Seminar on Axiomatic Metaphysics
Lecture 7
Leibnizian Modal Metaphysics

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Leibniz's Modal Picture I

From the *Theodicy*:

> *I will now show you some [worlds], wherein shall be found, not absolutely the same Sextus as you have seen (that is not possible, he carries with him always that which he shall be) but several Sextuses resembling him, possessing all that you know already of the true Sextus, but not that is already in him imperceptibly, nor in consequence all that shall yet happen to him. You will find in one world a very happy and noble Sextus, in another a Sextus content with a mediocre state, . . .*

(Leibniz 1714, source in G.vi 363)

Leibniz's Modal Picture II

Letter to Landgraf Ernst von Hessen-Rheinfels of April 12, 1686:

> *For by the individual notion of Adam I undoubtedly mean a perfect representation of a particular Adam, with given individual conditions and distinguished thereby from an infinity of other possible persons very much like him, but yet different from him... There is one possible Adam whose posterity is such and such, and an infinity of others whose posterity would be different; is it not the case that these possible Adams (if I may so speak of them) are different from one another, and that God has chosen only one of them, who is exactly our Adam?*

Translation in Parkinson, PW 51. The source is G.ii 20

Leibnizian Scholarship

- Mondadori 1973: introduces the suggestion of using counterpart theory to model Leibniz's views.
- Whereas for Lewis the counterpart relation is a relation on individuals, "in Leibniz's case, it is best regarded as being a relation between (complete) concepts" (1973, 248).
- This is explicitly built into the Leibnizian system described in Fitch 1979.
- So in Leibnizian modal metaphysics, the possible worlds are not inhabited by Lewis's possibilia, but rather by complete individual concepts.
- See also Wilson 1979, Vailati 1986, and Lloyd 1978.
- We'll have both a Kripkean and a Lewisian component in our reconstruction of Leibniz (Zalta 2000).
- Our Picture: intuitions sketched out diagrammatically.

Some Definitions and Lemmas We'll Need

- $D! \equiv_{df} [\lambda x\, \Box\forall z(z \neq x \rightarrow \exists F\neg(Fz \equiv Fx))]$
- $\vdash D!x \rightarrow \Box D!x$
- $=_D \equiv_{df} [\lambda xy\; D!x \;\&\; D!y \;\&\; x = y]$
- $\vdash x =_D y \rightarrow \Box x =_D y$
- $\vdash \Diamond x =_D y \rightarrow x =_D y$
- $\vdash [A!x \;\&\; A!y \;\&\; \forall F(xF \equiv yF)] \rightarrow x = y$
- $\vdash w \models p \;\equiv\; w \models [\lambda y\, p]x$
- $\vdash w \models (p \vee q) \equiv (w \models p \vee w \models q)$

Proof of a Lemma

- $\vdash x =_D y \rightarrow \Box x =_D y$
- *Proof*. Assume $x =_D y$. Then by the definition of $=_D$,

  $D!x \;\&\; D!y \;\&\; x = y$

  But it is a theorem that $D!x \rightarrow \Box D!x$ (exercise), and so the first two conjuncts imply, respectively, $\Box D!x$ and $\Box D!y$. And by the necessity of identity, the third conjunct implies $\Box x = y$. Hence:

  $\Box D!x \;\&\; \Box D!y \;\&\; \Box x = y$

  So by a basic theorem of modal logic:

  $\Box(D!x \;\&\; D!y \;\&\; x = y)$

  Since it is a modally strict theorem that $x =_D y \equiv D!x \;\&\; D!y \;\&\; x = y$, it follows by a Rule of Substitution that $\Box x =_D y$. $\bowtie$

Proof of a Lemma

- $\vdash \Diamond x\!=_D\! y \rightarrow x\!=_D\! y$
- *Proof*. By applying RN to the previous lemma, we know $\Box(x\!=_D\! y \rightarrow \Box x\!=_D\! y)$. But in S5, it is a modally strict theorem that $\Box(\varphi \rightarrow \Box\psi) \equiv \Box(\Diamond\varphi \rightarrow \psi)$. Hence, by a Rule of Substitution $\Box(\Diamond x\!=_D\! y \rightarrow x\!=_D\! y)$. So by the T schema, $\Diamond x\!=_D\! y \rightarrow x\!=_D\! y$. ⋈

Proof of a Lemma

- $\vdash [A!x \,\&\, A!y \,\&\, \forall F(xF \equiv yF)] \rightarrow x{=}y$
- *Proof*. Suppose $A!x$, $A!y$, and $\forall F(xF \equiv yF)$. By the definition of $=$, we only have to show $\Box\forall F(aF \equiv bF)$. Pick an arbitrary property, say P. So $xP \equiv yP$. But, by the rigidity of encoding, $xP \equiv \Box xP$ and $yP \equiv \Box yP$. So $\Box xP \equiv \Box yP$. This implies, by propositional logic, either $\Box xP \,\&\, \Box yP$ or $\neg\Box xP \,\&\, \neg\Box yP$. If the former, then by modal logic, $\Box(xP \,\&\, yP)$. If the latter, then again by the rigidity of encoding, it follows that $\neg\Diamond xP \,\&\, \neg\Diamond yP$,* i.e., $\Box\neg xP \,\&\, \Box\neg yP$, which by modal logic, implies $\Box(\neg xP \,\&\, \neg yP)$. So either $\Box(xP \,\&\, yP)$ or $\Box(\neg xP \,\&\, \neg yP)$, which means that necessarily, xP and yP have the same truth value, i.e., $\Box(xP \equiv yP)$. But P was arbitrary, so $\forall F\Box(xF \equiv yF)$. Thus, by the Barcan formula, $\Box\forall F(xF \equiv yF)$.

* Axiom: $xF \rightarrow \Box xF$. By RN: $\Box(xF \rightarrow \Box xF)$. By the principle: $\Box(\varphi \rightarrow \psi) \rightarrow (\Diamond\varphi \rightarrow \Diamond\psi)$, it follows that $\Diamond xF \rightarrow \Diamond\Box xF$. By the S5 principle $\Diamond\Box\varphi \rightarrow \Box\varphi$, this implies $\Diamond xF \rightarrow \Box xF$. So $\neg\Box xF \rightarrow \neg\Diamond xF$.

Proof of a Lemma

- $\vdash w \models p \;\equiv\; w \models [\lambda y\, p]x$
- Since $[\lambda y\, p]\downarrow$, it follows from λ-conversion that $[\lambda y\, p]x \equiv p$. So by RN, $\Box([\lambda y\, p]x \equiv p)$. This implies both $\Box([\lambda y\, p]x \to p)$ and $\Box(p \to [\lambda y\, p]x)$. $(\to)$ Suppose $w \models p$. Then since $\Box(p \to [\lambda y\, p]x)$ and worlds are modally closed, it follows that $w \models [\lambda y\, p]x$. $(\leftarrow)$ Suppose $w \models [\lambda y\, p]x$. Then since $\Box([\lambda y\, p]x \to p)$ and worlds are modally closed, it follows that $w \models p$.
- Exercise: To obtain a simpler proof, establish the lemma: $w \models (p \equiv q) \equiv ((w \models p) \equiv (w \models q))$. Then instantiate q to $[\lambda y\, p]x$. The left-side of the result, $w \models (p \equiv [\lambda y\, p]x)$ follows from the Fundamental Theorem $[\Box p \equiv \forall w(w \models p)]$ and the fact that the fact that $p \equiv [\lambda y\, p]x$ is provably necessary.

Proof of a Lemma

- $\vdash w \models (p \vee q) \equiv (w \models p \vee w \models q)$
- *Proof*. ($\rightarrow$) Suppose $w \models (p \vee q)$. Suppose, for reductio, that $w \not\models p \;\&\; w \not\models q$. Then, by the maximality of possible worlds, we know both $w \models \neg p$ and $w \models \neg q$. Now by propositional logic, we know that $(p \vee q)$, $\neg p$, and $\neg q$ jointly imply a contradiction, say $r \;\&\; \neg r$. But since worlds are modally closed, we know that any proposition necessarily implied by propositions true at w is also true at w. Since we already have the facts that $w \models (p \vee q)$, $w \models \neg p$, and $w \models \neg q$, it follows from the fact that worlds are modally closed that $w \models (r \;\&\; \neg r)$, which contradicts the fact that worlds are possible. ($\leftarrow$) Exercise.

Realization I

- Let u, v be restricted to discernible individuals
- $RealizesAt(u, c, w) \; =_{df} \; \forall F(w \models Fu \equiv cF)$
- Facts about realization:
- $\exists u \exists w(RealizesAt(u, c, w) \; \& \; RealizesAt(u, d, w)) \rightarrow c = d$
- *Proof*. Assume the antecedent, and let a and w_1 be witnesses: $RealizesAt(a, c, w_1)$ and $RealizesAt(a, d, w_1)$. Then by the definition of realization, we know both $\forall F(w_1 \models Fa \equiv cF)$ and $\forall F(w_1 \models Fa \equiv dF)$. So, by the laws of quantified biconditionals, it follows that $\forall F(cF \equiv dF)$. Since c and d are concepts, they are abstract. So, by the definition of identify, $c = d$.

Realization II

- Fact: $\exists c \exists w(RealizesAt(u,c,w) \,\&\, RealizesAt(v,c,w)) \rightarrow u = v$
- *Proof*. Assume the antecedent and let c_1 and w_1 be witnesses: $RealizesAt(u,c_1,w_1)$ and $RealizesAt(v,c_1,w_1)$, where c_1 is an arbitrary concept and w_1 an arbitrary world (to show $u = v$). Then, by the definition of realization, we know both $\forall F((w_1 \models Fu) \equiv c_1 F)$ and $\forall F((w_1 \models Fv) \equiv c_1 F)$. So by the laws of quantified biconditionals, we know: $\forall F((w_1 \models Fu) \equiv (w_1 \models Fv))$. [At this point, there are multiple ways to go: (a) instantiate $\forall F$ to $[\lambda y\, u =_D y]$ and show that $w_1 \models u =_D v$, or (b) instantiate $\forall F$ to an arbitrary property and infer $w_1 \models (Fu \equiv Fv)$ from $w_1 \models Fu \equiv w_1 \models Fv$, and then show $w_1 \models u =_D v$.] Either way, $\exists w(w \models u =_D v)$. So $\Diamond u =_D v$, and hence $u =_D v$ and thus $u = v$.

Realization III

- Fact: $\exists u \exists c(RealizesAt(u,c,w) \;\&\; RealizesAt(u,c,w')) \rightarrow w = w'$
- *Proof*. For witnesses a and c_1, assume $RealizesAt(a,c_1,w)$ and $RealizesAt(a,c_1,w')$. So we know, by the definition of realization, that $\forall F((w \models Fa) \equiv c_1F)$ and $\forall F((w' \models Fa) \equiv c_1F)$. So, by the laws of quantified biconditionals:
  - $\forall F((w \models Fa) \equiv (w' \models Fa))$. (ϑ)

  Suppose, for reductio, that $w \neq w'$. Then, since w, w' are worlds, and hence situations, we know from theorems of situation theory that there must be a proposition true at one and not the other. Without loss of generality, assume $w \models q$ and $\neg(w' \models q)$, where q is arbitrary. From the former, it follows by a Lemma about worlds that $w \models [\lambda y\, q]a$. So, in light of (ϑ), it follows that $w' \models [\lambda y\, q]a$. So again by our Lemma about worlds, $w' \models q$. Contradiction.

Appearance and Mirroring

- $AppearsAt(c, w) \;=_{df}\; \exists u RealizesAt(u, c, w)$
- $AppearsAt(c, w) \rightarrow \exists! u(RealizesAt(u, c, w))$
- *Proof*. Assume $AppearsAt(c, w)$. By the definition of appearance, it follows that for some discernible individual, say b, that $RealizesAt(b, c, w)$. To show uniqueness, assume $RealizesAt(a, c, w)$, where a is discernible (to show $a = b$). But this follows immediately by a fact about realization.
- $Mirrors(c, w) \;=_{df}\; \forall p(c\Sigma p \;\equiv\; w\Sigma p)$
- $AppearsAt(c, w) \rightarrow Mirrors(c, w)$
- *Proof*. Suppose $AppearsAt(c, w)$. So c is realized by some discernible object, say b, at w; i.e., $\forall F((w \models Fb) \equiv cF)$. By definition of $\models$, this is just:

  $$\forall F((w\Sigma Fb) \equiv cF) \qquad (\vartheta)$$

  We want to show, for an arbitrary proposition q, that $c\Sigma q \equiv w\Sigma q$. ($\rightarrow$) Assume $c\Sigma q$, i.e., $c[\lambda y\, q]$. So, by ϑ, $w\Sigma[\lambda y\, q]b$, i.e., $w \models [\lambda y\, q]b$. And by our Lemma about worlds, it follows that $w \models q$, i.e., $w\Sigma q$. ($\leftarrow$) Reverse the reasoning.

Another Fact About Realization

- $\exists c(RealizesAt(u,c,w)\ \&\ RealizesAt(v,c,w')) \rightarrow (w=w'\ \&\ u=v)$
- *Proof*. Assume $RealizesAt(u,c_1,w)$ and $RealizesAt(v,c_1,w')$. We show: (1) $w=w'$, and then (2) $u=v$. (1) From our two assumptions and the definition of appearance, we know that $AppearsAt(c_1,w)$ and $AppearsAt(c_1,w')$. So, by the previous theorem, it follows both that $Mirrors(c_1,w)$ and $Mirrors(c_1,w')$. We may infer from these, by the definition of mirroring, that $\forall p(c_1\Sigma p \equiv w\Sigma p)$ and $\forall p(c_1\Sigma p] \equiv w'\Sigma p)$. By the definition of Σ, we therefore know $\forall p(c_1[\lambda y\, p] \equiv w[\lambda y\, p])$ and $\forall p(c_1[\lambda y\, p] \equiv w'[\lambda y\, p])$. So by the laws of quantified biconditionals, we know $\forall p(w[\lambda y\, p] \equiv w'[\lambda y\, p])$, i.e., $\forall p((w \models p) \equiv (w' \models p))$. But since w and w' are both worlds, and hence situations, it follows by a fact about the identity of situations, that $w=w'$. (2) From (1) and the second of our hypotheses, it follows that $RealizesAt(v,c_1,w)$. From this, and the first of our hypotheses, it follows by a fact about realization that $u=v$.

Fact About Appearance

- $\exists c(AppearsAt(c,w) \,\&\, AppearsAt(c,w')) \rightarrow w\!=\!w'$
- *Proof*. Assume $AppearsAt(c_1,w)$ and $AppearsAt(c_1,w')$. Then, by the definition of appearance, we know that there is some discernible individual, say b, such that $RealizesAt(a,c_1,w)$, and some discernible individual, say b, such that $RealizesAt(b,c_1,w')$. So by the last fact about realization, it follows that $w\!=\!w'$.

Individual Concepts

- $IndividualConcept(c) \equiv_{df} \exists w AppearsAt(c, w)$
- $IndividualConcept(\boldsymbol{c}_u)$
- *Proof.* (There is a ⋆-proof from ⋆-facts $\boldsymbol{w}_\alpha \models p \equiv p$ (Lecture 5) and $\boldsymbol{c}_u G \equiv Gu$ (Lecture 6).) Instead: use $(\boldsymbol{w}_\alpha \models p) \equiv \mathcal{A}p$ (exercise). It suffices to show $AppearsAt(\boldsymbol{c}_u, \boldsymbol{w}_\alpha)$. Let P be an arbitrary property. By definition of $\boldsymbol{c}_u$ and strict Abstraction, we know $\boldsymbol{c}_u P \equiv \mathcal{A}Pu$. And by our exercise, $(\boldsymbol{w}_\alpha \models Pu) \equiv \mathcal{A}Pu$. Hence, $(\boldsymbol{w}_\alpha \models Pu) \equiv \boldsymbol{c}_u P$. Since P was arbitrary, $RealizesAt(u, \boldsymbol{c}_u, \boldsymbol{w}_\alpha) \therefore \exists u RealizesAt(u, \boldsymbol{c}_u, \boldsymbol{w}_\alpha) \therefore AppearsAt(\boldsymbol{c}_u, \boldsymbol{w}_\alpha)$ $\therefore \exists w AppearsAt(\boldsymbol{c}_u, w)$.
- Let $\hat{c}, \hat{d}, \hat{e}, \ldots$ range over individual concepts
- $\exists! w AppearsAt(\hat{c}, w)$
- *Proof.* By definition of $\hat{c}$, $\exists w AppearsAt(\hat{c}, w)$, say w_1. For reductio, suppose $AppearsAt(\hat{c}, w_2)$, where $w_2 \neq w_1$. Without loss of generality, assume that $w_1 \models p$ and $w_2 \not\models p$. By maximality, $w_2 \models \neg p$. But, by a previous theorem, $\hat{c}$ mirrors w_1, since it appears there. So since $w_1 \models p$ (i.e., $w_1 \Sigma p$), we know $\hat{c}\Sigma p$. But $\hat{c}$ also mirrors w_2, since it appears there as well. So, from our last fact, $w_2 \Sigma p$, i.e., $w_2 \models p$, contradicting the consistency of w_2.
- $\boldsymbol{w}_{\hat{c}} =_{df} \imath w AppearsAt(\hat{c}, w)$

Completeness

- $Complete(c) =_{df} \forall F(cF \vee c\overline{F})$ (where $\overline{F} = [\lambda x\, \neg Fx]$)
- $Complete(\hat{c})$
- *Proof*. By definition, $\hat{c}$ appears at some world and so is realized by some discernible object, say b, at some world, say w_1. Consider an arbitrary property P. By logic alone, $\Box(Pb \vee \neg Pb)$. Furthermore, the following is a consequence of λ-conversion: $\Box(\overline{P}b \equiv \neg Pb)$. Now by modal propositional logic, it follows that $\Box(Pb \vee \overline{P}b)$. So by a fundamental theorem of world theory, $w_1 \models (Pb \vee \overline{P}b)$. By a previous Lemma, it follows that $w_1 \models Pb \vee w_1 \models \overline{P}b$. But since $\hat{c}$ is realized by b at w_1, we know that $\forall F(\hat{c}F \equiv w_1 \models Fb)$, from which it follows that $\hat{c}P \vee \hat{c}\overline{P}$. Since P was arbitrary, $\hat{c}$ is complete.

Compossibility

- The intuitions sketched out diagrammatically.
- $Compossible(\hat{c}, \hat{e}) \ =_{df} \ \exists w(AppearsAt(\hat{c}, w) \ \& \ AppearsAt(\hat{e}, w))$
- $Compossible(\hat{c}, \hat{e}) \equiv \boldsymbol{w}_{\hat{c}} = \boldsymbol{w}_{\hat{e}}$ (Lemma)
- *Proof*. Assume $Compossible(\hat{c}, \hat{e})$. Then by definition, there is a world, say w_1, where they both appear. But then, since every individual concept appears at a unique world, and *the* world where an individual concepts appears is well-defined, it follows that $w_1 = w_{\hat{c}}$ and $w_1 = w_{\hat{e}}$. So $w_{\hat{c}} = w_{\hat{e}}$. ($\leftarrow$) Clearly, if $w_{\hat{c}} = w_{\hat{e}}$, there is a world where they both appear.
- Compossibility is an equivalence condition on individual concepts:
  - $Compossible(\hat{c}, \hat{c})$
  - $Compossible(\hat{c}, \hat{e}) \rightarrow Compossible(\hat{e}, \hat{c})$
  - $Compossible(\hat{c}, \hat{e}) \ \& \ Compossible(\hat{e}, \hat{d}) \rightarrow Compossible(\hat{c}, \hat{d})$

Compossibility is an Equivalence Condition

- *Proof of Reflexivity*: Let $\hat{c}$ be any individual concept. Then, by definition, $\exists w\, Appear(\hat{c}, w)$. So $\exists w(Appear(\hat{c}, w)\ \&\ Appear(\hat{c}, w))$. So, by definition, $Compossible(\hat{c}, \hat{c})$.
- *Proof of Symmetry*: Suppose $Compossible(\hat{c}, \hat{e})$. Then, by definition, $\exists w(Appear(\hat{c}, w)\ \&\ Appear(\hat{e}, w))$. So, by predicate logic and the laws of conjunction, $\exists w(Appear(\hat{e}, w)\ \&\ Appear(\hat{c}, w))$. So $Compossible(\hat{e}, \hat{c})$. ⋈
- *Proof of Transitivity*: Suppose $Compossible(\hat{c}, \hat{e})$ and $Compossible(\hat{e}, \hat{d})$. Then, by the compossibility Lemma, $w_{\hat{c}} = w_{\hat{e}}$ and $w_{\hat{e}} = w_{\hat{d}}$. So, by transitivity of identity, $w_{\hat{c}} = w_{\hat{d}}$. So, again by a previous theorem, $Compossible(\hat{c}, \hat{d})$.

Counterparts

- $CounterpartOf(\hat{e},\hat{c}) \;\; \equiv_{df}$
  $$\exists u \exists w \exists w' (RealizesAt(u,\hat{c},w) \,\&\, RealizesAt(u,\hat{e},w'))$$
- *CounterpartOf* is an equivalence condition on individual concepts.
- $CounterpartOf(\hat{c},\hat{c})$
- *Proof*. Suppose $IndividualConcept(\hat{c})$. Then by the definitions of individual concepts and appearance, we know there is an discernible individual, say b and a world, say w_1, such that $RealizesAt(b,\hat{c},w_1)$. So, conjoining this fact with itself, we have $RealizesAt(b,\hat{c},w_1) \,\&\, RealizesAt(b,\hat{c},w_1)$. By three applications of existential generalization, we have:
  $$\exists u,w,w' (RealizesAt(u,\hat{c},w) \,\&\, RealizesAt(u,\hat{c},w')).$$
  So, by the definition of counterparts, $CounterpartOf(\hat{c},\hat{c})$.

CounterpartOf is a Partition: Symmetry

- $CounterpartOf(\hat{e}, \hat{c}) \rightarrow CounterpartOf(\hat{c}, \hat{e})$
- *Proof*. Assume $CounterpartOf(\hat{e}, \hat{c})$. Then there is an discernible object, say b, and worlds w_1, w_2, such that $RealizesAt(b, \hat{e}, w_1)$ & $RealizesAt(b, \hat{c}, w_2)$. So, reversing the order of the conjuncts, we know:

  $$RealizesAt(b, \hat{c}, w_2) \ \& \ RealizesAt(b, \hat{e}, w_1)$$

  It follows therefore that:

  $$\exists u, w, w'(RealizesAt(u, \hat{c}, w) \ \& \ RealizesAt(u, \hat{e}, w'))$$

  So by the definition of counterparts, $CounterpartOf(\hat{c}, \hat{e})$.

CounterpartOf is a Partition: Transitivity

- $CounterpartsOf(\hat{e}, \hat{d})$ & $CounterpartOf(\hat{d}, \hat{c}) \rightarrow$
  $CounterpartOf(\hat{e}, \hat{c})$
- *Proof of Transitivity*. Assume $CounterpartOf(\hat{e}, \hat{d})$ and $CounterpartOf(\hat{d}, \hat{c})$. Then for some discernible objects a, b and worlds w_1, w_2, w_3, w_4, we know following facts:

  $$RealizesAt(a, \hat{e}, w_1) \; \& \; RealizesAt(a, \hat{d}, w_2) \qquad (\vartheta)$$

  $$RealizesAt(b, \hat{d}, w_3) \; \& \; RealizesAt(b, \hat{c}, w_4) \qquad (\xi)$$

  From the 2nd conjunct of (ϑ) and the 1st conjunct of (ξ), we may apply a Another Fact About Realization, to conclude: $w_2 = w_3$ and $a = b$. So substituting b for a in the 1st conjunct of (ϑ), we may conjoin the result with the 2nd conjunct of (ξ) to obtain:

  $$RealizesAt(b, \hat{e}, w_1) \; \& \; RealizesAt(b, \hat{c}, w_4)$$

  It therefore follows that:

  $$\exists u, w, w'(RealizesAt(u, \hat{e}, w) \; \& \; RealizesAt(u, \hat{c}, w')),$$

  from which it follows that $CounterpartOf(\hat{e}, \hat{c})$, by definition.

Lemmas concerning Counterparts

- $\vdash \textit{CounterpartOf}(\hat{e},\hat{c}) \equiv$
$\exists! u \exists w \exists w'(\textit{RealizesAt}(u,\hat{c},w) \;\&\; \textit{RealizesAt}(u,\hat{e},w'))$
- *Proof*. Assume $\textit{CounterpartOf}(\hat{e},\hat{c})$. Then by the definition of counterparts, there is an discernible object, say a, and worlds, say w_1 and w_2, such that:

$$\textit{RealizesAt}(a,\hat{e},w_1) \;\&\; \textit{RealizesAt}(a,\hat{c},w_2)$$

To prove uniqueness, assume for an arbitrary discernible object b, $\textit{RealizesAt}(b,\hat{e},w_1)$ and $\textit{RealizesAt}(b,\hat{c},w_2)$ (to show $b\!=\!a$). But since we have both $\textit{Realizes}(a,\hat{e},w_1)$ and $\textit{RealizesAt}(b,\hat{e},w_1)$, it follows that $b\!=\!d$, by a Fact about Realization.

The Concept of u at World w

- $ConceptOfAt(c,u,w) \equiv_{df} \forall F(cF \equiv w \models Fu)$
- The concept of u at w (‘$\boldsymbol{c}_u^w$’) $=_{df}$ $\imath c ConceptOfAt(c,u,w)$
- Lemmas:
  - $\boldsymbol{c}_u^w G \equiv w \models Gu$
  - $RealizesAt(u, \boldsymbol{c}_u^w, w)$
  - $AppearsAt(\boldsymbol{c}_u^w, w)$
  - $IndividualConcept(\boldsymbol{c}_u^w)$
  - $Mirrors(\boldsymbol{c}_u^w, w)$
  - $\boldsymbol{c}_u^{\boldsymbol{w}_\alpha} = \boldsymbol{c}_u$
  - $\boldsymbol{c}_u^w G \equiv \boldsymbol{c}_u^w \succeq \boldsymbol{c}_G$
  - $\boldsymbol{c}_u^w = \boldsymbol{c}_v^w \rightarrow u = v$
  - $\boldsymbol{c}_u^w = \boldsymbol{c}_u^{w'} \rightarrow w = w'$
  - $\boldsymbol{c}_u^w = \boldsymbol{c}_v^{w'} \rightarrow (w = w' \;\&\; u = v)$
  - $Compossible(\boldsymbol{c}_u^w, \boldsymbol{c}_v^w)$
  - $CounterpartOf(\boldsymbol{c}_u^w, \boldsymbol{c}_u^{w'})$

The Fundamental Theorems

- Fundamental Theorem 1 (instance): If Alexander is a king but might not have been, then both (1) the individual concept of Alexander contains the concept king, and (2) there is a (complete) individual concept that is a counterpart of the concept of Alexander that doesn't contain the concept king and which appears at some other possible world.
- $\star\vdash (Fu \;\&\; \Diamond\neg Fu) \rightarrow [\boldsymbol{c}_u \succeq \boldsymbol{c}_F \;\&\; \exists\hat{c}(CounterpartOf(\hat{c}, \boldsymbol{c}_u) \;\&\; \hat{c} \not\succeq \boldsymbol{c}_F \;\&\; \exists w(w \neq \boldsymbol{w}_\alpha \;\&\; AppearsAt(\hat{c}, w)))]$
- Fundamental Theorem 2 (instance): If Alexander isn't a philosopher but might have been, then both (1) the individual concept of Alexander doesn't contain the concept philosopher, and (2) there is a (complete) individual concept that is a counterpart of the concept of Alexander that does contain the concept philosopher and which appears at some other possible world.
- $\star\vdash (\neg Fu \;\&\; \Diamond Fu) \rightarrow [\boldsymbol{c}_u \not\succeq \boldsymbol{c}_F \;\&\; \exists\hat{c}(CounterpartOf(\hat{c}, \boldsymbol{c}_u) \;\&\; \hat{c} \succeq \boldsymbol{c}_F \;\&\; \exists w(w \neq \boldsymbol{w}_\alpha \;\&\; AppearsAt(\hat{c}, w)))]$

Proof of Fundamental Theorem 1

- Assume $Fu \mathbin{\&} \Diamond\neg Fu$, to show:
  a. $\boldsymbol{c}_u \succeq \boldsymbol{c}_F$
  b. $\exists\hat{c}(CounterpartOf(\hat{c}, \boldsymbol{c}_u) \mathbin{\&} \hat{c} \not\succeq \boldsymbol{c}_F \mathbin{\&}$
  $\exists w(w \neq \boldsymbol{w}_\alpha \mathbin{\&} AppearsAt(\hat{c}, w)))$
- $\boldsymbol{c}_u \succeq \boldsymbol{c}_F$ By 1st conjunct and a prior theorem.
- $\exists w(w \models \neg Fu)$ By 2nd conjunct and world theory.
- Suppose $w_1 \models \neg Fu$ (w_1 arbitrary)
- Consider $\boldsymbol{c}_u^{w_1}$.
- $IndividualConcept(\boldsymbol{c}_u^{w_1})$ (and so $Complete(\boldsymbol{c}_a^{w_1})$) (theorems)
- $CounterpartOf(\boldsymbol{c}_u^{w_1}, \boldsymbol{c}_u)$, since $\boldsymbol{c}_u = \boldsymbol{c}_u^{\boldsymbol{w}_\alpha}$ and $CounterpartOf(\boldsymbol{c}_u^{w_1}, \boldsymbol{c}_u^{\boldsymbol{w}_\alpha})$
- $\boldsymbol{c}_u^{w_1} \not\succeq \boldsymbol{c}_F$: $w_1 \not\models Fu$, so $\neg\boldsymbol{c}_u^{w_1}F$, by a previous lemma. But $c_F F$, so c_F encodes F and $\boldsymbol{c}_u^{w_1}$ doesn't.
- $w_1 \neq \boldsymbol{w}_\alpha$: Fu implies $\boldsymbol{w}_\alpha \models Fu$ ($\star$-theorem); $w_1 \not\models Fu$ (assumed)
- $AppearsAt(\boldsymbol{c}_u^{w_1}, w_1)$: (instance of a previous lemma).

Observations

- We've captured the distinguishing features of Leibniz's metaphysics of individual concepts.
- We can get modally strict theorems by adding actuality operator.
- The metaphysics has a Kripkean and a Lewisian component. Kripkean: an object has properties at all worlds; Lewisian: counterpart theory explains truth conditions of modal claims (though, for us, it relates concepts and is an equivalence condition).
- If monads are complete individual concepts, then an ambiguity in predication explains why L thinks they 'are alive', etc.

Bibliography

- Alama J., P. Oppenheimer, and E. Zalta, 2015, ‘Automating Leibniz’s Theory of Concepts’, in A. Felty and A. Middeldorp (eds.), *Automated Deduction – CADE 25: Proceedings of the 25th International Conference on Automated Deduction* (Lecture Notes in Artificial Intelligence: Volume 9195), Berlin: Springer, pp. 73–97. https://doi.org/10.1007/978-3-319-21401-6_4
- Fitch, G., 1979, ‘Analyticity and Necessity in Leibniz’, *Journal of the History of Philosophy*, 17(1): 29-42.
- Gerhardt, C. I. (ed.), *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*, Volumes i – vii, Berlin, 1875–90.
- Kripke, S., 1963, ‘Semantical Considerations on Modal Logic’, *Acta Philosophica Fennica*, 16: 83–94.
- Leibniz, G.W.F., 1690, ‘A Study in the Calculus of Real Addition’, in G. H. R. Parkinson (ed. and trans.), *Leibniz: Logical Papers*, Oxford: Clarendon, 1966, 131–144.
- Leibniz, G.W.F., [**PW**] *Philosophical Writings*, G. H. R. Parkinson (ed.), M. Morris and G. Parkinson (trans.), London: Dent & Sons, 1973.
- Leibniz, G.W.F., 1714, *Theodicy*, translated by E. Huggard, New Haven: Yale University Press, 1952.
- Lewis, D., 1986, *On the Plurality of Worlds*, Oxford: Blackwell.

Bibliography

- Lloyd, G., 1978, 'Leibniz on Possible Individuals and Possible Worlds', *Australasian Journal of Philosophy*, 56(2): 126-142.
- Mondadori, F., 1973, 'Reference, Essentialism, and Modality in Leibniz's Metaphysics', *Studia Leibnitiana*, 5(1): 74-101.
- Vailati, E., 1986, 'Leibniz on Necessary and Contingent Predication', *Studia Leibnitiana*, 18(2): 195-210.
- Wilson, M., 1979, 'Possible Gods', *Review of Metaphysics*, 32(4): 717-733.
- Zalta, E., 1993, 'Twenty-Five Basic Theorems in Situation and World Theory', *Journal of Philosophical Logic*, 22: 385–428.
- Zalta, E., 2000, 'A (Leibnizian) Theory of Concepts', *Philosophiegeschichte und logische Analyse / Logical Analysis and History of Philosophy*, 3: 137–183.
- Zalta, E., m.s., *Principia Logico-Metaphysica*, https://mally.stanford.edu/principia.pdf