Basis:

$x \preceq y \equiv \exists z(x \oplus z = y)$	Definition 3 ($\preceq$)
$x \oplus y = y \oplus x$	Axiom 1 (Commutativity)
$x \oplus x = x$	Axiom 2 (Idempotence)
$x \oplus (y \oplus z) = (x \oplus y) \oplus z$	(Axiom 3) (Associativity)

Theorems Provable from Laws of Identity Alone

1. $x = y \rightarrow y = x$
2. $x \neq y \rightarrow y \neq x$
3. $[x = y \,\&\, y = z] \rightarrow x = z$

   Corollary: $[x = y \,\&\, y = z \,\&\, z = w] \rightarrow x = w$
4. $x = y \,\&\, y \neq z \rightarrow x \neq z$
5. $[x \preceq y \,\&\, x = z] \rightarrow z \preceq y$
6. $[x \preceq y \,\&\, z = y] \rightarrow x \preceq z$
9. $x = y \rightarrow [x \oplus z = y \oplus z]$
10. $[x = z \,\&\, y = w] \rightarrow x \oplus y = z \oplus w$
11. $[x = u \,\&\, y = v \,\&\, z = w] \rightarrow x \oplus y \oplus z = u \oplus v \oplus w$

Interesting Theorems:

7. $x \preceq x$
8. $x = y \rightarrow x \preceq y$
12. $y \preceq z \rightarrow [x \oplus y \preceq x \oplus z]$
13. $x \oplus y = x \rightarrow y \preceq x$
14. $y \preceq x \rightarrow x \oplus y = x$
15. $[x \preceq y \,\&\, y \preceq z] \rightarrow x \preceq z$

    Corollary: $[x \oplus z \preceq y] \rightarrow z \preceq y$
16. $[x \preceq y \,\&\, y \preceq z \,\&\, z \preceq w] \rightarrow x \preceq w$
17. $[x \preceq y \,\&\, y \preceq x] \rightarrow x = y$
18. $[x \preceq z \,\&\, y \preceq z] \rightarrow x \oplus y \preceq z$
19. $[x \preceq z \,\&\, y \preceq z \,\&\, w \preceq z] \rightarrow x \oplus y \oplus w \preceq z$
20. $[x \preceq z \,\&\, y \preceq w] \rightarrow x \oplus y \preceq z \oplus w$
21. $[x \preceq u \,\&\, y \preceq v \,\&\, z \preceq w] \rightarrow x \oplus y \oplus z \preceq u \oplus v \oplus w$
22. $[x \not\preceq y \,\&\, y \not\preceq x] \rightarrow$

    $\exists z(z \neq x \,\&\, z \neq y \,\&\, (x \oplus z \preceq y \oplus z \;\vee\; y \oplus z \preceq x \oplus z))$
23. $(x \not\preceq y \,\&\, y \not\preceq x) \rightarrow \exists z(z \neq x \,\&\, z \neq y \,\&\, x \oplus y = x \oplus z)$
24. Exercise.