On Anselm's Ontological Argument in *Proslogion* II*

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Abstract

Formulations of Anselm's ontological argument have been the subject of a number of recent studies. We examine these studies in light of Anselm's text and (a) respond to criticisms that have surfaced in reaction to our earlier representations of the argument, (b) identify and defend a more refined representation of Anselm's argument on the basis of new research, and (c) compare our representation of the argument, which analyzes *that than which none greater can be conceived* as a definite description, to a representation that analyzes it as an arbitrary name.

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1 Introduction

St. Anselm's *Proslogion* II is a locus classicus for an ontological argument for the existence of God. But there is still some question about how to best reconstruct the argument there. In 1991, we argued that Anselm needed 2 premises, a minimal condition on the *greater than* relation, and a definition of *God*, to give a valid argument for God's existence. In 2011, we showed how computational investigations established that one of the premises and the definition of *God* from the 1991 paper, were sufficient, just by themselves, to validly conclude God's existence. In the analysis section of our 2011 paper, we offered a number of observations about the differences between the original 1991 formulation and the simplified 2011 representation of the argument.

The goals in what follows are to: (1) respond to criticisms that have surfaced in reaction to our papers of 1991 and 2011, (2) identify a somewhat improved representation of Anselm's argument, (3) show that this new version is immune to the criticisms of the earlier version (even if one were to grant that the criticisms are valid), and (4) show that Anselm's term *that than which nothing greater can be conceived* is more accurately analyzed as a definite description than as an arbitrary name. Along the way, we point out how certain criticisms of our 2011 paper insufficiently attend to the analysis and observations we made in that paper.

2 Review

We begin by reviewing the relevant lines of *Proslogion* II. In what follows, all inline '(page, line)' numbers refer to the Latin source in Anselm, *Opera Omnia*, F. Schmitt (ed.), 1946. However, we use the English translation (Barnes 1972) in the text (with some modifications) and quote the Latin source in footnotes. Anselm (101, 4–5) takes God to be *something than which nothing greater can be conceived.*[1] Then he argues (101, 5–14) that even the fool agrees that there is in the understanding something than which nothing greater can be conceived.[2] So Anselm takes himself to have

[1]This starts at the end of line 4 and concludes on line 5 and reads "*Et quidem credimus te esse aliquid quo nihil maius cogitari possit.*"

[2]The argument (101, 5–14) starts with "*An ergo non est aliqua talis natura, quia 'dixit insipiens in corde suo: Non est Deus?'*" and continues on through to "*Convincitur ergo etiam insipiens esse vel in intellectu aliquid, quo nihil maius cogitari potest* [...] ."

established, at this point, the premise that there is in the understanding something than which nothing greater can be conceived.[3] Then he develops a *reductio* argument (101, 15 – 102, 3), from the *reductio* assumption that this object exists in the mind alone,[4] to an apparent conclusion which occurs at the end of chapter 2 (102, 2–3):

> **Apparent Conclusion**
> Something than which nothing greater can be conceived exists in reality.[5]

So, there is a puzzle: where is the explicit conclusion that everyone agrees is contained in *Proslogion* II, namely, that *God* exists? One might suggest that it follows from the Apparent Conclusion *given* the opening context in which Anselm is directing his remarks to God (101, 3–4).[6] But with a little effort, we can see why this doesn't really work.

Let C be the property *being conceivable*, let G be the *greater than* relation, and consider the formula that we used in our 1991 paper:

$$Cx \mathbin{\&} \neg\exists y(Gyx \mathbin{\&} Cy) \qquad (\phi_1)$$

This says that x is conceivable and such that nothing greater is conceivable. Now since the passage at 101, 4–5 asserts that "you are" (*te esse*) such that ϕ_1, then given that the antecedent of "you" (101, 4) is "Lord" ('*Domine*', 101, 3), the passage at 101, 4–5 is in fact asserting that *God* is such that ϕ_1. So where 'g' is a constant or name of God, we can represent the claim that God is such that ϕ_1 as:

$$Cg \mathbin{\&} \neg\exists y(Gyg \mathbin{\&} Cy) \qquad (1)$$

But even under this assumption, the claim that *God exists* doesn't follow from the Apparent Conclusion. The Apparent Conclusion is:

$$\exists x(\phi_1 \mathbin{\&} E!x) \qquad (2)$$

i.e., there is something (a) that is conceivable and such that nothing greater is conceivable, and (b) that exists. The conclusion that God exists

[3] I.e., "[...] *esse* [...] *in intellectu aliquid quo nihil maius cogitari potest* [...] ."

[4] I.e., lines 16–17 begin "*Si enim vel in solo intellectu est,* [...] ."

[5] Lines 2–3 on p. 102 are: "*Existit ergo procul dubio aliquid quo maius cogitari non valet, et intellectu, et in re.*"

[6] *Ergo, Domine, qui das fidei intellectum, da mihi, ut, quantum scis expedire, intelligam quia es, sicut credimus; et hoc es, quod credimus.*

($E!g$) doesn't validly follow from (1) and (2).[7]

So, is there a valid argument to the conclusion that God exists in *Proslogion* II? In our paper of 1991, we attempted to show that there is, and that all Anselm really needs to derive God's existence, in addition to what he says in *Proslogion* II is, (a) to regard the phrase "that than which nothing greater can be conceived" (*id quo maius cogitari non potest*) as a definite description, and (b) to assume, as a meaning postulate, that *greater than* is a connected relation. The following summary of our 1991 argument explains why.

In 1991, we started with a 1st-order logic extended with primitive definite descriptions of the form $\imath x\phi$. We assumed that the classical laws of quantification theory govern constants and variables, but that a free logic governs definite descriptions. We also assumed that quantified formulas are *not* existentially loaded: we read $\exists x\phi$ as "*there is* an x such that ϕ" (not as "there exists an x such that ϕ"). And we defined the uniqueness quantifier ($\exists!x\phi$) in the usual way. We also used an existence predicate: $E!$. So a formula of the form $\exists x(E!x \mathbin{\&} \phi)$ asserts "there is an x such that that x exists and is such that ϕ", i.e., *there exists* an x such that ϕ.

Then we used ϕ_1 (identified above) to represent the claim: x is conceivable and such that nothing greater than x is conceivable. We then proposed a formalization of the ontological argument in *Proslogion* II on which Anselm used two nonlogical premises and the previously mentioned meaning-postulate about the *greater than* relation:

> Premise 1: $\exists x\phi_1$
>
> Meaning Postulate: $\forall x\forall y(Gxy \vee Gyx \vee x = y)$
>
> Premise 2: $\neg E!\imath x\phi_1 \rightarrow \exists y(Gy\imath x\phi_1 \mathbin{\&} Cy)$

[7] In the absence of any assumptions about *greater than*, we can see that this inference is invalid by considering a model in which there are distinct, equally great objects, say a and b, such that both are conceivable and such that none greater are conceivable. And suppose, in this model, that a exists (i.e., makes the claim '$E!a$' true), b doesn't exist (i.e., makes the claim '$\neg E!b$' true), and the denotation of God (i.e., the denotation of 'g') is the object b. Then, in this model, (1) is true (since b is conceivable and such that none greater is conceivable and $g = b$, it follows that g is conceivable and such that none greater is conceivble); (2) is true (since something, namely, a, is conceivable and such that nothing greater is conceivable, and that exists); and *God* exists (i.e., $E!g$) is false (since $\neg E!b$ and $g = b$). So, *God*'s existence doesn't yet follow and we're still at a loss as to how to see *Proslogion* II as containing an ontological argument for the existence of God.

Premise 1 captures the preliminary conclusion at line 14 of *Proslogion* II; it asserts merely that *there is* a conceivable object such that nothing greater is conceivable. The Meaning Postulate for *greater than* asserts merely that *greater than* is a connected relation, i.e., for any two distinct objects, either the first is greater than the second or the second is greater than the first. From the Meaning Postulate, we derived a lemma (Lemma 2): $\exists x\phi_1 \rightarrow \exists! x\phi_1$. In other words, the mere connectedness of *greater than* allows us to infer, from Premise 1, that there is a *unique* conceivable thing such that nothing greater is conceivable. By the classical principles governing definite descriptions, it follows that the description $\imath x\phi_1$ has a denotation, i.e., that $\exists y(y = \imath x\phi_1)$. This fact justifies Anselm's use of the definite description *that than which nothing greater can be conceived* (*id quo maius cogitari non potest*) in his reasoning in *Proslogion* II. Moreover, from this conclusion and Premise 2, the ontological argument easily proceeds to the conclusion that God exists, if given the definition that God ('g') is, by definition, $\imath x\phi_1$. In other words, with the connectedness of *greater than*, Anselm can *strengthen* line 2 of *Proslogion* II; instead of the claim that God *is* an object such that ϕ_1, line 2 can be strengthened to the identity statement that God *is* (identical to) the conceivable thing such that nothing greater can be conceived and so be treated as a definition.[8]

The actual reasoning in the argument cites logical theorems governing definite descriptions, which are explained in the 1991 paper.[9] But, the conclusion of the argument is that *God* exists ($E!g$) and so this representation explains why it appears that *Proslogion* II contains an ontological argument for the existence of God.

[8]Of course, if line 2 is understood as a definition, then Anselm has introduced this definition before establishing that the description $\imath x\phi_1$ is well-defined. But that is perfectly benign as long as he doesn't make use of the description until after he's established that there is a unique thing satisfying the matrix of the description.

[9]In tabular form:

1.	$\exists x\phi_1$	Premise 1
2.	$\exists! x\phi_1$	from (1), by Lemma 2 (1991)
3.	$\exists y(y = \imath x\phi_1)$	from (2), by Description Thm 1 (1991)
4.	$C\imath x\phi_1 \,\&\, \neg\exists y(Gy\imath x\phi_1 \,\&\, Cy)$	from (3), by Description Thm 2 (1991)
5.	$\neg E!\imath x\phi_1$	Assumption, for *Reductio*
6.	$\exists y(Gy\imath x\phi_1 \,\&\, Cy)$	from (5), by Premise 2
7.	$\neg\exists y(Gy\imath x\phi_1 \,\&\, Cy)$	from (4), by &E
8.	$E!\imath x\phi_1$	from (5), (6), and (7), by *Reductio*
9.	$E!g$	from (8), by the definition of 'g'

We subsequently discovered (2011), by computational means, that the conclusion that God exists can be derived solely from Premise 2 and the definition of God. Again, the proof cites logical theorems governing descriptions, which are explained in the 2011 paper.[10] From this brief review, it is clear why the title and focus of our 2011 paper had to be on (a) how Premise 1 and the Meaning Postulate for *greater than* are redundant given the strength of Premise 2, and (b) the discovery of this redundancy by computational means. But our paper also contained a number of observations and some discussion of the soundness of the argument; this analysis, and in particular, our doubts about Premise 2 and our view about how one might tweak the 1991 argument to avoid the redundancies, seem to have been overlooked by the critics. In Section 4 of our 2011 paper, we compared the original 1991 version with the simplified 2011 version and in Section 5, we concluded our paper with reasons for objecting to Premise 2.

We show, in what follows, that these observations and reasons already anticipate and address some of the criticisms that have been raised about our work.[11] This is not to say that no valid criticisms have been raised. We agree with the section of Garbacz's 2012 paper (Section 3), where he raises some genuine methodological issues about our translation of the 1991 argument into Prover9 notation. He noted, for example, that in the context of Prover9, our use of the constant 'g' implicitly validated Premise 1 simply by *representing* the definition that g is identical to the conceivable x such that none greater can be conceived.

We are happy to acknowledge these points from Section 3 of Garbacz's paper. In 2009–2010, while doing research for our 2011 paper, we were in the early stages of learning how to use resolution-based automated rea-

[10]In tabular form:

1.	$\neg E!\imath x\phi_1$	Assumption, for *Reductio*
2.	$\exists y(Gy\imath x\phi_1 \,\&\, Cy)$	from (1), by Premise 2 and MP
3.	$Gh\imath x\phi_1 \,\&\, Ch$	from (2), by $\exists$E, 'h' arbitrary
4.	$Gh\imath x\phi_1$	from (3), by &E
5.	$\exists y(y = \imath x\phi_1)$	from (4), by Desc. Thm. 3 (2011)
6.	$C\imath x\phi_1 \,\&\, \neg\exists y(Gy\imath x\phi_1 \,\&\, Cy)$	from (5), by Desc. Thm. 2 (1991)
7.	$\neg\exists y(Gy\imath x\phi_1 \,\&\, Cy)$	from (6), by &E
8.	$E!\imath x\phi_1$	from (1), (2), (7), by *Reductio*
9.	$E!g$	from (8), by the definition of 'g'

[11]A notable exception is Parent (2015), who recognized that we were suspicious and, indeed, critical of the strength of Premise 2 in our 2011 paper.

soning systems to represent statement schemata for a first-order, natural deduction logic extended with definite descriptions. There are a number of subtleties that we had not recognized and so our methodology was not as refined as it might have been. We've therefore revised our webpage on the computational investigation of the ontological argument to reflect our enhanced understanding of how to implement the 1991 premises in a first-order reasoning environment. The new representation no longer defines 'g' by deploying a creative definition. We've acknowledged Garbacz's concerns on that webpage.[12]

Nevertheless, despite these flaws, we deployed the automated reasoning engine with enough sophistication to discover that Premise 2 makes Premise 1 and the Meaning Postulate redundant.[13] This hadn't been noticed in the 20 years subsequent to the publication of our 1991 paper. As we shall see, however, other criticisms raised in Garbacz's paper are not as effective and we shall address these in Section 4 below. We first address the charge of begging the question, however, which was raised by Rushby.

3 Does the Argument Beg the Question?

Rushby (2018, 1475–1478, 1484) claims that the version of the argument in our 1991 paper *begs the question*. He introduces his claim by saying (1475):

> In the next section, I introduce a strict definition of "begging the question" and show that a rendition of the Argument due to Oppenheimer and Zalta is vulnerable to this charge.

His subsequent argument has two parts: first there is a definition of *begging the question* and then there is the claim that an analogue of Premise 2 begs the question according to that definition. We challenge both the definition and the claim that Premise 2 begs the question.

Here is Rushby's definition of *begging the question*, where he uses the equality sign = as a biconditional sign (2018, 1476):

[12]See http://mally.stanford.edu/cm/ontological-argument/.

[13]It is worth mentioning work by Blumson (2017), who implemented our 1991 and 2011 representations of the argument in the higher-order reasoning system Isabelle/HOL.

> [...] if C is our conclusion, Q our "questionable" premise (which may be a conjunction of simpler premises) and P our other premises, then Q begs the question if C is equivalent to Q, assuming P: i.e., $P \vdash C = Q$.

Note here that Rushby has defined what it is for a *premise* to beg the question. Of course, it should be observed that the fallacy of begging the question traditionally applies to *arguments*, not to *premises*, yet for some reason, Rushby takes it to apply to premises.[14] Let's put this aside for the moment, since we'll later argue that *begging the question* is a charge that can be leveled only against an argument relative to a dialogical context. Let's focus for now solely on Rushby's definition of *begging the question.*

We think it is easy to undermine his definition by showing that it categorizes premises and arguments that clearly don't beg the question as ones that do. We do this by considering the following two arguments, neither of which is question-begging.

$$\begin{array}{ll} A \equiv B & A \equiv B \\ B \equiv C & B \equiv C \\ \therefore\ C \equiv A & A \\ & \therefore\ C \end{array}$$

In the first argument, a biconditional conclusion follows from two biconditional premises. The argument is non-question begging—both premises play a role in the derivation of the conclusion. In the second argument, we derive a conclusion C by adding the premise A to the first argument.

But according to Rushby's definition, the premise A in the second argument is question-begging. To see this, let us assign elements of the second argument as values to the variables in Rushby's definition of *question-begging*: let C be the conclusion, let Q be the premise A, and let P (the other premises) stand for the first two premises. Then plugging in these values, Rushby's definition makes the following claim about the second argument:

[14]Parent (2015, 477) suggests that *we* would accept that Premise 2 is question-begging, for he says "Nevertheless, O&Z go on to give an independent case that (P) [Premise 2] is either false or question-begging, and as things currently stand, their verdict strikes me as correct". But it is important to correct this misattribution, since otherwise it counts against what we say below. Though we did develop (2011, Section 5) reasons against adopting Premise 2 we didn't claim there that Premise 2 is question-begging or that the argument with Premise 2 is question-begging.

> If C is our conclusion, A our "questionable" premise (which may be a conjunction of simpler premises) and P our other premises ($A \equiv B$ and $B \equiv C$), then A begs the question if C is equivalent to A, assuming P, i.e., if $P \vdash C \equiv A$, i.e., if $A \equiv B$, $B \equiv C \vdash C \equiv A$.

Thus, Rushby's definition of question-begging leads to the incorrect conclusion that the premise A in the second argument begs the question, since the first argument does indeed show that from the premises $A \equiv B$ and $B \equiv C$, we can derive the conclusion $C \equiv A$.

Of course, it may be that our 1991 representation of the ontological argument *is* question-begging, but one can't establish this from reasoning that makes use of Rushby's incorrect definition. A better definition of *question-begging* is one that applies to arguments as a whole. Indeed, it applies to arguments *relative to* a dialogical situation.[15] We don't have an uncontroversial definition of question-begging to offer in this paper. But we can say, to a first approximation, that our versions of the ontological argument do *not* assume what they are trying to prove, and do *not* help themselves to information to which they are not entitled (in the dialogical context). So if this is what is meant by begging the question, our version of the argument doesn't do so.

Moreover, to a second approximation, we might say that an argument A begs the question if and only if (1) A is valid, and (2) in the dialogical situation in which A is presented, the arguer is not entitled to one of the premises (even if only for the sake of the argument). This, at least, explains why the valid argument $P \therefore P$ is question-begging: in any dialogical situation in which the arguer is presenting an argument for P, they are not entitled to the premise P *even if only for the sake of the argument*. For if one were entitled to P (even if only for the sake of the argument), one wouldn't need an argument, since the premise is sufficient and no further argument is called for.[16] Using this rough definition, then, we see no reason to think that the 1991 version is question-begging.

[15]See the literature on the dialectical/dialogical approach to fallacies, for instance Hamblin (1970) and Walton (1994).

[16]Note that this is one of the reasons why the ontological argument from Premise 2 alone is unsatisfactory. As we noted at the end of our 2011 paper, one cannot argue for Premise 2 by asserting: assume the description $\imath x\phi_1$ denotes and that what it denotes exists; then the antecedent of Premise 2 is false, since it asserts $\neg E!\imath x\phi_1$, in which case Premise 2 is true. Such an argument for Premise 2 helps itself to the claim that $E!\imath x\phi_1$, something which a theist is not entitled to when arguing for the existence of God.

We emphasize here that we are not defending the *soundness* of the ontological argument. But we're happy to suppose, for the purposes of this paper, that the argument is being presented in a dialogical situation where the interlocutor is an atheist or agnostic.[17] We still think that one is entitled to use the premises of all the versions of the argument we've put forward in such situations, if only for the sake of the argument. This doesn't require that the premises be true, but only that using all of the premises doesn't defeat the arguer's purpose in the situation of arguing for a conclusion. Of course, one can continue the debate after the argument is presented by considering whether the premises are true or false. And that is where we think the focus should be with respect to the ontological argument as we've presented it, not on whether the argument begs the question. Indeed, in the next section, we'll see that one can regard the definition of God ($g =_{df} \imath x\phi_1$) as an additional premise needed for the argument. The addition of another needed premise makes it even harder to conclude that the argument begs the question. We'll reconsider this at the end of the next section, after we present a more refined version of Premise 2.

4 Other Criticisms of Premise 2

Since Premise 2 makes Premise 1 and the Meaning Postulate for *greater than* otiose, the revised version of the argument presented in 2011 has a single premise (Premise 2) and a single conclusion ($E!g$). The 2011 version was criticized by Garbacz (2012), who claimed, in Section 2 of his paper, that Premise 2 is equivalent to the conclusion that God exists and so using it in this argument is "epistemically inefficient" (2012, 588). His criticism was developed on the basis of a certain understanding of our paper that he puts forward, namely, "The main contribution of Oppenheimer and Zalta 2011 is a discovery that [...] *Premise 1* and *Connectedness* are obsolete, i.e., that you can reach the same conclusion assuming only Premise 2" (Garbacz 2012, 586). But in developing his criticisms, Garbacz missed certain crucial facts about the 2011 argument; we

[17]The Fool mentioned in *Proslogion* II could be seen as an atheist interlocutor. Gaunilo, who is not mentioned in *Proslogion* II, could also be seen as representing an atheist or agnostic point of view. But Gaunilo's reply to Anselm was written after the *Proslogion* was circulated, and so can't be considered, strictly speaking, as part of the dialogical context internal to the *Proslogion*.

included some observations about Premise 2 in that paper that addressed his concerns.

Let's first look at exactly what Garbacz says (2012, 586–587). He notes that Premise 2 is equivalent to the claim that $E!\imath x\phi_1$. That is, Garbacz notes that:

$$E!\imath x\phi_1 \equiv (\neg E!\imath x\phi_1 \rightarrow \exists y(Gy\imath x\phi_1 \,\&\, Cy)) \tag{A}$$

Clearly, Garbacz is correct about this. The right-to-left direction of the above biconditional was established by our 2011 version of the argument. And the left-to-right direction follows by propositional logic: from p it follows that $\neg p \rightarrow q$.

However, Garbacz uses this fact as part of his argument to the conclusion that "Premise 2 cannot be used in any proper argument for the existence of God" (2012, 586). His first reason is that "It [Premise 2] not only implies that God exists but it is (logically) equivalent to the latter claim", where the 'latter claim' refers to 'God exists'. But the equivalence (A) above doesn't allow one to infer that Premise 2 is logically equivalent to the claim that God exists. For Garbacz's claim to be true, Premise 2 would have to be logically equivalent to the claim $E!g$. It is not; that is, the claim $E!g$ is not equivalent to the claim on the left side of equivalence (A): it is not equivalent to 'the x such that ϕ_1 exists'. Rather, for $E!g$ and $E!\imath x\phi_1$ to be equivalent, you need the *definition* $g =_{df} \imath x\phi_1$. And this is a non-trivial part of the argument. One can't introduce this definition into the argument until it is justified. And the justification can be given by noting that the intermediate conclusion $E!\imath x\phi_1$ implies that the definite description is well-defined. Specifically, if you examine the 2011 argument reproduced in n. 10 above, you will see that the intermediate conclusion $E!\imath x\phi_1$ occurs on line 8; this implies, by Russell's theory of descriptions, that the definite description is well-defined (i.e., has a unique denotation). So to reach the conclusion that God exists from Premise 2, you need the non-trivial step of justifying and introducing the definition of 'God' as the x such that ϕ_1.

We think this shows that Premise 2 is not (logically) equivalent to the claim that God exists. Thus far, though, we've only shown that Premise 2 doesn't imply that God exists. But the converse implication also fails: the claim that God exists doesn't imply Premise 2. Again, to draw the inference from the claim that God exists to Premise 2, one must first convert $E!g$ to $E!\imath x\phi_1$ by the definition of God. Then one could correctly

claim, as Garbacz does, that the latter is equivalent to Premise 2. But one can't thereby conclude that *God exists* ($E!g$) implies Premise 2 without appealing to, and justifying, the definition of God. And that can't be done in this direction; there is no justification for stipulating that $g =_{df} \imath x\phi_1$ simply on the basis of $E!g$.

This observation undermines Garbacz's first reason for suggesting that our 2011 version is 'epistemically inefficient'. For if the question is 'Does God exist?' and the formal representation of that claim is $E!g$, then Premise 2, despite its power, does not imply $E!g$ *without* the definition of God.

But Garbacz gives another other reason why Premise 2 shouldn't be used in the ontological argument (587–588), namely, that the consequent of Premise 2 is logically inconsistent and so the claim $E!\imath x\phi_1$ becomes equivalent to a tautology. But, clearly, the logical inconsistency of the consequent of Premise 2 is not obvious and it takes both some reasoning (indeed a diagonal argument) *and* the logic of definite descriptions to show that one can derive a contradiction from the assumption that $\exists y(Gy\imath x\phi_1 \,\&\, Cy)$. This is precisely how Anselm's argument gets its purchase—he coupled a consequent that subtly implies a contradiction to the antecedent $\neg E!\imath x\phi_1$.

Moreover, this reason that Garbacz offers against Premise 2 doesn't go beyond the reservations we had already expressed about this premise in 2011. The fourth observation in Section 4 (2011, 346) included the lines:

> [...] it is interesting to note that one can (i) abandon the definition of God as $\imath x\phi_1$, (ii) generalize Premise 2 to the claim that $\neg E!x \rightarrow \exists y(Gyx \,\&\, Cy)$, and still (iii) develop a valid argument to the conclusion that anything that satisfies ϕ_1 exemplifies existence.

And in Section 5 (2011, 348), we gave an extended argument against Premise 2. The last paragraph of our paper (2011, 348–349) included the following lines:

> Thus, arguments ... above show that the defender of the ontological argument needs independent support for two claims: that the definite description denotes and that Premise 2 is true. Our 1991 analysis of the argument is still relevant, since it shows how the ontological arguer could justify Anselm's use of the definite description. [n. 14: Given the argument outlined above against Premise 2,

> a defender of Anselm might consider whether the ontological argument can be strengthened by using our original formulation as in 1991, but with the *general* form of Premise 2 discussed earlier: $\neg E!x \rightarrow \exists y(Gyx \,\&\, Cy)$. The justification of this more general premise may not be subject to the same circularity that infects the justification of Premise 2 (though, of course, it may have problems of its own).] The present analysis shows why the use of the definite description needs independent justification. Consequently, though the simplified ontological argument is valid, Premise 2 is questionable and to the extent that it lacks independent justification, the simplified argument fails to demonstrate that God exists. The use of computational techniques in systematic metaphysics has illuminated the relationship between Premise 2 of the ontological argument and the conclusion that God exists.

It is clear from these passages not only that we gave reasons why Premise 1 and the connectedness of *greater than* are *not* obsolete, but also that we questioned Premise 2. Thus, we anticipated the conclusion of the extended argument that Garbacz develops in Section 2 of his paper, where he concludes that (589):

> [...] Thus, it is little wonder that one can dispense with Premise 1 and Connectedness in Oppenheimer and Zalta's [1991] ontological argument as their logical contribution (to this argument) is covered by Premise 2. Alas, this third premise seems to be too strong to be an acceptable basis for any argument for the existence of God.

Garbacz here makes it seem that we were advocating for Premise 2 in our paper. While the specific criticisms he puts forward do offer some additional reasons for not accepting Premise 2 (though see below), we did not argue in our paper that, in a proper reconstruction of the argument, Premise 2 should *replace* Premise 1 and the Meaning Postulate.

Indeed, in the fourth observation and the final paragraph (n. 14) quoted above from our 2011 paper, we suggested that the 1991 paper should have weakened Premise 2 so that it becomes the following universal claim:

Premise 2′: $\forall x(\neg E!x \rightarrow \exists y(Gyx \,\&\, Cy))$

This asserts, in essence, that *if a thing fails to exist then something greater than it can be conceived.* We pointed out that Premise 1, the Meaning

Postulate, and Premise 2′ yield a perfectly good argument for God's existence without appealing to Premise 2. The resulting argument is a variant of the argument in the 1991 paper—once one establishes that the description $\imath x\phi_1$ denotes, one can instantiate it into Premise 2′ to obtain the original Premise 2. And then the *reductio* proceeds in the same manner as in the 1991 paper, i.e., the revised version of the argument goes exactly like the original 1991 argument except at the point where Premise 2 is invoked.

We think it is easy to see that Premise 2′ alone does not yield the intermediate conclusion that $E!\imath x\phi_1$, nor does any pairwise combination of Premise 1, Premise 2′ and the Meaning Postulate for *greater than.* The reader might wish to use automated reasoning tools to establish these last claims, as we shall not argue for them further here.[18]

However, to forestall an objection to Premise 2′, we can weaken it even further, so that it asserts: for any x, if x is a conceivable thing such that nothing greater is conceivable and x fails to exist, then something greater than x can be conceived. Formally:

Premise 2″: $\forall x((\phi_1 \,\&\, \neg E!x) \rightarrow \exists y(Gyx \,\&\, Cy))$

This would forestall the objection that Premise 2′ is consistent with, and even licenses, the view that an existing conceivable evil thing is greater than a nonexistent one. A variant of the 1991 ontological argument still goes through with Premise 2″:

1.	$\exists x\phi_1$	Premise 1
2.	$\exists! x\phi_1$	from (1), by Lemma 2 (1991)
3.	$\exists y(y=\imath x\phi_1)$	from (2), by Desc. Thm. 1 (1991)
4.	$(C\imath x\phi_1 \,\&\, \neg\exists y(Gy\imath x\phi_1 \,\&\, Cy) \,\&\, \neg E!\imath x\phi_1) \rightarrow \exists y(Gy\imath x\phi_1 \,\&\, Cy)$	from (3) and Premise 2″, by ∀E
5.	$C\imath x\phi_1 \,\&\, \neg\exists y(Gy\imath x\phi_1 \,\&\, Cy)$	from (3), by Desc. Thm. 2 (1991)
6.	$\neg E!\imath x\phi_1$	Assumption, for *Reductio*
7.	$C\imath x\phi_1 \,\&\, \neg\exists y(Gy\imath x\phi_1 \,\&\, Cy) \,\&\, \neg E!\imath x\phi_1$	from (5), (6), by &I
8.	$\exists y(Gy\imath x\phi_1 \,\&\, Cy)$	from (4), (7), by MP
9.	$\neg\exists y(Gy\imath x\phi_1 \,\&\, Cy)$	from (5), by &E
10.	$E!\imath x\phi_1$	from (6), (7), and (8), by *Reductio*
11.	$E!g$	from (9), by the definition of g

[18]See Parent 2015 (478) for another way of weakening Premise 2.

This represents our best analysis of the argument; had we seen the issue with Premise 2 in 1991, we would have eliminated Premise 2 in favor of Premise 2″. Note that neither Premise 2′ nor Premise 2″ has a consequent that is a contradiction—the consequent of the conditionals embedded in the universal claims doesn't invoke $\imath x\phi_1$, and as such, a contradiction can't be derived. This undermines an objection Garbacz raised against Premise 2, which we discussed earlier. Note here how Premise 1 and the Meaning Postulate for *greater than* still play a role. The latter is needed for the derivation of Lemma 2, which moves us from the former to the claim that there is a unique x such that ϕ_1. These are key to the justification of the use of the description $\imath x\phi_1$ which, in turn, is needed to instantiate Premise 2″ to obtain line 5. In free logic, one may not instantiate a term into a universal claim until one has established that the term has a denotation.

Consequently, by substituting Premise 2″ for Premise 2 in the 1991 argument, we obtain a *valid* argument in which all of the premises are needed to derive the intermediate conclusion $E!\imath x\phi_1$.[19] None of the premises are redundant and the argument doesn't beg the question. We leave the argument that the premises aren't redundant to a footnote, but close this section with a brief discussion of why the new version of the argument is not question-begging.[20]

As mentioned earlier, universal agreement about the definition of *begging the question* has not been reached in the recent literature (Iacona & Marconi (2005); Hazlett (2006); Betz (2010); Sgaravatti (2013); and Šuster (2020), for example). In our view, none of these papers has a completely clear and unassailable analysis of the fallacy; some come dangerously close to requiring that the premises of an argument have to be true for the argument to avoid begging the question. That said, however, we simply note the following:[21]

- Premises 1, 2″, the meaning postulate for *greater than* and the definition of God do *not* depend for their truth on the existence of God, nor do they offer support for the conclusion that God exists that can be discredited solely on the grounds that God doesn't exist (cf. Šuster 2020).

- One could have a justified belief in Premises 1, 2″, the meaning postulate for *greater than* and the definition of God *without* having a justified belief in the claim that God exists (cf. Sgaravatti 2013).

- The argument presented above is not a *petitio principii* either in the narrow sense or the broad sense of Betz 2010; an evaluation of the dialectical structure and dialectical patterns of the argument, when considered as a distinctive case, yields no reason to think that the reasoning is redundant or circular in any of the senses Betz suggests.

- Our version of the argument doesn't beg the question in the sense of Hazlett (2006, 357). It doesn't violate the 'Submaxim of Relation': Anselm could reasonably believe that his premises (as formulated above) are accepted (as reasonable background assumptions) in his community.

Our position, therefore, is that no definition of *begging the question* has yet established that the version of the argument presented above does so.

5 Does Anselm Use a Definite Description?

In a recent paper, Eder & Ramharter (2015) also recognize that question-begging is not an accurate charge to bring against any of the versions of

[19] Note how our reconstruction using Premise 2″ differs from Garbacz's proposed weakening of Premise 2 (2012, 591). His proposed weakening asserts that, for any x, if it is not the case that both ϕ_1 and $E!x$, then there is something y such that (a) y is greater than x, (b) ϕ_1, and (c) x is conceivable. Formally, $\forall x(\neg(\phi_1 \;\&\; E!x) \rightarrow \exists y(Gyx \;\&\; \phi_1 \;\&\; Cy))$. We think Premise 2″ is a more elegant weakening of Premise 2; ϕ_1 is not required in both the antecedent and the consequent to derive the desired conclusion. And the antecedent of Premise 2″ considers only the case of a ϕ_1 object that doesn't exist (the antecedent of Garbacz's proposed weakening considers the cases where either x isn't such that none greater can be conceived or x doesn't exist). Modulo the definition of God (see below), Premise 2″ is sufficient to yield the existence of God given Premise 1 and the connectedness of *greater than*.

[20] To see this with respect to the argument that includes Premise 2″, notice two things: (1) line 5 depends on Premise 1 and the Meaning Postulate for *greater than* even though those weren't cited as the justification. But these two principles are needed to conclude that the description has a denotation and can thereby be instantiated, via the free logic version of ∀E, into Premise 2″. (2) When one assumes, for reductio, at line 6, that $\neg E!\imath x\phi_1$, one cannot then reach line 7 from Premise 2″ alone; one must additionally have Premise 1 and the Meaning Postulate for *greater than* in order to establish the second and third conjuncts of line 7, which come from line 4.

[21] We omit, in what follows, a discussion of Iacona & Marconi (2005) because it isn't clear whether they offer a preferred analysis of *begging the question* (2005, 30–31).

the ontological argument of the kind that we've been discussing.[22] But they suggest that the regimentation of the argument using a definite description doesn't properly capture the reasoning in *Prolosgion* II. While our versions of the argument represent Anselm's term *that than which nothing greater can be conceived* as a definite description, Eder & Ramharter argue that it is not. They introduce two abbreviations and put them in boldface: **id quo** abbreviates "*id quo maius cogitari non potest*" ("that than which nothing greater can be conceived"), and **aliquid quo** abbreviates "*aliquid quo nihil maius cogitari potest*" ("something than which nothing greater can be conceived"). Then they say (2802):[23]

> Whether or not a reconstruction of Anselm's argument is valid may crucially depend on whether **id quo** has to be understood as a definite description. But we think that it is not just that we do not *have to* understand **id quo** as a definite description, but that we *should* not. For one thing, if **id quo** had to be read as a definite description, Anselm would be committed to presupposing the *uniqueness* of **aliquid quo** already in Chap. II, which seems to be in conflict with the fact that only in Chap. III does Anselm mention God's uniqueness for the first time. Rather, it seems to us that Anselm is using this diction only as a device to refer back to *something* 'than which nothing greater can be conceived'. In other words, we think that Anselm's **id quo** is best understood as an *auxiliary name*, which is used to prove something from an existence assumption.

Eder and Ramharter are suggesting here that "*id quo maius cogitari non potest*" is not being used as a definite description in *Proslogion* II but rather functions more like an *arbitrary name*.[24] Later, in their paper, they further abbreviate **id quo** as the *arbitrary constant* 'g', but it is

[22]They say (2015, 2797, n. 4): "Sometimes, proofs of the existence of God are accused of being question-begging, but this critique is untenable. It is odd to ask for a deductive argument whose conclusion is not contained in the premisses. Logic cannot pull a rabbit out of the hat."

[23]In the following quote and the remainder of the paper, we have preserved the boldface that Eder & Ramharter use to format the Latin expressions 'id quo' and 'aliquid quo'. This also applies to their use of boldface to format the label for the principles abbreviated as 'ExUnd', 'Def C-God', 'God!', and 'Realization', which are discussed below.

[24]Their position has recently been endorsed in Campbell (2018, 55).

important to remember that this is an abbreviation of a phrase with descriptive content. On the face of it, there is a huge difference between a simple constant and an expression like *id quo maius cogitari non potest*. According to standard logical practice, an *arbitrary name* is a simple constant that has *no* descriptive content, whereas the phrase "*id quo maius cogitari non potest*" is a complex expression that has descriptive content. The descriptive content is expressed by the arrangement of the operators (negation, quantification, and conjunction) and Latin predicates (for *greater than* and *conceives*). So it is not clear how to reconcile Eder & Ramharter's hypothesis (that we can introduce 'g' as an abbreviation for a phrase with descriptive content) with current logical practice.[25]

But more importantly, the suggestion that **id quo** is being used by Anselm as an arbitrary name can't be sustained given their representation of the argument. To develop our objection to this suggestion, note that when a number theorist introduces an arbitary name for a prime number by saying 'let τ be an arbitrary prime', they may go on to reason about τ using the known principles of number theory, including known principles about primes. But what they cannot do is, at some point in the argument, assert or appeal to new principles (i.e., new axioms, premises, etc.) governing τ.

This understanding is made clear by the classical rule for using arbitrary names within an argument to derive a conclusion from some premises. The classical rule requires that one choose an arbitrary name that doesn't already occur in the premises or in the conclusion. In a standard logic text, e.g., Enderton (2001, 124), Rule EI is stated as follows, where Γ is a set of premises, $\exists x\phi$ is an existentially quantified premise, ψ is the conclusion, and ϕ^x_c is the result of substituting c for the free occurrences of x in ϕ:[26]

[25]One might, with Rushby (2013, m.s.), suggest that **id quo** is a Hilbert-style ϵ-term, that is, a term of the form $\epsilon x\phi_1$, which might be read: an x such that ϕ_1. But the semantic interpretation of such terms requires a *choice function*. We leave it as an open question as to whether Anselm used language that requires a choice function for its interpretation. Though we recognize that Anselm lived near the beginning of a period that saw a number of interesting developments in logic, the project that we, and Eder & Ramharter, are engaged in is not to systematize 11^{th} century logic, but rather understand 11^{th} century reasoning in terms that are today recognized as valid.

[26]Note that Enderton uses 'EI' to stand for Existential Instantiation (2001, 125), because the reasoning goes from $\exists x\phi$ to an instantiation with an arbitrary name c. But this rule should not be confused with the rule labeled 'EI' in natural deduction or Gentzen-style systems. This label stands for Existential Introduction, and the rule

Rule EI
Assume that the constant symbol c does not occur in ϕ, ψ or Γ, and that $\Gamma, \phi^x_c \vdash \psi$. Then $\Gamma, \exists x\phi \vdash \psi$, and there is a deduction of ψ from Γ and $\exists x\phi$ in which c does not occur.

Thus, the rule says: if you can derive ψ from some premises Γ in which c doesn't occur and from the fact that c is such that ϕ, then you can derive ψ from Γ and the claim $\exists x\phi$. By requiring that the constant c be new to the proof, one guarantees that the reasoning is about an arbitrary entity such that ϕ, i.e., that no other information about that entity is available in the premises or conclusion. This implies that if one assumes ϕ^x_c as an instance of $\exists x\phi$, one can't then introduce new premises or assumptions that govern c.

But Eder & Ramharter's analysis of the argument seems to violate Rule EI in a number of ways. First, their conclusion contains the arbitrary name. In the formal proof in their paper (2813), they introduce constant 'g' as the arbitrary constant (instead of the more cumbersome **id quo**). From the principles **ExUnd** ($\exists xGx$) and **Def C-God** ($Gx :\leftrightarrow \neg\exists y(y > x)$) they conclude $\exists y\neg\exists x(x > y)$ and then say "let g be such that $\neg\exists x(x > g)$." Then they assume $\neg E!g$ for reductio, reach a contradiction, and so conclude $E!g$. But this conclusion of the argument contains the arbitrary name. The reasoning doesn't really conform to Rule EI; the conclusion of the argument doesn't seem to have the right form.

But suppose they reformulate the argument to address this issue. The second problem is that it appears that they have used premises in their argument *in which the arbitrary name occurs*, in violation of the condition that the arbitrary constant can't occur in any premises used in the derivation. Let's try to match up their argument with the requirements of Rule EI. At the point where they say "let g be such that $\neg\exists x(x > g)$", the rule doesn't allow them to reason from any other premise containing the arbitrary name g. But their argument does precisely that. After introducing g, they then define (2813) a new notion ($\mathcal{F}_{E!}(F)$) by means of a biconditional in which g occurs, and then use that definition to derive facts about g. The definition in question is:

$$\mathcal{F}_{E!}(F) :\leftrightarrow Fg \vee F = E!$$

allows one to infer $\exists x\phi$ from ϕ^x_c, where c is a constant in classical logic. In terms of introduction and elimination rules, Enderton's Rule EI corresponds to Existential Elimination!

Note first that the definiendum appears to depend on the choice of g, but this isn't acknowledged by indexing the definiendum to g. Note second that a definition is equivalent to a biconditional axiom, and so an appeal to this definition in their proof becomes an appeal to a premise in which the arbitrary name occurs, something that is expressly prohibited by Rule EI. Because the definition uses the arbitrary name in the definiens, there is a real question here about the propriety of such a definition.

Of course, it may be that the authors can discharge the definition so that no premise involving the arbitrary name is used in violation of Rule EI. For example, they might be able to (a) use a definition in which a variable replaces the arbitrary name, (b) prove general, universally quantified, theorems about the notion defined, and (c) appeal to those theorems when reasoning with the arbitrary name in the ontological argument. For example, instead of the above, they could offer the following definition, indexing the definiendum to x and $E!$:

$$\mathcal{F}_{x,E!}(F) :\leftrightarrow Fx \vee F = E!$$

With this definition, they might be able to prove theorems about the defined notion $\mathcal{F}_{x,E!}(F)$ that hold for arbitrary x. Then, when reasoning in the ontological argument with respect to the arbitrarily chosen object g, they could instantiate these theorems to g without the theorems counting as premises that would violate Rule EI.[27] But the classical literature on the logic of the existential quantifier and the theory of definition suggests that, in absence of such changes to the reasoning, the argument Eder & Ramharter presented in their paper is in violation of the conditions for the use of Rule EI.[28]

We conclude that the argument in their paper has to be reformulated much more carefully, to make sure that the reasoning with arbitrary names is valid. But, again, let's suppose for the sake of argument that they've redeveloped the reasoning to address our concerns. The final problem for their representation is the fact that the name of God never makes an appearance in the argument. Examination of their formal representation

[27]See Rushby (2016), who reformulated the Eder & Ramharter argument in PVS, his proof verification system. We shall not address the question of whether Rushby faithfully reconstructed their argument.

[28]For the theory of definition, see Frege (1879, §24); Padoa (1900); Frege (1903a, §§55–67, §§139–144, and §§146–147); Frege (1903b, Part I); Frege (1914, 224–225); Suppes (1957); Mates (1972, 197–203); Dudman (1973); Belnap (1993); Hodges (2008); and Gupta (2019).

shows that they introduce (2808) the label **God!** to stand for the formal claim $\exists x(Gx \;\&\; E!x)$ ("there is a x such that x is a God and x exists"), where Gx is defined by the statement **Def C-God** identified above. Then they say (2813), "Now that everything is in place, we are in a position to prove **God!** as follows."

Putting aside the fact that they use both second-order and third-order logic in the argument,[29] the problem is that the formal representation doesn't show that *Proslogion* II has an argument for the existence of *God*. Nowhere is the *name* of God introduced into the argument. As we've seen, the constant g is not a name of God, but rather an arbitrary name which they use to represent '*id quo maius cogitari non potest*'. Thus, the conclusion of their argument, $E!g$, doesn't use a name of God. So once you grant them that the phrase 'that than which nothing greater can be conceived' is an arbitrary name and not a description, they have only established a fact about an arbitrarily chosen object of the kind *nothing greater is conceivable*, namely, that such an object exists. It isn't clear how *Proslogion* II contains an ontological argument for the existence of *God*, on their representation.

We don't think it would be a good response to suggest: since uniqueness isn't discussed until *Proslogion* III, Anselm can't conclude that God exists until that next chapter. Such a response won't work for the following reason. In the *opening* of *Proslogion* II, Anselm directly uses the name 'God' ('*Deus*') and the vocative case for 'Lord' ('*Domine*', vocative case of '*Dominus*'). So Anselm clearly takes the conclusion of the argument to apply to God. And that is how Eder & Ramharter understand *Proslogion* II. They say (2015, 2800):

> Having established in Chap. II that God exists in reality from the assumption that God exists at least in the understanding, Anselm proceeds in Chap. III by proving it is inconceivable that God does not exist.

[29] The second-order quantifiers appear in (*) and (**), and third-order logic is used in the statement of **Realization**:

(*) $\forall_{\mathcal{P}} F((Fg \rightarrow Fa) \wedge (F = E! \rightarrow Fa))$

(**) $\forall_{\mathcal{P}} F(Fa \rightarrow (Fg \vee F = E!))$

Realization: $\forall_P \mathcal{F} \exists x \forall_P F(\mathcal{F}(F) \leftrightarrow Fx)$

We don't see any textual justification or other reason for thinking that higher-order machinery is needed for Anselm's argument in *Proslogion* II.

So Eder & Ramharter themselves agree that in *Proslogion* II, there is an argument that establishes something about *God*, and not just about some arbitrarily chosen object such that nothing greater can be conceived.

Indeed, we can't accept the concluding clause of Eder & Ramharter's claim (quoted earlier) that (2802):

> [...] if **id quo** had to be read as a definite description, Anselm would be committed to presupposing the *uniqueness* of **aliquid quo** already in Chap. II, which seems to be in conflict with the fact that only in Chap. III does Anselm mention God's uniqueness for the first time.

Their evidence for the concluding clause is given in n. 20 (2802), where they quote Anselm as saying, in *Proslogion* III (Anselm of Canterbury 2008, 88), "In fact, everything else there is, except You alone, can be thought of as non existing. You, alone then, ..." [ellipsis theirs]. But this hardly counts as a statement that God is a unique thing such that nothing greater can be conceived. Here Anselm is saying only that God uniquely has existence in the highest degree, and this is a claim that plays no role in the ontological argument, as far as we can tell. This is why we don't accept their conclusion that our account, which uses a definite description for the argument in *Proslogion* II, "seems to be in conflict with the fact that only in Chap. III does Anselm mention God's uniqueness for the first time." By using 'God' as a proper name in *Proslogion* II, he is already presupposing uniqueness, and that presupposition, together with Premise 1 and the meaning postulate for *greater than*, justifies his move from **aliquid quo** to **id quo**, as suggested by our representation of the argument.

We conclude that one can not so easily dismiss the suggestion that "*id quo maius cogitari non potest*" is used as a definite description in *Proslogion* II. At present, Eder & Ramharter's suggestion that **id quo** is being used as an arbitrary name leads to the list of problems just outlined. They would have to make a much stronger case before one should be willing to accept this analysis.

6 Conclusion

We conclude that the representation of Anselm's argument in terms of a definite description still has a lot to offer those trying to understand

Proslogion II. To our way of thinking, the interesting questions concern the truth of the premises and the justification of the definition of God. Given what we've now learned, the premises in question are Premise 1, Premise 2$''$, the Meaning Postulate for *greater than*, and the definition of God (g) as $\imath x \phi_1$.

In our paper of 2007, we argued that Premise 1 is the real culprit in the argument. We tried to show that Premise 1 is too strong because it yields the existence of an object that *exemplifies* the property of being a conceivable thing such that nothing greater is conceivable. We argued that Anselm's subsidiary argument for Premise 1 involves two assumptions: (1) that the mere understanding of the phrase 'conceivable thing such that nothing greater is conceivable' requires one to grasp an *intensional* object, and (2) any such intensional object has to exemplify the property *being a conceivable thing such that nothing greater is conceivable*. We then challenged the second assumption on the grounds that the intentionality involved in understanding the phrase only requires that the intensional object (which is thereby grasped) *encode* the property *being a conceivable thing such that nothing greater is conceivable*. Here we appealed to the notion of *encoding* used in the theory of abstract objects (Zalta 1983, 1988).

Interestingly, this is a point of contact with the work in Eder & Ramharter's paper, since their principle **Realization** (which we discussed in n. 29 above) is a kind of comprehension principle that underlies Anselm's assertion that there is something in the understanding such that nothing greater is conceivable. Eder & Ramharter write (2015, 2812–2813):

> So, bearing in mind that first-order quantifiers are ranging over objects existing in the understanding, **Realization** seems plausible. It appears to be an analytic truth that any consistent set of (primitive, positive) conditions is realized by some object in the understanding. This seems to be confirmed by passages like (II.8), where Anselm claims that 'whatever is understood is in the understanding'. Bearing in mind that by 'understanding something' Anselm means understanding what its properties are, we can see that whenever we conceive of a certain set of (non-contradictory) properties, this set gives rise to an object that exists in the understanding—and this is just what **Realization** says. So even though Anselm does not state **Realization** explicitly, we think that it is implicit in how Anselm thinks about objects.

The authors here are placing a lot of weight on this third-order principle (**Realization**) and it isn't clear to us that we should accept that Anselm is committed to this principle simply on the basis of the fact that he takes whatever is understood to be in the understanding. Nor is it clear to us that "by 'understanding something' Anselm means understanding what its properties are". We don't see any textual evidence for this claim. But more importantly, they take **Realization** to be an integral premise of the ontological argument.

By contrast, our 2007 paper shows that whereas the comprehension principle for intensional objects might help us to see why Anselm thought Premise 1 is true, such a principle doesn't need to be added as a premise in the ontological argument itself. It might be needed to justify Premise 1, but it doesn't make an appearance in *Proslogion* II. To formulate the ontological argument, one shouldn't need, as a premise, that for any primitive condition on properties, there is an object that *exemplifies* just the properties satisfying that condition. But this is what **Realization** intuitively asserts. Thus, the work in Oppenheimer & Zalta (2007) bears on this question, and we suggest that further study be focused on Premise 1 and the implicit comprehension principle that Anselm must be relying upon to conclude that it is true.

It remains only to discuss the meaning postulate for *greater than* and whether Anselm might have accepted that this relation is connected. Here, we can at least point to the fact that *greater than* is usually understood in natural language to be an *ordering* relation. But we've not required that there be such an ordering. So the only question concerns whether Anselm would have accepted connectedness, i.e., whether Anselm would have agreed that *any two things* can be compared in terms of *greater than*. We think he would have accepted this principle without question, as part of his Augustinian neo-Platonism. It is part of this world-view that there is the great chain of Being (Rogers 1997; Moran 2004), with God being a unique thing at the top of the chain. As we've seen, the only way for there to be a unique conceivable thing such that nothing greater can be conceived is if *greater than* is, at a minimum, connected.

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