The Power of Predication and Quantification*

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Abstract

In this paper, I show how two modes of predication and quantification in a modal context allow one to (a) define what it is for an individual or relation to exist, (b) define identity conditions for properties and relations conceived hyperintensionally, (c) define identity conditions for individuals and prove the necessity of identity for both individuals and relations, (d) derive the central definition of free logic as a theorem, (e) define the essential properties of abstract objects and provide a framework for defining the essential properties of ordinary objects, and (f) derive a theory of truth. I also describe my indebtedness to the work of Terence Parsons, and take the opportunity to advance the discussion in connection with an objection raised to the theory of essential properties.

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1 Introduction

In this paper, I bring together a number of philosophical definitions and theorems of *object theory* (henceforth 'OT') that are often overlooked because they are usually presented in passing, as part of the conceptual framework needed to develop one of the theory's applications. These definitions and theorems demonstrate what can be achieved with the addition, *encoding* mode of predication in a 2nd-order, quantified modal logic. Readers familiar with OT will find a number of new refinements that improve and extend the theory.

1.1 Specific Goals

In what follows, I shall try to show that, in the context of 2nd-order quantified modal logic, two forms of predication and a primitive quantifier '∀' suffice to:

- define what it is for an individual or relation to exist, have being, or be something (Section 2),
- define identity conditions for properties and relations conceived hyperintensionally (Section 3),
- define identity conditions for individuals and prove the necessity of identity for both individuals and relations (Section 4),
- derive the central definition of free logic as a theorem (Section 5),
- define the essential properties of abstract objects and provide a framework for defining the essential properties of ordinary objects (Section 6), and
- derive a theory of truth (Section 7).

In Section 6, I also take the opportunity to move the discussion about essential properties forward, in light of a criticism in Wildman 2016.

These results help to establish that, within a modal framework, the primitive notions of predication and quantification, and the basic principles that govern them, are sufficient for the analysis of a variety of apparently disparate, but fundamental philosophical notions and principles.

1.2 Parsons' Version of Object Theory

Before we start working towards to the goals just stated, it is important to acknowledge indebtedness to the work of Terence Parsons. The first axiomatic theory of objects I encountered was the one developed in Parsons' book of 1980, which I initially read in manuscript form in 1978. Parsons reconstructed Meinong's (1904) object theory by couching it in a formal language with object variables ($x, y, z, \ldots$); *extranuclear* relation variables ($F^n, G^n, \ldots$), with a distinguished 1-place extranuclear unary predicate of *existence* ($E!$); *nuclear* relation variables ($f^n, g^n, \ldots$); one form of predication ($\Pi^n x_1 \ldots x_n$), where Π is either an extranuclear or nuclear n-place relation term; negation ($\neg$); a conditional ($\rightarrow$), and a quantifier ($\exists$), which is read "there is". (Henceforth I suppress the superscript indicating the arity of a relation term in a formula, since it can be inferred from the number of arguments.) In this framework, Parsons asserted a comprehension principle stipulating that for any condition on *nuclear* properties, there is an object that exemplifies all and only the nuclear properties satisfying φ:

$$\exists x \forall f (fx \equiv \varphi), \text{ provided } x \text{ isn't free in } \varphi$$

Parsons then showed how the objects whose being is asserted by instances of this comprehension schema aren't subject to the criticisms Russell (1905) raised against Meinong (1904). Parsons also offer an analysis of the data involving names of fictional and mythical characters that, in contrast to Russell's theory of descriptions, doesn't turn truths into falsehoods.

I shall not rehearse here either the details of Parsons' defense of (his reconstruction of) object theory against the Russellian objections, or the details of his truth-preserving analysis of the data concerning the names of fictional and mythical characters. Nor will I rehearse the problems that crop up for Parsons' theory. Instead, I simply want to focus on one limitation that Parsons expressly admits. He writes (1980, 10):

> When discussing problems of existence and nonexistence, I'll limit myself entirely to a discussion of concrete objects. So, when I say that some objects don't exist, I mean that some concrete objects don't exist – I don't have in mind propositions, or numbers, or sets.

Parsons' work, however, lead me to a version of object theory that isn't limited to concrete objects and that doesn't require one to interpret the quantifier $\exists$ as "there is" and interpret $E!$ as an existence predicate. Nevertheless, the results I describe in what follows were made possible by the study of Parsons' work.

1.3 Extending Object Theory to Abstract Objects

It is still not all that widely known that Meinong himself (1915, 176) attributed the distinction between nuclear and extranuclear properties to his student Ernst Mally. But Mally also had a rather different idea for addressing Russell's objections and analyzing the data. Mally introduced what appears to be a second mode of predication (Mally 1912, 63–64, 76).[1] He suggested that the fact that an abstract object *x is determined by* (*sein determiniert*) a property F by which we conceive of x doesn't imply that x exemplifies (*erfüllt*) F. In my own work, I've reconstructed this suggestion formally as a distinction between the atomic formula xF ("*x encodes F*") and Fx ("*x exemplifies F*"), with the latter generalized to $F^n x_1 \ldots x_n$ ($x_1, \ldots, x_n$ *exemplify* F^n). (Again, I henceforth suppress the superscript on the relation term that indicates arity.) Both xF and Fx can be used to disambiguate natural language predications of the form "x is F".

I shall not rehearse here the way in which this distinction addresses the problems Russell set for the existing golden mountain and for the round square, or show how we can analyze the data involving names of fictional and mythical characters without turning truths into falsehoods. Instead I want to focus on the benefits of reconstructing Mally's suggestion within the context of a 2nd-order, S5 quantified modal language without identity. Assume that this language includes object variables ($x, y, \ldots$); n-ary relation variables for $n \geq 0$ ($F^n, G^n, \ldots$), with a distinguished 1-place predicate $E!$ (not necessarily interpreted as an existence predicate); n-ary exemplification formulas ($F^n x_1 \ldots x_n$); and n-ary encoding formulas ($x_1 \ldots x_n F^n$). (For most of this paper, we'll need only unary encoding formulas of the form xF.) Then, using negation ($\neg$), a conditional ($\rightarrow$), a quantifier *every* ($\forall$), a modal operator for necessity ($\Box$), and the usual definition $\exists \alpha \varphi \equiv_{df} \neg \forall \alpha \neg \varphi$, the following have formed the core principles of OT (the first two are definitions, the next two are axioms, and the last is an axiom schema):

$$x \text{ is } \textit{ordinary} \text{ (`}O!x\text{')} \equiv_{df} \Diamond E!x \qquad (1)$$

[1] See Findlay 1933 [1963], 110–12, 182–84, for the first discussion in English of Mally's view. I cite Mally (and Findlay) in Zalta 1983 (11), when I discuss the origins of OT.

$$x \text{ is } \textit{abstract} \text{ (`}A!x\text{')} \equiv_{df} \neg\Diamond E!x \tag{2}$$

$$O!x \rightarrow \neg\exists F xF \tag{3}$$

$$xF \rightarrow \Box xF \tag{4}$$

$$\exists x(A!x \,\&\, \forall F(xF \equiv \varphi)), \text{ provided } x \text{ isn't free in } \varphi \tag{5}$$

In the next subsection, we'll discuss various ways in which one might interpret some of the key formal expressions in the above principles. Clearly, though, (5) constitutes a comprehension principle for abstract objects, no matter how we read it. It should also be mentioned, before we begin, that OT is usually extended to include both complex individuals, namely, rigid definite descriptions of the form $\imath x\varphi$, as as well complex n-ary relation terms ($n \geq 0$), namely, λ-expressions of the form $[\lambda x_1 \ldots x_n\, \varphi]$. We'll assume that a negative free logic governs both kinds of terms. These facts will come into play below.

1.4 Interpretations of the Quantifier/Formalism

Quine's arguments (1948, 23) about "ruining the good old word 'exist'" notwithstanding, I shall suppose in what follows that in natural language, one may assert that there *are* things of a certain sort without implying that there *exist* things of that sort.[2] And, if the noneists are right, then one can assert *some* object is such that φ without asserting that *there is* an object such that φ. Given these differences in the meanings of *there exists*, *there are*, and *some* in natural language, the reader may interpret the theory outlined in previous subsection in one of three ways, depending on how one interprets the formal symbols $E!$ and $\exists$:

- Platonist: Interpret the quantifier '$\exists$' as *there exists* and the predicate '$E!$' as *being concrete*. Then (1) stipulates that ordinary objects are possibly concrete, while (2) stipulates that abstract objects couldn't be concrete. Moreover, since $\exists x\psi$ asserts that *there exists* an object such that ψ, (5) asserts that for any condition φ on properties, there exists an abstract object that encodes all and only the properties such that φ.

[2]For example, we can say, without contradiction, that there are fictional characters that inspire us even though they don't exist. Or, to be more exact, one may assert that there are fictional characters that inspire us without having asserted that there exist fictional characters that inspire us.

- Meinongian: Interpret the quantifier '$\exists$' as *there is* and the predicate '$E!$' as *existence*. Then (1) stipulates that ordinary objects possibly exist, while (2) stipulates that abstract objects couldn't exist. Moreover, since $\exists x\psi$ asserts that *there is* an object such that ψ, (5) asserts that for any condition φ on properties, there is an abstract object that encodes all and only the properties such that φ.

- Noneist: Interpret the quantifier '$\exists$' as *some* and the predicate '$E!$' as *existence*. Then (1) stipulates that ordinary objects possibly exist, while (2) stipulates that abstract objects couldn't exist. However, since $\exists x\psi$ asserts that *some* object is such that ψ, (5) asserts that for any condition φ on properties, some abstract object encodes all and only the properties such that φ, without implying that there is or there exists such an object.

I'll use the Platonist interpretation in what follows. But the reader may choose to give principles (1) – (5) one of the other interpretations. How one regiments and represents natural language depends on the choice. Henceforth, we'll call (1) – (5), and any supporting principles, *object theory* and abbreviate this as 'OT'.

2 Existence Defined via Predication

2.1 Existence of Individuals

In this section, we work our way towards a definition of what it is for an individual or a relation to exist. I'll assume the Platonist interpretation our formalism, i.e., in which $\exists$ is read as *there exists*. Where τ is any individual term or relation term, then we'll use the formal expression $\tau\downarrow$ to assert that τ exists. This will be defined below. However, if you take the Meinongian interpretation and read $\exists$ as *there is*, then the definition of $\tau\downarrow$ below will assert what it is for τ to have being. And if you take the Noneist interpretation and read $\exists$ as *some*, then the definition of $\tau\downarrow$ below will assert that τ is something. But we'll not discuss these other interpretations further.

Now in the 1st-order predicate calculus with identity, one often sees the following definition of what it is for an individual to exist:

$$E!x \equiv_{df} \exists y(y = x)$$

However, we'll write this as:

$$x{\downarrow} \equiv_{df} \exists y(y = x)$$

The advantage of this notation is that we can use ↓ to also define $F{\downarrow}$, so that we can later define, without going to a third-order logic, what it is for a relation F to exist.

For readability, I've used the object language variable x as the free variable in the above definition. But, strictly speaking, x should be replaced by a metavariable. That's because we allow the definition to be instanced not just by individual constants and variables, but also by (non-denoting) descriptions of the form $\imath x\varphi$. So, in the above definition, the free variable x is functioning as a metavariable.[3]

A problem with the above definition is that it uses a primitive or defined notion of identity to define existence. One might wonder whether the notion of identity is really required to define existence. Isn't there an alternative definition that uses only the notions of the predicate calculus without identity? One natural candidate comes to mind, namely:

$$x{\downarrow} \equiv_{df} \exists F(Fx) \tag{6}$$

In other words, x exists if and only if x exemplifies a property. This defines existence in terms of quantification, and it works even when the definition is instanced to a non-denoting definite description. If $\imath x\varphi$ doesn't denote, then in our negative free logic, the instance $\imath x\varphi{\downarrow} \equiv \exists F(F\imath x\varphi)$ is true because both sides of the biconditional are false.

I think (6) is a good definition, but there is a concern: it doesn't generalize to a definition of property existence when we move to the 2nd-order predicate calculus. We can't use quantification to define the existence of properties as follows:

$$F{\downarrow} \equiv_{df} \exists xFx$$

This definition fails for unexemplified properties, since such properties exist even though they aren't exemplified.

[3]In a language in which every term denotes, one can use object language variables to formulate definitions. For then one can turn $\equiv_{df}$ into $\equiv$, universally generalize on the free variable, and instantiate any term for the free variable using Rule ∀E (universal instantiation). But when working in a language with non-denoting terms, one should use metavariables in the definition, so that even obtains instances of the definitions even for non-denoting terms; you don't need either Rule ∀E or its restricted version in free logic, which allows one to instantiate terms into universal claims only if those terms denote.

But in OT, you can generalize (6) and define what it is for an n-ary relation to exist ($n \geq 1$) in terms of predication; you just need the right mode of predication. If we make use of n-ary encoding formulas, the definition of existence for relations is ($n \geq 1$):[4]

$$F^n{\downarrow} \equiv_{df} \exists x_1 \ldots \exists x_n(x_1 \ldots x_n F^n) \tag{7}$$

When $n = 1$, this definition reduces to:

$$F{\downarrow} \equiv_{df} \exists x(xF) \tag{8}$$

In other words, if given some property term Π (such as a simple predicate or a λ-expression), then to assert that Π exists is to assert that Π is encoded by some object.[5,6]

[4]These n-ary encoding formulas have other uses as well. In the latest developments of OT (Zalta m.s.), they are used, for example, to represent true encoding readings of relational claims that take place within the context of a story or a theory. For example, when we drop the story prefix "In the story" from a claim such as "In the Conan Doyle novels, Holmes is a friend of Watson", we are left with the claim "Holmes is a friend of Watson". This has both a true reading and a false reasing. The false reading is Fhw, which asserts that Holmes and Watson exemplify the *friendship* relation. This is false because abstract objects don't exemplify friendship. But the true reading is hwF, which asserts that Holmes and Watson encode the *friendship* relation. Object theory then requires both that Holmes encodes the property $[\lambda x\, Fxw]$ and that Watson encodes the property $[\lambda x\, Fhx]$. We also use these n-ary encoding formulas to give true readings of relational claims made within the context of a mathematical theory; outside the context of ZF, for example, the claim that $\emptyset$ is an element of $\{\emptyset\}$ becomes analyzed in OT as the claim that $\emptyset$ and $\{\emptyset\}$ *encode* the membership relation instead of exemplifying it. Indeed, when we formula OT in relational type theory, we would say that the membership relation of ZF encodes the higher-order ZF-property: being a relation R that relates $\emptyset$ to $\{\emptyset\}$, i.e., that $\in_{ZF}$ encodes $[\lambda R\, \emptyset R\{\emptyset\}]_{ZF}$.

[5]Once we have seen a few more principles of object theory, we can actually prove that (8) is a good definition. For example, once we have seen, in Section 3.1, that property identity can be defined and that property identity is reflexive, then one can give the following argument that shows why (8) correctly defines property existence: (→) Assume $F{\downarrow}$. Then, to avoid a clash of variables, consider the following instance of an alphabetic variant of (5), which asserts that there is an abstract object that encodes just the single property F:

$$\exists x(A!x \,\&\, \forall G(xG \equiv G = F))$$

Suppose a is such an object. Then since $F = F$, it follows that aF. So $\exists x(xF)$. (←) Assume $\exists x(xF)$, and suppose b is such an object, so that we know bF. Then by the axioms of negative free logic (one of which asserts that if a term appears in a true atomic formula, then it denotes), it follows that $F{\downarrow}$.

[6]Definition (8) works not just for primitive property constants and λ-expressions that denote exemplified and unexemplified properties, but also for λ-expressions that don't denote at all. OT now allows for non-denoting λ-expressions and if $[\lambda y\, \varphi]$ is such an expression, then in our negative free logic, the instance $[\lambda y\, \varphi]{\downarrow} \equiv \exists x(x[\lambda y\, \varphi])$ is true, since both sides of the biconditional are false.

When $n = 0$ and $p, q, \ldots$ are used as propositional variables instead of $F^0, G^0, \ldots$, we may define the existence of propositions (= 0-ary relations) as follows:

$$p\downarrow \equiv_{df} [\lambda x\, p]\downarrow \quad (9)$$

This reduces proposition existence to property existence and, hence, to n-ary relation existence. So OT lets us define the conditions under which objects and n-ary relations exist ($n \geq 0$) in terms of predication, without invoking the notion of identity.

3 Hyperintensionality

In this section, we first spell out the theory of relations available in OT and then explain why the resulting theory is hyperintensional.

3.1 The Theory of Relations

As noted earlier, OT is formulated with complex relation terms of the form $[\lambda x_1 \ldots x_n\, \varphi]$, for $n \geq 0$, where φ can be *any* formula.[7] Let us say that a *core* λ-expression is one in which none of the variables bound by the λ occur in encoding position anywhere in the matrix φ.[8] OT now uses the follow principle as part of its free logic:

> Whenever τ is a primitive (object or relation) variable, a primitive (object or relation) constant, or a core λ-expression, $\tau\downarrow$ is an axiom. (10)

This axiom ensures that the predicate logic of constants and variables is classical, but that the logic of definite descriptions and λ-expressions is free; some descriptions and λ-expressions fail to denote.[9] So we can immediately instantiate constants and variables into universal claim, but if τ is a description or a λ-expression, we need the assumption that $\tau\downarrow$; see the discussion of OT's free logic in Section 5. However, if a term τ occurs as a relation or object term in a true exemplification or encoding formula, then $\tau\downarrow$.[10]

Intuitively, n-ary relations terms denote relations, not functions, when they denote. (Though one can build models of OT in which the n-ary relation terms denote functions, these aren't the intended interpretations.) Whereas functions simply map arguments to values, relations are predicable entities. This is made clear not just by the primitive notions couched in the atomic exemplification and encoding formulas, and by the existence conditions for relations defined in Section 2, but also by the identity conditions for relations. Identity conditions for relations are definable by cases. Where $n = 1$ and $n = 0$, we have, respectively:[11]

$$F = G \;\equiv_{df}\; F\downarrow \;\&\; G\downarrow \;\&\; \Box\forall x(xF \equiv xG) \quad (11)$$

$$p = q \;\equiv_{df}\; p\downarrow \;\&\; q\downarrow \;\&\; [\lambda x\, p] = [\lambda x\, q] \quad (12)$$

We leave the case where $n \geq 2$ to a footnote.[12] Thus, identity conditions for n-ary relations ($n \geq 0$) are also definable in terms of predication and

[7] This is a change from previous formulations of OT. In earlier versions, for a λ-expression to be well-formed, its matrix φ couldn't contain *any* encoding subformulas. But now, φ can be any formula. However, though the predicates that give rise to paradox are now well-formed, OT doesn't assert that they denote a property and this forestalls the paradoxes. See the discussion that follows and especially footnote 9.

[8] A variable bound by the λ occurs in *encoding position* in φ just in case it occurs as one of $\kappa_1, \ldots, \kappa_n$ in an encoding formula $\kappa_1 \ldots \kappa_n \Pi$ somewhere in φ. Thus, there are λ-expression that denote properties even though their matrix φ contains an encoding subformula. For example, it is axiomatic that $[\lambda x\, \neg Px \,\&\, aQ]\downarrow$, since the variable x bound by the λ doesn't occur in encoding position; this denotes the property *being an x such that x fails to exemplify P and a encodes Q*.

[9] Importantly, the λ-expressions that give rise to paradox fail to denote a property. For example, $[\lambda x\, \exists F(xF \,\&\, \neg Fx)]$, which gives rise to the Clark-Boolos paradox, is well-formed, but (10) doesn't assert $[\lambda x\, \exists F(xF \,\&\, \neg Fx)]\downarrow$. Indeed it is provable that $\neg[\lambda x\, \exists F(xF \,\&\, \neg Fx)]\downarrow$. This new development in object theory emerged as a result of the work of Daniel Kirchner (2017, 2022).

[10] Formally, this is captured by the following axioms, where the κ_i are any individual terms and Π is any n-ary relation term ($n \geq 1$):

$$\Pi\kappa_1 \ldots \kappa_n \to \Pi\downarrow \;\&\; \kappa_1\downarrow \;\&\; \ldots \;\&\; \kappa_n\downarrow$$

$$\kappa_1 \ldots \kappa_n\Pi \to \Pi\downarrow \;\&\; \kappa_1\downarrow \;\&\; \ldots \;\&\; \kappa_n\downarrow$$

The contrapositives tell us that if a term (i.e., either Π or one of the κ_i) fails to denote, then any atomic formula containing such a non-denoting term is false.

[11] We've added the existence clauses $F\downarrow$ and $G\downarrow$ to the definiens of $F = G$ to avoid degenerate cases where the variables are instanced by non-denoting λ-expressions. Without those clauses, we could prove that an identity holds between non-denoting λ-expressions. For if, say, $[\lambda z\, \varphi]$ and $[\lambda z\, \psi]$ are non-denoting λ-expressions, then by one of the principles of negative free logic (namely, that atomic formulas with non-denoting terms are false), one can prove both $\neg x[\lambda z\, \varphi]$ and $\neg x[\lambda z\, \psi]$, making the biconditional $x[\lambda z\, \varphi] \equiv x[\lambda z\, \psi]$ provable. So by the rules GEN and RN, it would follow that $\Box\forall x(x[\lambda z\, \varphi] \equiv x[\lambda z\, \psi]x)$. Thus, without the existence clauses, we would be able to prove $[\lambda z\, \varphi] = [\lambda z\, \psi]$ when both terms fail to denote. The addition of the existence clauses avoids this result.

[12] Intuitively, relations F^n and G^n are identical just in case the result of 'plugging up'

quantification in our modal setting.

Before we examine why this constitutes a hyperintensional conception of relations, it is worth mentioning that when we adapt and assert the principles of the λ-calculus in OT's relational setting, we may *derive*, in the form of a comprehension principle, conditions under which relations exist. The derivation goes by way of the following principle of the λ-calculus, which has been adapted not only so that it clearly governs n-ary relational expressions having a denotation ($n \geq 0$):

$$\beta\text{-Conversion: } [\lambda x_1 \ldots x_n\, \varphi]\!\downarrow\, \rightarrow ([\lambda x_1 \ldots x_n\, \varphi]x_1 \ldots x_n \equiv \varphi) \tag{13}$$

Now consider any formula φ in which $x_1, \ldots, x_n$ ($n \geq 1$) may or may not be free and for which none of the x_i occur in encoding position anywhere in φ. Then by (10), it is axiomatic that $[\lambda x_1 \ldots x_n\, \varphi]\!\downarrow$. So it follows from β-Conversion that:

> $[\lambda x_1 \ldots x_n\, \varphi]x_1 \ldots x_n \equiv \varphi$, provided none of the x_i occur free in encoding position anywhere in φ

By applying the Rule of Generalization to the free variables, applying the Rule of Necessitation, and then existentially generalizing on the λ-expression (which we can do because we know that it denotes), we obtain the following Comprehension Principle for Relations as a theorem:

> $\exists F^n \Box \forall x_1 \ldots x_n (F^n x_1 \ldots x_n \equiv \varphi)$, provided F^n is not free in φ and none of the x_i occur free in encoding position anywhere in φ

This yields all of the complex relations, properties, and propositions whose existence is assertible by the comprehension principle in classical second-order (quantified modal) logic *without* encoding. Since we now have existence and identity conditions for relations, we have rehearsed enough of OT's theory of relations to examine the question of hyperintensionality.

F^n and G^n in the same way with $n-1$ objects yields identical properties. Formally, we say:

$$\begin{aligned} F^n = G^n \equiv_{df}\ & F\!\downarrow \,\&\, G\!\downarrow \,\&\, \\ & \forall y_1 \ldots \forall y_{n-1}([\lambda x\, F^n x y_1 \ldots y_{n-1}] = [\lambda x\, G^n x y_1 \ldots y_{n-1}]\ \&\\ & [\lambda x\, F^n y_1 x y_2 \ldots y_{n-1}] = [\lambda x\, G^n y_1 x y_2 \ldots y_{n-1}]\ \& \ldots \&\\ & [\lambda x\, F^n y_1 \ldots y_{n-1} x] = [\lambda x\, G^n y_1 \ldots y_{n-1} x]) \end{aligned} \qquad (n \geq 2)$$

This reduces relation identity to property identity, where the latter is defined in terms of predication and quantification in a modal context.

3.2 Hyperintensionality of Relations

The theory of relations just developed allows us to consistently assert there are necessarily equivalent properties that aren't identical. That is, one may consistently assert, using OT's language and modal framework, where $n \geq 0$:

$$\exists F \exists G\big(\Box \forall x_1 \ldots \forall x_n (F x_1 \ldots x_n \equiv G x_1 \ldots x_n)\ \&\ F \neq G\big) \tag{14}$$

When $n = 1$, (14) reduces to:

$$\exists F \exists G\big(\Box \forall x (Fx \equiv Gx)\ \&\ F \neq G\big) \tag{15}$$

And when $n = 0$, (14) reduces to:

$$\exists p \exists q (\Box (p \equiv q)\ \&\ p \neq q)$$

These are desirables results since it seems unintuitive to identify properties, relations, and propositions that are necessarily equivalent.

Let's discuss the case of properties. Clearly, if one tells a story about a barber who shaves all and only those who don't shave themselves, no one would conclude that this is a story about a brown and colorless dog. One can consistently assert that *being a barber who shaves all and only those who don't shave themselves* ($[\lambda x\, Bx\ \&\ \forall y(Sxy \equiv \neg Syy)]$) is distinct from *being a brown and colorless dog* ($[\lambda x\, Dx\ \&\ Bx\ \&\ \neg Cx]$). OT doesn't force them to be identical and, in general, doesn't force necessarily equivalent relations of any arity to be identical.

Notice how our identity conditions leave us with an *extensional* theory of the identity of hyperintensional properties! Given that the modal logic of encoding is expressed by principle (4), it follows that when properties F and G are encoded by the same objects, i.e., when $\forall x(xF \equiv xG)$, then they are identical. For given (4) (i.e., $xF \rightarrow \Box xF$), the claim $\forall x(xF \equiv xG)$ implies $\Box \forall x(xF \equiv xG)$.[13] So by (11), it follows that $F = G$. One way

[13] To prove this, assume $\forall x(xF \equiv xG)$, to show $\Box \forall x(xF \equiv xG)$. By the Barcan Formula, it suffices to show $\forall x \Box (xF \equiv xG)$. and by GEN, it suffices to show $\Box(xF \equiv xG)$ and so that is our goal. To reach it, first note that it is a theorem of modal logic that:

$$(\zeta)\quad \big(\Box(\varphi \rightarrow \Box\varphi)\ \&\ \Box(\psi \rightarrow \Box\psi)\big) \rightarrow \big((\Box\varphi \equiv \Box\psi) \rightarrow \Box(\varphi \equiv \psi)\big)$$

Now where φ is xF and ψ is xG, we can establish the following instance of the antecedent of (ζ), both conjuncts of which are an immediate consequence of (4), by the Rule RN:

$$\Box(xF \rightarrow \Box xF)\ \&\ \Box(xG \rightarrow \Box xG)$$

It is therefore a consequence of (ζ) that:

to picture this intuitively is to consider that, semantically, the properties that the variables F and G can take as values are assigned two extensions: an extension of objects that exemplify them and an extension of objects that encode them. The fact that properties F and G have the same exemplification extension at every possible world doesn't entail their identity. But the fact that F and G in fact have the same encoding extension is sufficient for concluding that they have the same encoding extension at every possible world and so are identical. This puts to rest Quine's concern that properties (i.e., Quine calls them 'attributes' or 'intensions') are 'creatures of darkness' (1956, 180), whose principle of individuation is 'obscure' (1956, 184). In OT, the extensional character of their identity conditions makes them as clearcut as sets.

These remarks about properties generalize to n-ary relations for all $n \geq 0$. Given definitions (11), (12), and the definition in footnote 12, we have a theory of hyperintensional relations: the necessary equivalence of relations doesn't imply their identity. The term *hyperintensional* is now firmly established as the technical term that describes properties that are witnesses to (15). That means we should regard the abstract objects of OT as *hyper-hyperintensional* entities, for they are even more fine-grained than properties. Intuitively, there is an abstract object for every (expressible) set of properties and the resulting abstract objects are identical whenever they encode the same hyperintensional properties.

4 Object Identity and the Necessity of Identity

4.1 Object Identity Also Defined via Predication

Intuitively abstract objects are identical whenever they encode the same properties. But one might wonder whether they are identical whenever they exemplify the same properties or necessarily exemplify the same

$(\xi)\quad (\Box xF \equiv \Box xG) \to \Box(xF \equiv xG)$

So if we can establish $\Box xF \equiv \Box xG$, we can reach our goal $\Box(xF \equiv xG)$, by (ξ). But $\Box xF \equiv \Box xG$ is easy, since we know all of the following:

- $\Box xF \equiv xF$: the left-to-right direction follows by the T schema and the right-to-left direction follows by (4).
- $xF \equiv xG$: this follows from our assumption.
- $xG \equiv \Box xG$: again, by (4) and the T schema.

So by an extended biconditional syllogism, $\Box xF \equiv \Box xG$.

properties. In fact, the classical definition of object identity, namely, that $x = y$ just in case $\Box\forall F(Fx \equiv Fy)$, isn't sufficiently fine-grained for OT. For it is a theorem of OT that there are distinct abstract objects that exemplify the same properties:

$$\exists x \exists y(A!x \,\&\, A!y \,\&\, x \neq y \,\&\, \forall F(Fx \equiv Fy))$$

Consequently, identity for objects is defined as follows:

$$x = y \ \equiv_{df} \\ (O!x \,\&\, O!y \,\&\, \Box\forall F(Fx \equiv Fy)) \vee (A!x \,\&\, A!y \,\&\, \Box\forall F(xF \equiv yF)) \qquad (16)$$

We've now defined $\tau = \tau'$ for cases where τ and τ' are (a) both individual terms or (b) both n-ary relation terms, for some n, $n \geq 0$. Note that our definition of $\tau = \tau'$ immediately implies, in each case, that identity is reflexive. In the first case, it is not too difficult to show $x = x$:

Proof. Our strategy is to reason by disjunctive syllogism from the fact that $O!x \vee A!x$, which is an immediate consequence of $O!x \vee \neg O!x$ and definition (2). Assume $O!x$. Then note that since $Fx \equiv Fx$ is a logical theorem, we may infer $\forall F(Fx \equiv Fx)$ by GEN and infer $\Box\forall F(Fx \equiv Fx)$ by RN. Hence, by &I, we have established:

$$O!x \,\&\, O!x \,\&\, \Box\forall F(Fx \equiv Fx)$$

And by $\vee$I, one obtains the definiens of (16) for the instance $x = x$.

Alternatively, assume $A!x$. Then note that since $xF \equiv xF$ is a logical theorem, we may infer $\forall F(xF \equiv xF)$ by GEN and infer $\Box\forall F(xF \equiv xF)$ by RN. Hence, by &I, we have established:

$$A!x \,\&\, A!x \,\&\, \Box\forall F(xF \equiv xF)$$

And by $\vee$I, we again obtain the definiens of (16) for the instance $x = x$. ⋈

We prove $F^n = F^n$, for $n \geq 0$, by cases. When $n = 1$, then $F = F$ follows from definition (11) by establishing that $F\downarrow$ and $\Box\forall x(xF \equiv xF)$. But the first holds axiomatically; recall that when α is any variable of the language, it is an axiom that $\alpha\downarrow$. The second follows by GEN and RN from the logical theorem $xF \equiv xF$. And when $n = 0$, the fact that $p = p$ is derivable by (a) conjoining the axiom $p\downarrow$ with the result of instantiating $[\lambda x\, p]$ into $F = F$ and (b) applying definition (12). We leave the proof that $F^n = F^n$ as an exercise; it is relatively straightforward to prove this from the definition of $F^n = G^n$ given in footnote 12 and the facts that both $F\downarrow$ and $F = F$ holds for every property F.

By providing that identity for objects and relations is reflexive, it follows from the axiom asserting the substitution of identicals that identity for objects and relations is also symmetric and transitive. Proofs can be found in any standard text that include a discussion of the predicate calculus with identity. In the next section we show how our definition of identity implies the necessity of identity.

4.2 The Necessity of Identity

In this section we prove the necessity of identity for individuals and n-ary relations ($n \geq 0$). A single, schematic proof suffices. Let α and β be metavariables that range over distinct variables of the object language of the same type. So, for example, α and β might be the variables x and y or the variables F and G (where F and G are relation variables of *any* arity $n \geq 0$). Since we proved $x = x$ (Section 4.1) and $F = G$ for all n-ary relation variables $n \geq 0$ (Section 3.1), we may represent all of these results as a proof of the theorem schema:

$$\alpha = \alpha \tag{17}$$

Now with this theorem schema in hand, we may formulate and prove, in complete generality, the necessity of identity as the following theorem schema:

$$\alpha = \beta \rightarrow \Box(\alpha = \beta) \tag{18}$$

Proof. Assume $\alpha = \beta$, for conditional proof. Since $\alpha = \alpha$ (17) is a theorem, it follows by Rule RN that $\Box\alpha = \alpha$. From this and our assumption $\alpha = \beta$, we may infer $\Box\alpha = \beta$ by the substitution of identicals. ⋈

Observe here that this proof parallels the proof of $x = y \rightarrow \Box(x = y)$ in Kripke 1971 (136). But the above theorem schema has greater significance, in two ways. First, the theorem schema governs both objects and relations generally, and not just objects. Second, in Kripke's proof, identity is a *primitive* and the reflexivity of identity is *stipulated* as an axiom. By contrast, our theorem schema is derived from *defined* notions of identity and the reflexivity of identity is *derived* as a theorem schema.

5 Free Logic

Let us continue to use α, β as metavariables that range over variables of the same type and τ to range over terms in our 2nd-order system. Note that in free logic, the quantifier axiom $\forall\alpha\varphi \rightarrow \varphi^\tau_\alpha$ (provided τ is substitutable for α) is typically revised to one of the following principles (both having the same proviso), depending on whether = or ↓ is primitive:

- $\forall\alpha\varphi \rightarrow (\exists\beta(\beta = \tau) \rightarrow \varphi^\tau_\alpha)$
- $\forall\alpha\varphi \rightarrow (\tau\downarrow \rightarrow \varphi^\tau_\alpha)$ (19)

Of course, one only needs the former if the system takes identity as primitive and ↓ is defined as:

$$\tau\downarrow \equiv_{df} \exists\beta(\beta = \tau), \text{ provided } \beta \text{ doesn't occur free in } \tau \tag{20}$$

In OT, however, = isn't used to define ↓. Instead, ↓ and = are both defined in terms of predication and quantification, and (19) is taken as the basic axiom of the negative free logic. Given definitions (11), (12), (16), and the definition of relation identity in footnote 12, the traditional definition of ↓, i.e., (20), becomes provable as a theorem:

Proof. Consider any term τ in which β doesn't occur free. We prove the two directions of the biconditional separately. (→) Assume $\tau\downarrow$. Since $\alpha = \alpha$ is a theorem (17), it follows by GEN that $\forall\alpha(\alpha = \alpha)$. Moreover, by inspection, τ is substitutable for α in $\alpha = \alpha$ (it isn't captured by any variable binding operator upon). So it follows from an appropriate instance of (19) and the assumption $\tau\downarrow$ that $\tau = \tau$. Since β doesn't occur free in τ, it follows that $\exists\beta(\beta = \tau)$. (←) Assume $\exists\beta(\beta = \tau)$. Let σ be a simple constant of the same type as β and that denotes an arbitrary such entity, so that we know $\sigma = \tau$. But then the definitions of identity in (16), (11), (12) and in footnote 12 all imply $\tau\downarrow$.[14] ⋈

[14] Here is a proof, by cases, that $\sigma = \tau$ implies both $\sigma\downarrow$ and $\tau\downarrow$. Suppose σ and τ are both individual terms. The $\sigma = \tau$ implies, by definition (16):

$$(O!\sigma \,\&\, O!\tau \,\&\, \Box\forall F(F\sigma \equiv F\tau)) \vee (A!\sigma \,\&\, A!\tau \,\&\, \Box\forall F(\sigma F \equiv \tau F))$$

But both disjuncts include an atomic formula involving σ and an atomic formula involving τ. So by OT's axiom for negative free logic mentioned earlier, both $\sigma\downarrow$ and $\tau\downarrow$.

If σ and τ are both relations terms, then the definition of $\sigma = \tau$ in (11), (12) and in footnote 12, all imply both $\sigma\downarrow$ and $\tau\downarrow$.

The interest in this theorem lies not only in the fact that it constitutes a derivation of a definition, namely (20), assumed in free logic, but also in the fact (20) itself has now been defined in terms of predication and quantification.

6 Essential Properties

In this section, it is important to recall that OT is formulated in an S5, quantified modal logic in which the Barcan and Converse Barcan Formulas apply to the necessity operator $\Box$ and both quantifiers $\forall x$ and $\forall F$. So the domain of objects and the domain of n-ary relations, for each n, don't vary from possible world to possible world. Moreover *being abstract* ($A!$) and *being ordinary* ($O!$) are mutually exclusive and jointly exhaustive properties, given definitions (1) and (2)

The axioms, definitions, and theorems of OT make it clear that abstract and ordinary objects are fundamentally distinct kinds of objects. Ordinary objects do not encode properties and their identity conditions are tied to the properties they exemplify. By contrast, abstract objects both encode and exemplify properties, and their identity conditions are tied to the properties they encode. Moreover, the existence conditions for abstract objects are describable by a comprehension principle, namely (5), whereas the existence conditions for ordinary objects are not. Indeed, (5) asserts there is a plenitude of abstract objects, since for *every* (expressible) condition on properties, it asserts the existence of an abstract object that encodes just those properties. No such principle holds for ordinary objects.[15]

I take the foregoing to show that there is a difference in kind, i.e., a categorial difference, between abstract and ordinary objects. So one should expect to find that the conditions under which a property is essential to an abstract object are distinct from the conditions under which a property is essential to an ordinary object. It is straightforward to define what it is for a property to be essential to an abstract object. Let us temporarily restrict the variable x to range over abstract objects. Then in Zalta 2006 (687), we defined *F is essential to x*, for abstract objects x, as follows:

$$F \textit{ is essential to } x \equiv_{df} xF \tag{21}$$

It is easy to see why the properties essential to an abstract object x are its encoded properties and not, say, the properties that x exemplifies necessarily. Consider the null set $\emptyset$, as systematized by the axioms and theorems of Zermelo-Fraenkel set theory (ZF). OT identifies the emptyset of ZF as the abstract object that encodes all and only the properties that are exemplified by $\emptyset$ according to the theorems of ZF. So it encodes properties such as: having no members, being an element of $\{\emptyset\}$, etc. That is, if F is a property such that it is a theorem of ZF that $F\emptyset$, then $\emptyset$ encodes F. Those are the only properties that the emptyset of ZF encodes. In some sense, the emptyset of ZF is just what the structuralist imagined – its only (encoded) properties are its mathematical properties. And this object simultaneously reifies the inferential role of the expression '$\emptyset$' in the body of theorems of ZF.

Now what properties does the emptyset of ZF exemplify? In OT, it doesn't exemplify any of the properties it encodes. Instead, it exemplifies their negations and exemplifies such properties as: failing to have a shape, failing to have an extension in space, failing to be a building, failing to be colored, etc. Indeed, these properties are necessarily exemplified by the emptyset, and so is the property of being abstract. But notice that none of the properties that the emptyset necessarily exemplifies are properties by which it is defined, conceived, or identified. They are not part of its nature. Its nature is given solely by the properties it encodes; these properties are more central to its identity than are the properties it necessarily exemplifies.

So OT rejects both directions of the classical view when it is applied to abstract objects; that is, for abstract x, OT rejects the biconditional that F is essential to an abstract object x if and only if $\Box Fx$ (or $\Box(x{\downarrow} \to Fx)$). Thus, the case of abstract objects is consistent with the argument in Fine 1994; the essential properties of such objects as the singleton of Socrates, the emptyset of ZF, the number 1 of Peano Arithmetic, The Triangle, etc., are not the properties that these objects necessarily exemplify. The essential properties of such objects are their encoded properties, and the

[15] The ordinary objects further divide into the contingently concrete, the contingently non-concrete, and the necessarily concrete. OT doesn't assert the existence of objects that are contingently concrete (i.e., objects x such that $E!x \,\&\, \Diamond\neg E!x$) or necessarily concrete (i.e., objects x such that $\Box E!x$). However, the $\Diamond$-form of the Barcan Formula ($\Diamond\exists x\varphi \to \exists x\Diamond\varphi$) is a *conditional* existence principle that has a number of consequences with respect to those objects x that are contingently non-concrete (i.e., $\neg E!x \,\&\, \Diamond E!x$).

fact that they are essential is provable from their definition.[16]

These observations hold not just for mathematical objects, but also for all the other abstract objects definable in object theory, such as situations and possible worlds, impossible worlds, moments of time, fictions, world-indexed Leibnizian concepts, Fregean senses, etc.[17] These are all objects whose essential properties are precisely the ones by which we define them as unique abstract objects. Their encoded properties, not the properties they necessarily exemplify, are their essential properties. Thus, it should be clear that to discover the essential properties of any particular abstract object, we don't have to discover the essential properties of any other object.

Now what about the essential properties of ordinary objects? This case is more difficult, since it isn't as clear what the nature of an ordinary object consists of. In what follows, let's use u as a restricted variable ranging over ordinary objects. In Zalta 2006 (679), I distinguished the *weakly* essential properties of an ordinary object u (i.e., those properties F such that, necessarily, if u exemplifies *being concrete* then u exemplifies F), from the *strongly* essential properties of u (i.e., those weakly essential properties that u doesn't necessarily exemplify). If, for the purposes of discussion, we suppose that *being human* (H) is strongly essential to Socrates (s), the definition tells us that (a) $\Box(E!s \to Hs)$, and (b) $\neg\Box Hs$.[18] In worlds where Socrates is not concrete, he isn't a human, since being a human necessarily implies being concrete. So the property *being human*

[16]Each instance of (5) yields a unique abstract object, given definition (16). So the description $\iota x(A!x \,\&\, \forall F(xF \equiv \varphi))$ is always well-formed, provided x doesn't occur free in φ. In previous work, we've used these *canonical* descriptions to define the following abstract objects, among others:

$$\textit{The Form of } G\ (\Phi_G) =_{df} \iota x(A!x \,\&\, \forall F(xF \equiv \Box\forall z(Fx \Rightarrow Gx)))$$

$$\textit{The actual world } (\boldsymbol{w}_0) =_{df} \iota x(A!x \,\&\, \forall F(xF \equiv \exists p(p \,\&\, F = [\lambda z\, p])))$$

$$\textit{The False } (\bot) =_{df} \iota x(A!x \,\&\, \forall F(xF \equiv \exists p(\neg p \,\&\, F = [\lambda z\, p])))$$

$$\textit{The complete individual concept of Socrates } (\boldsymbol{c}_s) =_{df} \iota x(A!x \,\&\, \forall F(xF \equiv Fs))$$

$$\textit{The sum of } y \textit{ and } z\ (x \oplus y) =_{df} \iota x(A!x \,\&\, \forall F(xF \equiv yF \vee zF))$$

Given such a definition for an abstract object x, one can one prove that xF by showing that F satisfies the matrix φ of the canonical description used as the definiens. Then F is essential to x, by (21).

[17]See, respectively, Zalta 1993, 1997, 1987, 2000a, 2000b, and 2001.

[18]I am not committed to this particular example. If you think that *being human* is not essential to Socrates, then pick some other property that you think is essential to Socrates and use that throughout the remainder of the paper.

is distinguished from necessary properties such as *being concrete if concrete* ($[\lambda x\, E!x \to E!x]$), i.e., properties that Socrates exemplifies even in worlds where he is not concrete.

I now think that the definiens of 'F is *strongly essential* to u' is not sufficient. It will take some time to understand why I've reached this conclusion and so let me begin by simplifying our terminology to eliminate 'strongly' from the defined notion. Consider the following definition:

$$F \textit{ is essential to } u \equiv_{df} \Box(E!u \to Fu) \,\&\, \neg\Box Fu \qquad (\vartheta)$$

Though I defended (ϑ) in Zalta 2006, I now think it is too strong. In order to see why, I'll review the objections raised in a paper by Wildman (2016). He objects to OT generally and, after uniting (21) and (ϑ) into a single, disjunctive definition, he attempts to show that the single definition is problematic. But, in the end, his objection to the single definition rests on an objection to just (ϑ). To see this, we have to first wade through some over-the-top rhetoric and separate out the parts of the argument that have merit from those that don't.

Wildman's general objection to OT is that it is 'costly' and requires us to "take on a lot of (highly debatable) metaphysical baggage" (2016, 186).[19] But I suggest that one can't validly draw such a conclusion without (a) considering all the applications of OT as a whole and (b) comparing OT's benefits with the costs and benefits of rival foundations having similar applications (many of which take on board a significant form of set theory).[20] Wildman doesn't do this, nor does he consider the arguments in Linsky & Zalta 1995, in which we show how (5) is consistent

[19]This is a running criticism in his paper, as can be seen from the following passages, all of which occur in Wildman 2016 (190):

> . . . why bother doing so after buying into Zalta's framework, with all of its prohibitive theoretical costs and counter-intuitive consequences?
>
> . . . a simple analysis of the theoretical costs shows Zalta's picture to be a worse deal. . .
>
> . . . if the [bullet-biting] strategy succeeds, then modalists can use it from the get-go, and Zalta's reply (with all of its costly metaphysical baggage) turns out to be fundamentally unnecessary.

And finally, Wildman concludes:

> His [Zalta's] account is costly, counter-intuitive, and, most worryingly, faces new counter-examples.

[20]Wildman counts, among the costs of OT, the fact that the notion of an essential property is bifurcated for abstract and ordinary objects (2016, 186). But, this is easily countered. For if there is indeed a fundamental distinction between abstract and ordinary objects,

with naturalism and is metaphysically cheap in many ways, especially when one counts the benefits of correctly deriving the principles governing a wide range of abstract objects (including mathematical objects) without assuming any mathematics. Nor does Wildman consider the arguments in Linsky & Zalta 1994, which shows how OT significantly simplifies quantified modal logic.

Another place where Wildman's argument goes wrong is in alleging that one cost of OT's theory of essential properties is that it:

> ... violates what is arguably *the* general inferential connection between claims of essence and claims of necessity, namely, that from 'Φ is essential to *x*' we can infer 'Φ is necessary to *x*'. (2016, 186)

Note that this objection doesn't apply to (21). If F is essential to an abstract object x, then (21) implies xF, and by the modal logic of encoding (4), this implies $\Box xF$. So if F is essential to an abstract object x, then one can indeed infer F is necessary to x in the encoding sense of 'is'.

But does this objection apply to (ϑ)? Wildman here seems to be in agreement with Fine, who says "I accept that if an object essentially has a certain property then it is necessary that it has the property (or has the property if it exist)" (1994, 3). The clause in parenthesis shows that Fine is being careful here. For if 'F is essential to x' simply implies 'necessarily, Fx', then from the fact that *being human* is an existence- (or concreteness-) entailing property that is essential to Socrates, it follows that Socrates necessarily exists (or is necessarily concrete). Even Fine would be hesitant, as the passage just quoted shows, to infer without reservation that Socrates is human in every possible world from the fact that *being human* is essential to Socrates. And so it isn't a cost, but rather a virtue, that (ϑ) doesn't force the inference from "*being human* is essential to Socrates" to "necessarily, Socrates is human".

Interestingly, Wildman (2016, footnote 11) cites Fine (2005, 332) in support of his objection to (ϑ). But I don't think that this passage in Fine 2005 supports this objection to (ϑ) either. The cited passage is part of an

and the two sorts of objects have fundamentally different natures, then a correct theory may need to bifurcate the definition. So, this objection takes us back to the question of whether OT has correctly identified an ontological distinction in kind between abstract and ordinary objects. Wildman doesn't address that question. He doesn't consider that a bifurcated definition is justified on the philosophical grounds that there is a categorial difference between the two kinds of object.

extended discussion of the following puzzling argument about nonexistence (2005, 328):

> (i) It is necessary that Socrates is self-identical.
> (ii) It is possible that Socrates does not exist.
> (iii) So it is possible that Socrates is self-identical and does not exist.

Fine suggests that this argument is puzzling because it appears to be valid but has true premises and a false conclusion. But this argument isn't puzzling in OT's framework.

In OT, (i) is true, whereas (ii) and (iii) are both false. Indeed, (i) is provable from the fact that Socrates is an ordinary object, given the special, definable relation of identity for ordinary objects available in OT. Consider (where x, y are general variables):

$$=_E \; =_{df} [\lambda xy\, O!x \,\&\, O!y \,\&\, \Box\forall F(Fx \equiv Fy)] \tag{22}$$

This yields, as theorems, both $O!x \rightarrow x =_E x$ and $x =_E y \rightarrow \Box x =_E y$.[21] So, the reflexivity of identity$_E$ implies (i). But in OT, (ii) and (iii) are false, since in OT's fixed domain modal logic, every object necessarily exists. However, the intuition underlying (ii) is preserved by reading "Socrates does not exist" as "Socrates is not concrete", since there are worlds where Socrates is not concrete. And the intuition underlying conclusion (iii) can be similarly preserved, for Socrates is self-identical even at worlds where he isn't concrete. That is what you would expect in a fixed domain framework.

So appeals to Fine's arguments in 1994 and 2005 don't justify Wildman's objection to (ϑ), i.e., that, for ordinary objects u, it fails to to preserve the inference from 'F is essential to u' to 'u is necessarily F'. If Wildman accepts that the inference is valid, then he needs some argument to justify it, since in the context of OT, one can reasonably assert that Socrates is essentially a human without asserting that he is a human

[21] To show $O!x \rightarrow x =_E x$, assume $O!x$. To show $x =_E x$ (where $=_E$ is in infix notation), we have to show $O!x \,\&\, O!x \,\&\, \Box\forall F(Fx \equiv Fx)$, by (22) and λ-Conversion. But this is straightforward, since $O!x$ by assumption and $\forall F(Fx \equiv Fx)$ is a theorem of logic and so necessary by the Rule of Necessitation.

To show $x =_E y \rightarrow \Box x =_E y$, assume $x =_E y$. Then by λ-Conversion, we know that this is necessarily equivalent to $O!x \,\&\, O!y \,\&\, \Box\forall F(Fx \equiv Fy)$. But $O!x$ is equivalent, by definition (1), to $\Diamond E!x$, and so in S5, $\Box\Diamond E!x$, i.e., $\Box O!x$, by definition (1). And, by the 4 axiom, $\Box\forall F(Fx \equiv Fy)$ implies $\Box\Box\forall F(Fx \equiv Fy)$. So we've established that all of $O!x$, $O!y$, and $\Box\forall F(Fx \equiv Fy)$ are necessary. So their conjunction is necessary. Hence, $\Box x =_E y$.

in every possible world. Thus, OT doesn't incur the cost alleged by Wildman but rather seems to avoid a bad inference and to correctly explain an acknowledged philosophical puzzle.

However, Wildman (2016, 186) raises an objection that does have merit, namely, that (ϑ) seems to fail when we conjoin an essential property of Socrates (e.g., *being human*) with a necessary property. Though this is indeed the place to focus, Wildman's particular counterexample isn't definitive. He formulates the property *being human and such that the Eiffel Tower is essentially a tower* and argues that it both satisfies the definiens of (ϑ) yet fails to be essential to Socrates. But it isn't so clear that this property satisfies the definiens of (ϑ). On some interpretations, in which the phrase 'the Eiffel Tower' is represented as either a rigid name or a rigid definite description, "The Eiffel Tower is a essentially a tower" is false. That particular hunk of metal might have been cast into a concrete object other than a tower.

So let's minimally change the example to one that we're accepting for the purposes of this paper. Consider a different conjunctive property of Socrates, namely, *being human and such that being human is essential to Plato*, where the proposition *being human is essential to Plato* is defined as in (ϑ). If we apply (ϑ) and invoke λ-Conversion, then the property in question is necessarily equivalent to the following property, where 'p' denotes Plato:

$$[\lambda u\, Hu \,\&\, \Box(E!p \rightarrow Hp) \,\&\, \neg\Box Hp] \tag{A}$$

It seems uncontroversial that property (A) and Socrates jointly satisfy the definiens of (ϑ). Socrates exemplifies (A) in every world in which he is concrete, but he does not exemplify (A) necessarily, since he isn't a human at every possible world. And this result seems to allow Wildman his conclusion (2016, 186–7):

> But such a property is clearly non-essential to Socrates—otherwise, discovering Socrates's nature would involve discovering 'the natures of all things' (Fine 1994, 6).

I accept that philosophers do have intuitions that suggest *being human and such that being human is essential to Plato* isn't a part of Socrates' nature and that such intuitions imply that (A) constitutes a counterexample to (ϑ).

Before I try to advance the discussion, however, let me mention that in Zalta 2006, I tried to anticipate a similar counterexample, namely, the property *being human and distinct from the Eiffel Tower*. This is a conjunction of the property *being human* and the property *being distinct from the Eiffel Tower*, the latter which Socrates exemplifies necessarily. In OT, this property would be represented formally as $[\lambda u\; u \neq_E t]$, and so the conjunctive property would be represented formally as:

$$[\lambda u\, Hu \,\&\, u \neq_E t] \tag{B}$$

In Zalta 2006 (683–685), I proposed several ways one might respond to such apparent counterexamples. Wildman does correctly point out that one of the options I suggested won't work,[22] However, the fundamental reply remains: it isn't clear exactly what the nature of any given ordinary object is supposed to be. So I was willing to let the theory decide these cases.

But now, let me then accept that (ϑ) is only true in the left-to-right direction. Let's then reconsider one of the points of Fine's 1994 essay, which is to argue that we need a definition of an object in order to say what it's essential properties (or nature) is:

> We have seen that there exists a certain analogy between defining a term and giving the essence of an object; for the one results in a sentence which is true in virtue of the meaning of the term, while the other results ni a proposition which is true in virtue of the identity of the object. However, I am inclined to think that the two cases are not merely parallel but are, at bottom, the same. . . .
>
> . . . Thus we find again that in giving a definition we are giving an essence—though not now of the word itself, but of its meaning.
>
> (Fine 1994, 13)

I accept this conclusion only for the case of abstract objects, but not for the case of ordinary objects, since the former can be defined whereas the latter can not. Here's why.

Fine seems to suggest not only that we can define objects but that we can define both abstract and ordinary objects. In response to those

[22] I should not have said:

> One could place a constraint on the principles governing that notion [of Socrates' nature] so as to exclude any property which necessarily implies a property that Socrates has in every possible world.

Wildman correctly notes that this would rule out all properties as essential properties, since every property necessarily implies every necessary property.

who would suggest that concepts, but not objects, can be defined, he says (1994, 14):

> The difficulty with this position is to see what is so special about concepts. It is granted that the concept bachelor may be defined as unmarried man; this definition states, in the significant essentialist sense, what the concept *is*. But then why is it not equally meaningful to define a particular set in terms of its members or to define a particular molecule of water in terms of its atomic constituents?

The example of a defining a particular water molecule is his only example of defining an ordinary object, but I don't think this example is telling. For if a water molecule has a nature, then it seems likely that its nature resides not just in its atomic constituents but in the constituent's *chemical bonds* to one another. So Fine's only example seems to be a case in which the nature of one thing depends on the natures of other things. And I would argue that the same holds of ordinary objects generally. They can't be defined, and so we can't identify them *by a definition*.

By contrast, I think that (a) abstract objects have definitions (see the examples in footnote 16), and (b) that the *nature* of an ordinary object u is an abstraction and so has a chance of being defined. The nature of u is not the same object as u itself. Now let me say again: I don't have a definition of *the nature of* u to offer; as I've noted earlier, I don't know exactly what the nature of an ordinary object consists of. But presumably, one would have to identify the nature of u in terms of (some condition on) the properties of u. And once one has a condition φ on the properties of u (with free variables F and u), one could introduce a definition of *the nature of* u ('$\boldsymbol{n}_u$') in the following manner:

$$\boldsymbol{n}_u =_{df} \imath x(A!x \,\&\, \forall F(xF \equiv \varphi)) \tag{23}$$

Then we could use a variant of definition (21) to stipulate that F is essential to u just in case the nature of u encodes F:

$$F \textit{ is essential to } u \equiv_{df} \boldsymbol{n}_u F \tag{24}$$

This opens up a number of options. One could place some constraints on the definition of $\boldsymbol{n}_u$. For example, if Wildman's own theory of essential properties were preserved in OT's fixed domain framework, he could define:

$$\boldsymbol{n}_u =_{df} \imath x(A!x \,\&\, \forall F(xF \equiv \Box(E!u \rightarrow Fu) \,\&\, Sparse(F)))$$

That is, the nature of u is an abstract object that encodes all and only those sparse properties that u has in every world in which it is concrete.[23]

Notice that one could place a constraint on φ in (23), to ensure that the resulting definition has, as a consequence:

$$\boldsymbol{n}_u F \rightarrow (\Box(E!u \rightarrow Fu) \,\&\, \neg\Box Fu)$$

In other words, one might assume, as a principle, that if F is in the nature of u, then u exemplifies F in every world in which u is concrete but not in every world. Then (24) would ensure that if F is essential to u, then $\Box(E!u \rightarrow Fu) \,\&\, \neg\Box Fu$, thereby preserving the left-to-right direction of (ϑ). So Fine's suggestion about the relation of essence and definition can be preserved if we define the nature of an ordinary object and not the object itself. But I shall leave that task to others.

Though Wildman's discussion has led us to a new approach to the definition of the nature of ordinary objects, his blanket rejection of OT's theory of essential properties doesn't stand. His argument doesn't establish that (21) is incorrect. Nor does it establish that we don't need different definitions of 'F is essential to x' for abstract and ordinary objects, for the reasons outlined in footnote 20. In the end, the important question is whether one can give a proper definition of *the nature of* u for an ordinary object u, say by supplying the right φ in (23).

7 Truth

Finally, we turn to the notion of truth.[24] I think philosophical logicians have overlooked the axiomatic theory of truth available in second-order logic extended with λ-expressions. Recall that the n-ary axiom schema β-Conversion was formulated above as (13). When $n = 0$, the following is the 0-ary version of β-Conversion, where φ is any formula:

$$[\lambda\, \varphi]\!\downarrow \rightarrow ([\lambda\, \varphi] \equiv \varphi) \tag{25}$$

Before we consider what this asserts, note that in the most recent formulation of of OT, one can prove, for every formula φ (even those with

[23]Wildman adopts the notion of a sparse property from Lewis 1986, 59–60; see Wildman 2016, 194. I don't have a formal definition of a sparse property to offer, but Lewis gives a nice intuitive discussion.

[24]In this section, I rehearse the theory of truth describe in Zalta 2014.

encoding formulas): *that-*φ exists. That is, for every formula φ, it is a theorem that $[\lambda\, \varphi]\!\downarrow$.[25] Hence it follows from (25) that:

$$[\lambda\, \varphi] \equiv \varphi \qquad (26)$$

This asserts: *that-*φ is true if and only if φ (no matter what formula φ we consider). For example, *that Biden is president* is true if and only if Biden is president. This is a theory of truth derivable in second-order logic extended with λ-expressions.

You might wonder, why are we justified in introducing the phrase 'is true' in our readings of (25) and (26)? The answer is: *predication* reduces to *truth* in the 0-ary case. To see this, consider the following unary instance of (13):

$$[\lambda x\, \neg Rx]x \equiv \neg Rx$$

If R designates the property of being red, then this instance would assert: x exemplifies *not being red* if and only if x fails to exemplify being red. Notice that on both sides of the biconditional, we have formulas, i.e., expressions that have truth conditions and denote propositions. Similarly, (26) has formulas on both sides of the biconditional; that's why the biconditional symbol '$\equiv$' is appropriate. But how does one read '$[\lambda\, \varphi]$' as a formula? It looks like all it says is *that-*φ. But, in fact, since the expression is in formula position, you have to remember that exemplification reduces to truth, and so you read $[\lambda\, \varphi]$ in (26) as *that* φ *is true*.

Of course, there are contexts where the expression $[\lambda\, \varphi]$ is *term* position and not in formula position. For example, in the formula $[\lambda\, \varphi] = [\lambda\, \psi]$, the λ-expressions are in term position; the identity symbol is defined in such a way that the expression flanking it are, in the first instance, terms. Of course, they may also be formulas, but identity is defined generally over terms and not formulas. $[\lambda\, \varphi] = [\lambda\, \psi]$ is a formula asserting the identity of two propositions. So we may read $[\lambda\, \varphi] = [\lambda\, \psi]$ as: (the proposition) *that-*φ is identical to (the proposition) *that-*ψ.

I'm not sure why logicians don't typically discuss the axiomatic theory of truth that is expressible in second-order logic minimally extended with λ-expressions.[26] One reason may be that they have focused on developing an axiomatic theory of the *truth predicate*, in the tradition of

[25]This is an instance of axiom (10): since λ doesn't bind any variables in $[\lambda\ :\varphi]$, none of the variables bound by the λ appear in encoding position anywhere in φ.

[26]For example, it isn't mentioned in Halbach & Leigh 2022, Väänänen 2021, or Enderton 2019.

Tarski (1933, 1944). Another reason may be that most logicians view the λ-calculus as a calculus of complex function terms and not complex relation terms. In the functional λ-calculus, β-Conversion is an identity (or a 'reduction' of the complex λ-term to a term without the λ) and not a biconditional. So a Tarski-like biconditional theory of truth wouldn't be available.

But when one interprets λ-expressions as denoting primitive relations, not functions, then (13), i.e., β-Conversion, is assertible as a biconditional and is the key axiom for deriving the theory expressible as (26). This way of extending the second-order predicate calculus and applying it to the notion of truth demonstrates the potential that predication (with a complex relation term) has for philosophical analysis.

8 Conclusion

By showing how a number of important philosophical definitions and principles can be constructed or derived with the help of a second mode of predication in a quantified modal setting, it becomes clearer just how much additional and significant philosophical power the second mode of predication contributes to the modal predicate calculus. This second mode of predication has been proposed at various points in the history of philosophy, going back to Plato (Pelletier & Zalta 2000). But I first encountered it when studying T. Parsons' 1980 work on Meinong, which led me to Mally's (1912) suggestion that there are two ways for abstract objects to be characterized by their properties. Parsons' work thereby opened up vista onto axiomatic metaphysics and cleared one path for the systematic study of fundamental philosophical issues related to the problem of existence and nonexistence.

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