Seminar on Axiomatic Metaphysics
Lecture 4
Situations, Routley Star, and HYPE

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Situation Theory I

- $Propositional(F) \equiv_{df} \exists p(F = [\lambda y\, p])$
- $Situation(x) \;\equiv_{df}\; A!x \;\&\; \forall F(xF \rightarrow Propositional(F))$
- *p is true in x* (‘$x \models p$’) $\equiv_{df}$ $Situation(x) \;\&\; x\Sigma p$
- All the connectives dominate $\models$, e.g.,
  $x \models p \equiv p$ abbreviates $(x \models p) \equiv p$
- $\vdash Situation(x) \rightarrow \Box Situation(x)$:
  - So let $s, s', \ldots$ be *rigid*, restricted variables ranging over situations.
- *s is part of s'* (‘$s \trianglelefteq s'$’) $\equiv_{df}$ $\forall p(s \models p \rightarrow s' \models p)$
- *Part-of* ($\trianglelefteq$) is reflexive, anti-symmetric, and transitive on the situations:
  - $s \trianglelefteq s$
  - $s \trianglelefteq s' \;\&\; s \neq s' \rightarrow \neg(s' \trianglelefteq s)$
  - $s \trianglelefteq s' \;\&\; s' \trianglelefteq s'' \rightarrow s \trianglelefteq s''$
- $s = s' \equiv s \trianglelefteq s' \;\&\; s' \trianglelefteq s$
- $s = s' \equiv \forall s''(s'' \trianglelefteq s \equiv s'' \trianglelefteq s')$

Situation Theory II

- $NullSituation(x) \equiv_{df} Situation(x) \ \& \ \neg\exists p(x \models p)$

  $\exists! x NullSituation(x)$
- $TrivialSituation(x) \equiv_{df} Situation(x) \ \& \ \forall p(x \models p)$

  $\exists! x TrivialSituation(x)$
- Some situations encode falsehoods:

  $\exists s \exists p(\neg p \ \& \ s \models p)$ (exercise)
- Some situations are partial:

  $\exists s \exists p(s \not\models p \ \& \ s \not\models \neg p)$ (exercise)
- $Actual(s) \equiv_{df} \forall p(s \models p \rightarrow p)$

  $\exists s Actual(s)$
- $Consistent(s) \equiv_{df} \neg\exists p(s \models p \ \& \ s \models \neg p)$

  $Possible(s) \equiv_{df} \Diamond Actual(s)$
- $Actual(s) \rightarrow Consistent(s)$

  $Actual(s) \rightarrow Possible(s)$
- The theory makes a prediction at 15 of the 19 choice points in situation theory (Barwise 1989).

Introduction

- Routley & Routley (1972) studied the semantics of entailment by assuming the existence of situations ('set-ups') that are neither consistent nor maximal (*ibid*., 335–339).
- Some logicians use the term 'non-normal worlds', but we reserve 'world' for maximal situations.
- Routleys used 'H' to range over set-ups (i.e, "a class of propositions or wff") and used 'A' to range over propositions or wffs (*ibid*., 337).
- They defined the star (*) operation negatively (*ibid*., 338):

  (iv) $\sim A \in H$ iff $A \notin H^*$

  Positively:

  $$A \in H^* \text{ iff it is not the case that } \sim A \in H$$

- They subsequently stipulated that a set-up is $\sim$-*normal* if it satisfies (iv) for every A and $H = H^{**}$ (*ibid*., 338).

Preliminaries

- Definition: $\overline{p} =_{df} \neg p$
- $\vdash \exists s \forall p (s \models p \equiv \phi)$, provided s isn't free in ϕ
- Proof: We have to show: $\exists x(Situation(x) \ \& \ \forall p(x \models p \ \equiv \ \phi))$, provided x isn't free in ϕ. Pick ϕ where x isn't free, and consider a property variable that isn't free in ϕ, say G. Let ψ be $\exists p(\phi \ \& \ G = [\lambda z \, p])$. Then $\exists x(A!x \ \& \ \forall G(xG \equiv \psi))$, i.e.,
  - $\exists x(A!x \ \& \ \forall G(xG \equiv \exists p(\phi \ \& \ G = [\lambda z \, p])))$

  Suppose it is a. Then $A!a$ and $\forall G(aG \equiv \exists p(\phi \ \& \ G = [\lambda z \, p]))$ (A)
  Clearly, $Situation(a)$. So, by GEN, we only have to show $a \models p \ \equiv \ \phi$. Instantiate $a[\lambda z \, p]$ into the following alphabetic variant of (A), where q is a variable that is substitutable for p, and doesn't occur free, in ϕ: $\forall G(aG \equiv \exists q(\phi^q_p \ \& \ G = [\lambda z \, q]))$ (A′)
  to obtain $a[\lambda z \, p] \equiv \exists q((\phi^q_p)^{[\lambda z \, p]}_G \ \& \ [\lambda z \, p] = [\lambda z \, q])$. But since G isn't free in ϕ, $(\phi^q_p)^{[\lambda z \, p]}_G$ is just ϕ^q_p.
  (B) $a[\lambda z \, p] \equiv \exists q(\phi^q_p \ \& \ [\lambda z \, p] = [\lambda z \, q])$
  Now prove $a \models p \equiv \phi$. (See Zalta forthcoming, or PLM).

The Definition of Routley Star in OT

- *The Routley star image of* situation s, written s^*, is the situation s' that makes true all and only those propositions whose negations aren't true in s:
  - $s^* =_{df} \iota s' \forall p(s' \models p \equiv \neg s \models \overline{p})$
- Given that $\neg s \models \overline{p}$ is modally collapsed, it strictly follows:
  - $\vdash \forall p(s^* \models p \equiv \neg s \models \overline{p})$
- This is equivalent to:
  - $\vdash \forall p(s \models \overline{p} \equiv \neg s^* \models p)$

  which is (iv) in R&R 1972.
- Definition of gaps and gluts:
  - $GlutOn(s,p) \equiv_{df} s \models p \ \& \ s \models \overline{p}$
  - $GapOn(s,p) \equiv_{df} \neg s \models p \ \& \ \neg s \models \overline{p}$
- Three theorems validating R&R 1972 claims:
  - $\vdash s = s^{**} \rightarrow (GlutOn(s,p) \rightarrow GapOn(s^*,p))$
  - $\vdash s = s^{**} \rightarrow (GapOn(s,p) \rightarrow GlutOn(s^*,p))$
  - $\vdash (\neg GlutOn(s,p) \ \& \ \neg GapOn(s,p)) \rightarrow (s^* \models p \equiv s \models p)$

Further Theorems Validating the Definition

- $\vdash \forall p(\neg GlutOn(s,p) \ \& \ \neg GapOn(s,p)) \rightarrow s^* = s$
- $\vdash \forall p(\neg GlutOn(s,p) \ \& \ \neg GapOn(s,p)) \equiv \forall p(s \models p \equiv \neg s \models \overline{p})$
- The null and trivial situations:
  - $\boldsymbol{s}_\emptyset =_{df} \iota s' \forall p(s' \models p \equiv p \neq p)$
  - $\boldsymbol{s}_{\boldsymbol{V}} =_{df} \iota s' \forall p(s' \models p \equiv p = p)$
- It follows that:
  - $\vdash \forall s(s^{**} = s) \rightarrow \boldsymbol{s}_\emptyset{}^* = \boldsymbol{s}_{\boldsymbol{V}}$
  - $\vdash \forall s(s^{**} = s) \rightarrow \boldsymbol{s}_{\boldsymbol{V}}{}^* = \boldsymbol{s}_\emptyset$
  - $\vdash s^{**} = s \equiv \forall p(s \models p \equiv s \models \overline{\overline{p}})$
- The proofs are in Zalta forthcoming, and *Principia Logico-Metaphysica*.

An Alternative Definition

- Instead of defining s^* as the situation that makes true all and only the propositions whose negations aren't true in s, the alternative defines s^* as the situation that makes true all and only the negations of propositions that aren't true in s:
  - $s^* =_{df} \iota s' \forall p(s' \models p \equiv \exists q(\neg s \models q \,\&\, p = \overline{q}))$

  Since the condition $\exists q(\neg s \models q \,\&\, p = \overline{q})$ is modally collapsed:
  - $\vdash \forall p(s^* \models p \equiv \exists q(\neg s \models q \,\&\, p = \overline{q}))$
- But the key condition, $\exists q(\neg s \models q \,\&\, p = \overline{q})$, is *not* equivalent to the condition $\neg s \models \overline{p}$ used in R&R 1972. I.e., the following are not equivalent definitions of s^*:
  - $s^* =_{df} \iota s' \forall p(s' \models p \equiv \neg s \models \overline{p})$
  - $s^* =_{df} \iota s' \forall p(s' \models p \equiv \exists q(\neg s \models q \,\&\, p = \overline{q}))$

Counterexamples That Show Non-Equivalence

- Consider a simple situation, say s_1, in which a single proposition, say p_1, is true. Ignore all other propositions and consider what propositions are true in s_1^* according to the original definition and what propositions are true s_1^* according to the new definition.
- According to the original:
  - $s_1^* \models p_1$ (since $\neg s_1 \models \overline{p_1}$)
  - $s_1^* \models \overline{p_1}$ (since $\neg s_1 \models \overline{\overline{p_1}}$)
  - $s_1^* \models \overline{\overline{p_1}}$ (since $\neg s_1 \models \overline{\overline{\overline{p_1}}}$)
  - and so on.
- According to the new, neither p_1 nor $\overline{p_1}$ are true in s_1^* (neither is the negation of a proposition that s_1 *fails* to encode). Instead:
  - $s_1^* \models \overline{\overline{p_1}}$ (since $\neg s_1 \models \overline{p_1}$ and $\overline{\overline{p_1}}$ is the negation of $\overline{p_1}$)
  - $s_1^* \models \overline{\overline{\overline{p_1}}}$ (since $\neg s_1 \models \overline{\overline{p_1}}$ and $\overline{\overline{\overline{p_1}}}$ is the negation of $\overline{\overline{p_1}}$)
  - and so on.
- So the definitions aren't equivalent.

The Condition Under Which They Are Equivalent

- Consider the condition: $\forall p(\overline{\overline{p}} = p)$ (ζ)
- (ζ) plays a role in the proof of equivalence:
  - $\exists q(\neg s \models q \;\&\; p = \overline{q}) \equiv \neg s \models \overline{p}$ (ω)
  - *Proof*: ($\rightarrow$) Assume $\exists q(\neg s \models q \;\&\; p = \overline{q})$ and let r be such a proposition, so that we know both $\neg s \models r$ and $p = \overline{r}$. The latter implies that $\overline{p} = \overline{\overline{r}}$, for if propositions are identical, so are their negations. But by (ζ), $\overline{\overline{r}} = r$. Hence, $\overline{p} = r$ and so $\neg s \models \overline{p}$. ($\leftarrow$) Assume $\neg s \models \overline{p}$. Then by ($\zeta$), $\neg s \models \overline{p} \;\&\; p = \overline{\overline{p}}$. By existentially generalizing on $\overline{p}$ we have: $\exists q(\neg s \models q \;\&\; p = \overline{q})$. $\bowtie$
- OT does *not* imply (ζ) since the identity conditions of relations and propositions are hyperintensional.
- To model HYPE, we just need $\exists p(\overline{\overline{p}} = p)$ (call these Hype-propositions) and then study situations that constructed only out of Hype-propositions.

Basic HYPE Models

- HYPE (Leitgeb 2019, 321ff) starts with a propositional language $\mathcal{L}$: atomic propositional letters $p_1, p_2, \ldots$, and logical symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\top$ ($\rightarrow$ is not the material conditional).
- Note: $\overline{p_i}$ abbreviates $\neg p_i$, and $\overline{\overline{p_i}}$ abbreviates for p_i.
- Proposition letters and their negations constitute the *literals*.
- HYPE Model: $\langle S, V, \circ, \bot \rangle$:
  - S is a non-empty set of states.
  - V is a function (the valuation function) from S to the power set of the set of literals of the language $\mathcal{L}$, so that each state s in S is associated with a set of literals $V(s)$.
  - $\circ$ is a partial fusion function on states that is idempotent and, when defined, commutative and partially associative.
  - $\bot$ is a relation of incompatibility that relates states s and s' when some proposition p is true at one and its negation $\overline{p}$ is true at the other. ['$\bot$' sometimes denotes the proposition $\neg\top$ (2019, 321).]
- The Routley star operation constrains all the above elements (Leitgeb 2019, 321). We'll see exactly how later.

Hype Propositions and Hype States

- A *Hype*-proposition is any proposition p that is identical to its double negation:
  $$Hype(p) \equiv_{df} \overline{\overline{p}} = p$$
- Theorem: If p is a *Hype*-proposition, then so is its negation $\overline{p}$:
  $$\vdash Hype(p) \rightarrow Hype(\overline{p})$$
- Instead of stipulating '$\overline{\overline{p}}$' is to be an abbreviation of 'p', we take as an assumption that there are *Hype*-propositions:
  $$\exists p Hype(p)$$
- Definition: x is a *HypeState* just in case x is a situation such that every proposition true in x is a *Hype*-proposition:
  $$HypeState(x) \equiv_{df} Situation(x) \& \forall p(x \models p \rightarrow Hype(p))$$
- So when Leitgeb speaks of the members of $V(s)$ as the facts or states of affairs obtaining at s (2019, 322), we may interpret this in terms of our defined notion, *p is true in s*, as follows:
  - $p \in V(s) \equiv_{df} s \models p$
- Theorem: *HypeStates* exist. (Exercise)

Comprehension for Hype States

- Introduce rigid restricted variables:
  - $\mathsf{p}, \mathsf{q}, \ldots$ are restricted variables ranging over *Hype*-propositions.
  - $\mathsf{s}, \mathsf{s}', \ldots$ be are restricted variables ranging over *HypeStates*.
- Comprehension conditions for *HypeStates*:
  - $\vdash \exists \mathsf{s} \forall \mathsf{p}(\mathsf{s} \models \mathsf{p} \equiv \phi)$, provided s isn't free in ϕ
  - $\vdash \exists ! \mathsf{s} \forall \mathsf{p}(\mathsf{s} \models \mathsf{p} \equiv \phi)$, provided s isn't free in ϕ
  - $\iota \mathsf{s} \forall \mathsf{p}(\mathsf{s} \models \mathsf{p} \equiv \phi)\downarrow$, when s isn't free in ϕ.

Hype Fusion

- The HYPE fusion operation $\circ$ is *partial*, but we'll use the OT (total) *summation* operation $\oplus$ on situations:

  $$s \oplus s' \ =_{df} \ \imath s'' \forall p (s'' \models p \equiv (s \models p \vee s' \models p))$$

- Since $s \models p \vee s' \models p$ is modally collapsed:

  $$\vdash \forall p (s \oplus s' \models p \equiv (s \models p \vee s' \models p))$$

- s is a part of s' if and only if the sum of s and s' just is s':

  $$\vdash s \trianglelefteq s' \equiv s \oplus s' = s'$$

- $\oplus$ is idempotent, commutative, and associative w.r.t. situations *generally*. So, in particular:

  $\vdash \oplus$ is idempotent, commutative, and associative on *HypeStates*

  $$\vdash \mathsf{s} \oplus \mathsf{s} = \mathsf{s}$$

  $$\vdash \mathsf{s} \oplus \mathsf{s}' = \mathsf{s}' \oplus \mathsf{s}$$

  $$\vdash \mathsf{s} \oplus (\mathsf{s}' \oplus \mathsf{s}'') = (\mathsf{s} \oplus \mathsf{s}') \oplus \mathsf{s}''$$

  So interpret $s \circ s'$ as $\mathsf{s} \oplus \mathsf{s}'$ if we ignore partiality. But see Zalta forthcoming for an explanation (a) how to model the partiality, and (b) why OT doesn't needs $\oplus$ to be partial.

- The proofs of the above are in Zalta forthcoming, and PLM.

The HYPE Explicit Incompatibility Relation

- To define, in OT, the HYPE *explicit incompatibility* condition $\perp$, we first define it on situations generally: s is *explicitly incompatible* with s' ($s \,!\, s'$) just in case there is a proposition p such that s makes p true and s' makes the negation of p true:

  $$s \,!\, s' \equiv_{df} \exists p(s \models p \;\&\; s' \models \overline{p})$$

  We may write $\mathsf{s} \,!\, \mathsf{s}'$ when *HypeStates* are explicitly incompatible.
- We then derive the HYPE principle (Leitgeb 2019, 322):

  If there is a v with $v \in V(s)$ and $\overline{v} \in V(s')$, then $s \perp s'$.

  as:

  $$\vdash (\mathsf{s} \models \mathsf{p} \;\&\; \mathsf{s}' \models \overline{\mathsf{p}}) \rightarrow \mathsf{s} \,!\, \mathsf{s}'$$
- And we derive the HYPE principle (Leitgeb 2019, 322):

  If $s \perp s'$ and both $s \circ s''$ and $s' \circ s'''$ are defined, then $s \circ s'' \perp s' \circ s'''$.

  as:

  $$\vdash \mathsf{s} \,!\, \mathsf{s}' \rightarrow (\mathsf{s} \oplus \mathsf{s}'') \,!\, (\mathsf{s}' \oplus \mathsf{s}''')$$
- See Zalta forthcoming, or PLM, for the proofs.

Routley Star in HYPE

- In HYPE, Leitgeb 2019 (322) introduces Routley star as follows (using our restricted variable ‘s’). For every s in S,
  - (A) there is a unique $s^* \in S$ (the star image of s) such that:
  - (B) $V(s^*) = \{\, \overline{v} \mid v \notin V(s) \,\}$,
  - (C) $s^{**} = s$,
  - (D) s and s^* are not incompatible, i.e., $\neg(s \perp s^*)$, and
  - (E) s^* is the largest state compatible with s, i.e., if s is not incompatible with s', then the fusion of s' and s^* is defined and the fusion of $s' \circ s^* = s^*$.
- Leitgeb uses the alternative definition of Routley star, but since p and $\overline{\overline{p}}$ are collapsed in HYPE, his definition becomes equivalent.
- So we may capture (B) as follows:

$$s^* =_{df} \imath s' \forall p(s' \models p \equiv \exists q(\neg s \models q \,\&\, p = \overline{q}))$$

- Now although the HYPE principle (A) requires a unique s^* satisfying (B) – (E), s^* is already uniquely defined.
- So we may immediately conclude that s^* exists, for any s, and it remains to show s^* also satisfies constraints (C) – (E). But first:

Some Facts About Routley Star in HYPE

- We can simply our definition by a theorem: the *Hype* propositions true in s^* are precisely the negations of the *Hype*-propositions that fail to be true in s:

  $\vdash \forall p(s^* \models p \equiv \exists q(\neg s \models q \;\&\; p = \overline{q}))$

- Verify earlier result: p is the negation of some proposition that s fails to make true if and only if s fails to make $\overline{p}$ true:

  $\vdash \exists q(\neg s \models q \;\&\; p = \overline{q}) \equiv \neg s \models \overline{p}$

- It follows that:

  $\vdash \forall p(s^* \models p \equiv \neg s \models \overline{p})$

  $\vdash \forall p(s \models \overline{p} \equiv \neg s^* \models p)$

  The 2nd is a direct analogue of the R&R 1972 condition (iv).

- Validate: if s has a glut w.r.t. p, then s^* has a gap w.r.t. p; if s has a gap w.r.t. p, then s^* has a glut w.r.t. p; and if s has neither a glut nor a gap w.r.t. p, then s^* agrees with s^* on p:

  $\vdash \mathit{GlutOn}(s, p) \rightarrow \mathit{GapOn}(s^*, p)$

  $\vdash \mathit{GapOn}(s, p) \rightarrow \mathit{GlutOn}(s^*, p)$

  $\vdash (\neg \mathit{GlutOn}(s, p) \;\&\; \neg \mathit{GapOn}(s, p)) \rightarrow (s^* \models p \equiv s \models p)$

Completing the Derivation

- HYPE Principle (C) (Leitgeb 2019, 322):

  $\vdash \mathsf{s}^{**} = \mathsf{s}$
- HYPE Principle (D): s is not explicitly incompatible with s^*:

  $\vdash \neg \mathsf{s}!\mathsf{s}^*$
- HYPE Principle (E), simplified because $\oplus$ is total: if s is not incompatible with s', then the sum/fusion of s' and s^* just is s^*:

  $\vdash \neg \mathsf{s}!\mathsf{s}' \to (\mathsf{s}' \oplus \mathsf{s}^* = \mathsf{s}^*)$

  This guarantees that s^* is the largest state compatible with s.
- Finally, if we recall definition of $s \trianglelefteq s'$ and fact that $s \trianglelefteq s' \equiv \forall p(s \models p \to s' \models p)$, we may prove that the HYPE Routley star operation reverses $\trianglelefteq$:

  $\vdash \mathsf{s} \trianglelefteq \mathsf{s}' \to \mathsf{s}'^* \trianglelefteq \mathsf{s}^*$

  Cf. Observation 3, Leitgeb 2019 (325). This completes the derivation of the principles stipulated in HYPE for the Routley star operation, modulo the partiality of the HYPE fusion operation.

Deriving Principles of Restall and Berto

- In Restall 2000, semantic frames and a primitive relation of compatibility on points are introduced on the first page.
- In Berto 2015, frames are introduced (766ff), and negation is analyzed as a modality (767) that is interpreted by a distinguished accessibility relation on worlds, R_N, understood as a compatibility relation (768ff).
- In Berto & Restall 2019, the semantic analysis occurs in Section 3, where frames and the primitive compatibility relation on worlds are introduced (1127).
- They both assert a Heredity Principle (Restall 2000, Definition 1.2; Berto 2015, 767; and Berto & Restall 2019, 1128): if p is true at a situation s (i.e., a point, world), and s is a part of s', then p is true at s'.

Exercises

- Derive the reflexivity, anti-symmetry, and transitivity principles governing the relation $\sqsubseteq$ on the points of compatibility frames (Restall 2000, 853, Definition 1.1), except derive them for the defined $\trianglelefteq$ on situations s in OT.
- Prove the Persistence (Heredity) Principle:
  - $(s \models p \;\&\; s \trianglelefteq s') \rightarrow s' \models p$

  Cf. Zalta 1993, 413, Theorem 8; this settled a choice point in Barwise 1989 (265) in favor of Alternative 6.1.
- Define the 'compatibility' relation taken as primitive in Restall 2000 and Berto 2015 as follows (for this exercise, use x, y as situation variables):
  - $xCy \equiv_{df} \neg\exists p(x \models p \;\&\; y \models \overline{p})$

  Now derive the principle they stipulate to characterize that relation (Restall 2000, 853, Definition 1.1; Berto 2015, 768, 'Backward'; and Berto & Restall 2019, 1129, 'Backwards'). I.e., show that their principle:
  - for any x, y, x', and y', if xCy, $x' \sqsubseteq x$, and $y' \sqsubseteq y$, then $x'Cy'$,

  becomes derivable in OT, with $\trianglelefteq$ replacing $\sqsubseteq$, as:
  - $(xCy \;\&\; x' \trianglelefteq x \;\&\; y' \trianglelefteq y) \rightarrow x'Cy'$
- Hint: Some of these proofs can be found in Zalta forthcoming ("The Metaphysics of Routley Star")

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