device to refer back to something ‘than which nothing greater can be conceived’.

In other words, we think that Anselm’s id quo is best understood as an auxiliary name, which is used to prove something from an existence assumption.[21]

[In this quote, their footnote numbers are preserved in square brackets and the emphasis is in the original.] Eder and Ramharter are suggesting here that “id quo maius cogitari non potest” is not being used as a definite description in Proslogion II but rather functions more like an arbitrary name.[16] Later, in their paper, they further abbreviate id quo as the arbitrary constant ‘g’, but it is important to remember that this is an abbreviation of a phrase with descriptive content. On the face of it, there is a huge difference between a simple constant and an expression like id quo maius cogitari non potest. According to standard logical practice, an arbitrary name is a simple constant that has no descriptive content, whereas the phrase “id quo maius cogitari non potest” is a complex expression that has descriptive content. The descriptive content is expressed by the arrangement of the operators (negation, quantification, and conjunction) and Latin predicates (for greater than and conceives). So it is not clear how to reconcile Eder & Ramharter’s hypothesis (that we can introduce ‘g’ as an abbreviation for a phrase with descriptive content) with current logical practice.[17]

But more importantly, the suggestion that id quo is being used by Anselm as an arbitrary name can’t be sustained given their representation of the argument. To develop our objection to this suggestion, note that when a number theorist introduces an arbitrary name for a prime number by saying ‘let τ be an arbitrary prime’, they may go on to reason about τ using the known principles of number theory, including known principles about primes. But what they cannot do is, at some point in the argument, assert or appeal to new principles (i.e., new axioms, premises, etc.) governing τ.

This understanding is made clear by the classical rule for using arbitrary names within an argument to derive a conclusion from some premises. The classical rule requires that one choose an arbitrary name that doesn’t already occur in the premises or in the conclusion. In a standard logic text, e.g., Enderton (2001, 124), Rule EI is stated as follows, where Γ is a set of premises, ⋁xφ is an existentially quantified premise, ψ is the conclusion, and ϕc is the result of substituting c for the free occurrences of x in ϕ.


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Sometimes, proofs of the existence of God are accused of being question-begging, but this critique is untenable. It is odd to ask for a deductive argument whose conclusion is not contained in the premises. Logic cannot pull a rabbit out of the hat.

[16] Their position has recently been endorsed in Campbell 2018 (55).

[17] One might, with Rushby (2013, m.s.), suggest that id quo is a Hilbert-style ε-term, that is, a term of the form εxφ1, which might be read: an x such that φ1. But the semantic interpretation of such terms requires a choice function. We leave it as an open question as to whether Anselm used language that requires a choice function for its interpretation. Though we recognize that Anselm lived near the beginning of a period that saw a number of interesting developments in logic, the project that we, and Eder & Ramharter, are engaged in is not to systematize 11th century logic, but rather understand 11th century reasoning in terms that are today recognized as valid.

[18] Note that Enderton uses ‘EI’ to stand for Existential Instantiation (2001, 125),
Rule EI

Assume that the constant symbol \( c \) does not occur in \( \phi \), \( \psi \) or \( \Gamma \), and that \( \Gamma, \phi^c \vdash \psi \). Then \( \Gamma, \exists x \phi \vdash \psi \), and there is a deduction of \( \psi \) from \( \Gamma \) and \( \exists x \phi \) in which \( c \) does not occur.

Thus, the rule says: if you can derive \( \psi \) from some premises \( \Gamma \) in which \( c \) doesn’t occur and from the fact that \( c \) is such that \( \phi \), then you can derive \( \psi \) from \( \Gamma \) and the claim \( \exists x \phi \). By requiring that the constant \( c \) be new to the proof, one guarantees that the reasoning is about an arbitrary entity from \( \Gamma \) and the claim \( \psi \) doesn’t occur and from the fact that \( c \). Thus, the rule says: if you can derive \( \psi \) from some premises \( \Gamma \) in which \( c \) doesn’t occur and from the fact that \( c \) is such that \( \phi \), then you can derive \( \psi \) from \( \Gamma \) and the claim \( \exists x \phi \).

But Eder & Ramharter’s analysis of the argument seems to violate Rule EI in a number of ways. First, their conclusion contains the arbitrary name. In the formal proof in their paper (p. 2813), they introduce constant \( g \) as the arbitrary constant (instead of the more cumbersome id quo). From the principles ExUnd \( (\exists x Gx) \) and Def C-God \( (Gx :\leftrightarrow \neg \exists y(y > x)) \) they conclude \( \exists y \neg \exists z(x > y) \) and then say “let \( g \) be such that \( \neg \exists z(x > g) \).” Then they assume \( \neg E! g \) for reductio, reach a contradiction, and so conclude \( E! g \). But this conclusion of the argument contains the arbitrary name. The reasoning doesn’t really conform to Rule EI; the conclusion of the argument doesn’t seem to have the right form.

But suppose they reformulate the argument to address this issue. The second problem is that it appears that they have used premises in their argument in which the arbitrary name occurs, in violation of the condition that the arbitrary constant can’t occur in any premises used in the derivation. Let’s try to match up their argument with the requirements of Rule EI. At the point where they say “let \( g \) be such that \( \neg \exists z(x > g) \)”, the rule doesn’t allow them to reason from any other premise containing the arbitrary name \( g \). But their argument does precisely that. After introducing \( g \), they then define (p. 2813) a new notion \( (F_E! (F)) \) by means of a biconditional in which \( g \) occurs, and then use that definition to derive facts about \( g \). The definition in question is:

\[
F_{E!}(F) :\leftrightarrow Fg \lor F = E!
\]

Note first that the definiendum appears to depend on the choice of \( g \), but this isn’t acknowledged by indexing the definiendum to \( g \). Note second that a definition is equivalent to a biconditional axiom, and so an appeal to this definition in their proof becomes an appeal to a premise in which the arbitrary name occurs, something that is expressly prohibited by Rule EI. Because the definition uses the arbitrary name in the definiens, there is a real question here about the propriety of such a definition.

Of course, it may be that the authors can discharge the definition so that no premise involving the arbitrary name is used in violation of Rule EI. For example, they might be able to (a) use a definition in which a variable replaces the arbitrary name, (b) prove general, universally quantified, theorems about the notion defined, and (c) appeal to those theorems when reasoning with the arbitrary name in the ontological argument. For example, instead of the above, they could offer the following definition, indexing the definiendum to \( x \) and \( E! \):

\[
F_{x,E!}(F) :\leftrightarrow Fx \lor F = E!
\]

With this definition, they might be able to prove theorems about the defined notion \( F_{x,E!}(F) \) that hold for arbitrary \( x \). Then, when reasoning in the ontological argument with respect to the arbitrarily chosen object \( g \), they could instantiate these theorems to \( g \) without the theorems counting as premises that would violate Rule EI. But the classical literature on the logic of the existential quantifier and the theory of definition suggests that, in absence of such changes to the reasoning, the argument Eder & Ramharter presented in their paper is in violation of the conditions for the use of Rule EI.\(^{19}\)

We conclude that the argument in their paper has to be reformulated much more carefully, to make sure that the reasoning with arbitrary names is valid. But, again, let’s suppose for the sake of argument that they’ve redeveloped the reasoning to address our concerns. The final problem for their representation is the fact that the name of God never makes an

\(^{19}\)For the theory of definition, see Frege 1879, §24; Padoa 1900; Frege 1903a, §§55–67, §§139–144, and §§146–147; Frege 1903b, Part I; Frege 1914, 224–225; Suppes 1957; Mates 1972 (197–203); Dummett 1973; Belnap 1993; Hodges 2008; and Gupta 2019.
appearance in the argument. Examination of their formal representation shows that they introduce the label God! to stand for the formal claim $\exists x (Gx \land E!x)$ (“there is an $x$ such that $x$ is a God and $x$ exists”), where $Gx$ is defined by the statement Def C-God identified above. Then they say (p. 2813), “Now that everything is in place, we are in a position to prove God! as follows.”

Putting aside the fact that they use both second-order and third-order logic in the argument, the problem is that the formal representation doesn’t show that Proslogion II has an argument for the existence of God. Nowhere is the name of God introduced into the argument. As we’ve seen, the constant $g$ is not a name of God, but rather an arbitrary name which they use to represent ‘id quo maius cogitari non potest’. Thus, the conclusion of their argument, $E!g$, doesn’t use a name of God. So once you grant them that the phrase ‘that than which nothing greater can be conceived’ is an arbitrary name and not a description, they have only established a fact about an arbitrarily chosen object of the kind nothing greater is conceivable, namely, that such an object exists. It isn’t clear how Proslogion II contains an ontological argument for the existence of God, on their representation.

We don’t think it would be a good response to suggest: since uniqueness isn’t discussed until Proslogion III, Anselm can’t conclude that God exists until that next chapter. Such a response won’t work for the following reason. In the opening of Proslogion II, Anselm directly uses the name ‘God’ (= ‘Deus’) and the vocative case for ‘Lord’ (= ‘Dominus’ = vocative case of ‘Dominus’). So Anselm clearly takes the conclusion of the argument to apply to God. And that is how Eder & Ramharter understand Proslogion II. They say (2015, p. 2800):

Having established in Chap. II that God exists in reality from the assumption that God exists at least in the understanding, Anselm proceeds in Chap. III by proving it is inconceivable that God does not exist.[10]

The second-order quantifiers appear in (*) and (**) and, third-order logic is used in the statement of Realization:

(*) $\forall F ((Fg \rightarrow Fa) \land (F = E! \rightarrow Fa))$

(**) $\forall F (Fa \rightarrow (Fg \lor F = E!))$

Realization: $\forall p. F \exists x \forall y. p. F (F) \leftrightarrow Fx$

We don’t see any textual justification or other reason for thinking that higher-order machinery is needed for Anselm’s argument in Proslogion II.

So Eder & Ramharter themselves agree that in Proslogion II, there is an argument that establishes something about God, and not just about some arbitrarily chosen object such that nothing greater can be conceived.

Indeed, we can’t accept the concluding clause of Eder & Ramharter’s claim (quoted earlier) that (p. 2802):

... if id quo had to be read as a definite description, Anselm would be committed to presupposing the uniqueness[19] of aliquid quo already in Chap. II, which seems to be in conflict with the fact that only in Chap. III does Anselm mention God’s uniqueness for the first time.[20]

Their evidence for the concluding clause, given in footnote 20, is to quote Anselm as saying, in Proslogion III, “In fact, everything else there is, except You alone, can be thought of as non existing. You, alone then, ...” (Anselm [MW, 2008, 88]). But this hardly counts as a statement that God is a unique thing such that nothing greater can be conceived. Here Anselm is saying only that God uniquely has existence in the highest degree, and this is a claim that plays no role in the ontological argument, as far as we can tell. This is why we don’t accept their conclusion that our account, which uses a definite description for the argument in Proslogion II, “seems to be in conflict with the fact that only in Chap. III does Anselm mention God’s uniqueness for the first time.” By using ‘God’ as a proper name in Proslogion II, he is already presupposing uniqueness, and that presupposition, together with Premise 1 and the meaning postulate for greater than, justifies his move from aliquid quo to id quo, as suggested by our representation of the argument.

We conclude that one can not so easily dismiss the suggestion that ‘id quo maius cogitari non potest’ is used as a definite description in Proslogion II. At present, Eder & Ramharter’s suggestion that id quo is being used as an arbitrary name leads to the list of problems just outlined. They would have to make a much stronger case before one should be willing to accept this analysis.

5 Conclusion

We conclude that the representation of Anselm’s argument in terms of a definite description still has a lot to offer those trying to understand Proslogion II. To our way of thinking, the interesting questions concern
the truth of the premises and the justification of the definition of God. Given what we’ve now learned, the premises in question are Premise 1, Premise 2’, the Meaning Postulate for greater than, and the definition of God (g) as \( \exists y \phi_y \).

In our paper of 2007, we argued that Premise 1 is the real culprit in the argument. We tried to show that Premise 1 is too strong because it yields the existence of an object that exemplifies the property of being a conceivable thing such that nothing greater is conceivable. We argued that Anselm’s subsidiary argument for Premise 1 involves two assumptions: (1) that the mere understanding of the phrase ‘conceivable thing such that nothing greater is conceivable’ requires one to grasp an intensional object, and (2) any such intensional object has to exemplify the property being a conceivable thing such that nothing greater is conceivable. We then challenged the second assumption on the grounds that the intentional [with-an-s] object (which is thereby grasped) encode the property being a conceivable thing such that nothing greater is conceivable. Here we appealed to the notion of encoding used in the theory of abstract objects (Zalta 1983, 1988).

Interestingly, this is a point of contact with the work of Eder & Ramharter’s paper, since their principle Realization (which we discussed in footnote 20 above) is a kind of comprehension principle that underlies Anselm’s assertion that there is something in the understanding such that nothing greater is conceivable. Eder & Ramharter write:

So, bearing in mind that first-order quantifiers are ranging over objects existing in the understanding, Realization seems plausible. It appears to be an analytic truth that any consistent set of (primitive, positive) conditions is realized by some object in the understanding. This seems to be confirmed by passages like (II.8), where Anselm claims that ‘whatever is understood is in the understanding’.[41] Bearing in mind that by ‘understanding something’ Anselm means understanding what its properties are, we can see that whenever we conceive of a certain set of (non-contradictory) properties, this set gives rise to an object that exists in the understanding—and this is just what Realization says. So even though Anselm does not state Realization explicitly, we think that it is implicit in how Anselm thinks about objects.

The authors here are placing a lot of weight on this third-order principle (Realization) and it isn’t clear to us that we should accept that Anselm is committed to this principle simply on the basis of the fact that he takes whatever is understood to be in the understanding. Nor is it clear to us that “by ‘understanding something’ Anselm means understanding what its properties are”. We don’t see any textual evidence for this claim. More importantly, they take Realization to be an integral premise of the ontological argument.

By contrast, our 2007 paper shows that whereas the comprehension principle for intensional objects might help us to see why Anselm thought Premise 1 is true, such a principle doesn’t need to be added as a premise in the ontological argument itself. It might be needed to justify Premise 1, but it doesn’t make an appearance in Proslogion II. To formulate the ontological argument, one shouldn’t need, as a premise, that for any primitive condition on properties, there is an object that exemplifies just the properties satisfying that condition. But this is what Realization intuitively asserts. Thus, the work in Oppenheimer & Zalta 2007 bears on this question, and we suggest that further study be focused on Premise 1 and the implicit comprehension principle that Anselm must be relying upon to conclude that it is true.

It remains only to discuss the meaning postulate for greater than and whether Anselm might have accepted that this relation is connected. Here, we can at least point to the fact that greater than is usually understood in natural language to be an ordering relation. But we’ve not required that there be such an ordering. So the only question concerns whether Anselm would have accepted connectedness, i.e., whether Anselm would have agreed that any two things can be compared in terms of greater than. We think he would have accepted this principle without question, as part of his Augustinian neo-Platonism. It is part of this world-view that there is the great chain of Being (Rogers 1997, Moran 2004), with God being a unique thing at the top of the chain. As we’ve seen, the only way for there to be a unique conceivable thing such that nothing greater can be conceived is if greater than is, at a minimum, connected.

Appendix: Anselm’s Proslogion II

1. Therefore, Lord, who grant understanding to faith, grant me that, in so far as you know it beneficial, I understand that you are as we believe and
you are that which we believe. (Ergo, Domine, qui das fidei intellectum, da mihi, ut, quantum scis expedire, intelligam quia es, sicut credimus; et hoc es, quod credimus.)

2. Now we believe that you are something than which nothing greater can be imagined. (Et quidem credimus te esse aliquid, quo nihil majus cogitari possit.)

3. Then is there no such nature, since the fool has said in his heart: God is not? (An ergo non est aliqua talis natura, quia dixit insipiens in corde suo: Non est Deus?)

4. But certainly this same fool, when he hears this very thing that I am saying — something than which none greater can be imagined — understands what he hears; and what he understands is in his understanding, even if he does not understand that it is. (Sed certe idem ipse insipiens, cum audit hoc ipsum quod dico, aliquid quo majus nihil cogitari potest; intelligit quod audit, et quod intelligit in intellectu ejus est; etiamsi non intelligat illud esse.)

5. For it is one thing for a thing to be in the understanding and another to understand that a thing is. (Aliud est enim rem esse in intellectu; aliud intelligere rem esse.)

6. For when a painter imagines beforehand what he is going to make, he has in his understanding what he has not yet made but he does not yet understand that it is. (Nam cum pictor preconcipit quæ facturus est, habet quidem in intellectu; sed nondum esse intelligit quod nondum fecit.)

7. But when he has already painted it, he both has in his understanding what he has already painted and understands that it is. (Cum vero jam pinxit, et habet in intellectu, et intelligit esse quod jam fecit.)

8. Therefore even the fool is bound to agree that there is at least in the understanding something than which nothing greater can be imagined, because when he hears this he understands it, and whatever is understood is in the understanding. (Convincitur ergo etiam insipiens esse vel in intellectu aliquid, quo nihil majus cogitari potest; quia hoc cum audit, intelligit; et quidquid intelligitur, in intellectu est.)

9. And certainly that than which a greater cannot be imagined cannot be in the understanding alone. (Et certe id, quo majus cogitari nequit, non potest esse in intellectu solo.)

10. For if it is at least in the understanding alone, it can be imagined to be in reality too, which is greater. (Si enim vel in solo intellectu est, potest cogitari esse et in re; quod majus est.)

11. Therefore, if that than which a greater cannot be imagined is in the understanding alone, that very thing than which a greater cannot be imagined is something than which a greater can be imagined. But certainly, this cannot be. (Si ergo id, quo majus cogitari non potest, est in solo intellectu, id ipsum, quo majus cogitari non potest, est quo majus cogitari potest: Sed certe hoc esse non potest.)

12. There exists, therefore, beyond doubt something than which a greater cannot be imagined, both in the understanding and in reality. (Existit ergo procul dubio aliquid quo majus cogitari non valet, et intellectu, et in re.)

References


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